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ON SOME CLASSES OF MODULES AND THEIR ENDOMORPHISM RINGS

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door

Mai Hoang Bien
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in 1982

Samenstelling van de promotiecommissie:

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To my parents and my wife

Contents

Partial list of notations:

$\mathbb{N}$	the set of non-negative integers.
$\mathbb{Z}$	the ring of integers.
R	an associative ring with $1 \neq 0$.
$Z(R)$	$= \{ x \in R \mid xy = yx$ for every $y \in R\}$, the center of R.
$J(R)$	the Jacobson radical of a ring R.
M_R	a right module over a ring R.
$_RM$	a left module over a ring R.
$E(M_R)$	the (an) injective envelope of M_R.
$M_R^{(I)}$	the direct sum $\bigoplus_{i\in I} M_i$ with $M_i \cong M_R$ for every $i \in I$.
$\mathrm{rad}(M_R)$	the radical of M_R.
$\mathrm{soc}(M_R)$	the socle of M_R.
$\mathrm{End}(M_R)$	the endomorphism ring of M_R.
$\mathrm{End}(_RM)$	the endomorphism ring of $_RM$.
$\mathrm{Hom}(N_R, M_R)$	the group of all module homomorphisms of N_R to M_R.
$M \leq N$	M is a submodule of N.
$M < N$	M is a proper submodule of N.
$I \subseteq J$	I is a subset of J.
$I \subset J$	I is a proper subset of J.
$M_n(R)$	the ring of $n \times n$ matrices over a ring R.
$f\|_A$	the restriction of a mapping $f : B \to C$ to a subset A of B.
1_A or Id_A	the identity mapping $A \to A$, where A is a set.
$(A : B)_R$	$= \{ r \in R \mid Br \leq A \}$, where A, B are submodules of module M_R.
$\mathrm{ann}_R(A)$	$= (0 : A)_R$, where A is a submodule of module M_R.
$\mathrm{ann}_R(x)$	$= (0 : \{x\})_R$, where x is an element of module M_R.