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Title: The mixed Ax-Lindemann theorem and its applications to the Zilber-Pink conjecture

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Abstract

The Zilber-Pink conjecture is a diophantine conjecture concerning unlikely intersections in mixed Shimura varieties. It is a common generalization of the André-Oort conjecture and the Mordell-Lang conjecture. This dissertation is aimed to study the Zilber-Pink conjecture. More concretely, we will study the André-Oort conjecture, which predicts that a subvariety of a mixed Shimura variety having dense intersection with the set of special points is special, and the André-Pink-Zannier conjecture which predicts that a subvariety of a mixed Shimura variety having dense intersection with a generalized Hecke orbit is weakly special. The latter conjecture generalizes the Mordell-Lang conjecture as explained by Pink.

In the Pila-Zannier method, a key point to study the Zilber-Pink conjecture is to prove the Ax-Lindemann theorem, which is a generalization of the functional analogue of the classical Lindemann-Weierstrass theorem. One of the main results of this dissertation is to prove the Ax-Lindemann theorem in its most general form, i.e. the mixed Ax-Lindemann theorem. This generalizes results of Pila, Pila-Tsimerman, Ullmo-Yafaev and Klingler-Ullmo-Yafaev concerning the Ax-Lindemann theorem for pure Shimura varieties.

Another main result of this dissertation is to prove the André-Oort conjecture for a large class of mixed Shimura varieties: unconditionally for any mixed Shimura variety whose pure part is a subvariety of \mathcal{A}_6^N (e.g. products of universal families of abelian varieties of dimension 6 and the Poincaré bundle over \mathcal{A}_6) and under GRH for all mixed Shimura varieties of abelian type. This generalizes existing theorems of Klingler-Ullmo-Yafaev, Pila, Pila-Tsimerman and Ullmo concerning pure Shimura varieties.

As for the André-Pink-Zannier conjecture, we prove several cases when the ambient mixed Shimura variety is the universal family of abelian varieties. First we prove the overlap of André-Oort and André-Pink-Zannier, i.e. we study the generalized Hecke orbit of a special point. This generalizes results of Edixhoven-Yafaev and Klingler-Ullmo-Yafaev for \mathcal{A}_g . Secondly we prove the conjecture in the following case: a subvariety of an abelian scheme over a curve is weakly special if its intersection with the generalized Hecke orbit of a torsion point of a non CM fiber is Zariski dense. Finally for the generalized Hecke orbit of an arbitrary $\overline{\mathbb{Q}}$ -point, we prove the conjecture for curves. These generalize existing results of Habegger-Pila and Orr for \mathcal{A}_g .

In all these proofs, the o-minimal theory, in particular the Pila-Wilkie counting theorems, plays an important role.