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Universiteit Leiden



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ON SOME CLASSES OF MODULES
AND THEIR ENDOMORPHISM RINGS

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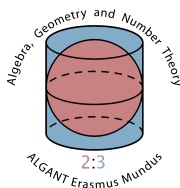
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To my parents and my wife

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Partial list of notations:

\mathbb{N}	the set of non-negative integers.
\mathbb{Z}	the ring of integers.
R	an associative ring with $1 \neq 0$.
$Z(R)$	$= \{x \in R \mid xy = yx \text{ for every } y \in R\}$, the center of R .
$J(R)$	the Jacobson radical of a ring R .
M_R	a right module over a ring R .
${}_R M$	a left module over a ring R .
$E(M_R)$	the (an) injective envelope of M_R .
$M_R^{(I)}$	the direct sum $\bigoplus_{i \in I} M_i$ with $M_i \cong M_R$ for every $i \in I$.
$\text{rad}(M_R)$	the radical of M_R .
$\text{soc}(M_R)$	the socle of M_R .
$\text{End}(M_R)$	the endomorphism ring of M_R .
$\text{End}({}_R M)$	the endomorphism ring of ${}_R M$.
$\text{Hom}(N_R, M_R)$	the group of all module homomorphisms of N_R to M_R .
$M \leq N$	M is a submodule of N .
$M < N$	M is a proper submodule of N .
$I \subseteq J$	I is a subset of J .
$I \subset J$	I is a proper subset of J .
$M_n(R)$	the ring of $n \times n$ matrices over a ring R .
$f _A$	the restriction of a mapping $f : B \rightarrow C$ to a subset A of B .
1_A or Id_A	the identity mapping $A \rightarrow A$, where A is a set.
$(A : B)_R$	$= \{r \in R \mid Br \leq A\}$, where A, B are submodules of module M_R .
$\text{ann}_R(A)$	$= (0 : A)_R$, where A is a submodule of module M_R .
$\text{ann}_R(x)$	$= (0 : \{x\})_R$, where x is an element of module M_R .

