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Hopf knot curves space-time

The basic issue is this: one can extend the notion of electric and magnetic fields to quite arbitrary coordinates in arbitrary space times, and one can extend the notion of the Poynting vector with it, but one does not understand what these things mean physically. In other words, the extension is purely formal. This is a very general problem in G[eneral] R[elativity], . . . [91].

In the final chapter of this thesis we set ourselves the ambitious task of solving the full nonlinear Einstein equations with an electromagnetic Hopf knot as its source. In doing so we audaciously neglect what can be considered as a warning or request, not to come up with new exact solutions but instead trying to interpret the ones that have already been found. This warning, that can be sensed for example in [92], expresses the fact that it can be very hard to give an interpretation for exact solutions. However, when our attempt will be successful, we do have an understanding of the source.
We will use the Newman-Penrose formalism for which the preparations appear in the previous chapter, that will be summarized first.
It should be noted that in the process of solving the equations every now and then a choice has to be made. The examples presented here are just two of the few options that have been considered, some of which, like the one presented first, lead to a contradiction, whereas other lead to a set of equations that have not been solved. The list of equations of the form $D(...) = f(x^\mu)$ partly presented in chapter 9 could be of help in solving said equations. However, the complexity of the equations to be solved is daunting. The purpose of present example is to convey the method used, for which these choices are best suited.

10.1 introduction (summary of chapter 9)

In the previous chapter we have seen that an electromagnetic Hopf knot can be represented in the Newman-Penrose formalism in the following way.

In M^4 we use the tetrad

$$\mathbf{l} = (1, \frac{\mathbf{S}}{S}), \quad \mathbf{n} = \frac{1}{2}(1, -\frac{\mathbf{S}}{S}), \quad \mathbf{m} = \frac{1+i(t-z)}{\sqrt{2S(1+(t-z)^2)}}(0, \mathbf{F}), \quad \bar{\mathbf{m}} = \frac{1-i(t-z)}{\sqrt{2S(1+(t-z)^2)}}(0, \mathbf{F}^*),$$

with $\mathbf{F}$ and $\mathbf{S}$ as in chapter 4, equations (4.1) and (4.2).

The spin coefficients that correspond with this tetrad are:

$$\kappa = \sigma = \epsilon = \pi = \lambda = 0, \quad \tau = 2\nu^*, \quad \beta = -\alpha^*, \quad \mu = \frac{1}{2}\rho^*,$$

$$\rho = -2\frac{t-z+i}{1+x^2+y^2+(t-z)^2},$$

$$\nu = \frac{-(1-i(t-z))(x+iy)}{\sqrt{2S(1+(t-z)^2)((t+i)^2-r^2)^3}},$$

$$\alpha = [3\frac{(t+i)^2-r^2}{(t-i)^2-r^2} - \frac{t-z+i}{t-z-i}]\nu,$$

$$\gamma = \frac{2i}{1+x^2+y^2+(t-z)^2} - \frac{6i(t^2+r^2+1)}{(t^2-r^2)^2+1+2(t^2+r^2)} + \frac{i}{1+(t-z)^2}$$

in which $r^2 = x^2 + y^2 + z^2$.

The electromagnetic Hopf knot is given by:

$$\phi_0 = \phi_1 = 0, \quad \phi_2 = -(1 - i(t - z))\sqrt{\frac{S}{2(1+(t-z)^2)}}$$

which leads to the only nonzero component of the energy-momentum tensor: $T_{(2)(2)} = -S$.

10.2 how does an electromagnetic Hopf knot curve space-time?

Associated with an electromagnetic field is an energy-momentum distribution. When considering Einstein's equations, electromagnetic fields should therefore be included as a source. We now attempt to solve the combined Einstein-Maxwell equations in case the energy-momentum tensor of the electromagnetic Hopf knot is the only source for curvature. Due to the complicated functional relationship of the various physical quantities in terms of Minkowski coordinates, it seems appropriate to use the Newman-Penrose formalism in which we can take advantage of the incorporation of these complicated structures in the tetrad vector fields, thereby simplifying the equations to be solved.

The combined system of Einstein and Maxwell equations in this formalism consists of 64 (real) equations [12] that contain the tetrad as derivative operators, $D, \Delta, \delta, \delta^*$, the spin coefficients, $\kappa, \sigma, ...$ and the tetrad representatives of the Weyl tensor, $\Psi_0, ..., \Psi_4$, the Ricci tensor, $\Phi_{00}, ..., \Phi_{22}, \Lambda$, and the Faraday tensor, ϕ_1, ϕ_2, ϕ_3.

If we would use the quantities that define an electromagnetic Hopf knot in M^4, given in the introduction, in the 64 equations, we would get back flat space-time. Of course, this is no surprise, since it has already been build in. This relates to the fact that, in general relativity, in order to describe the sources in detail we do need the metric first. There is a circularity, since the metric depends on the details of the sources. Apart from linearizing the theory, few circumventions to this circularity exists:
1. We could try to find an exact solution mathematically, without consideration to the physical sources. When the solution has been found we try to interpret the sources.
2. We could give up a detailed description of the source, and only demand for example axial symmetry. The more symmetry we demand, the less likely we find a solution.
3. In the present context, we should "declare" what really is essential to the source, and only demand those properties for the source. This then *defines* a generalization of a "Minkowski electromagnetic Hopf knot" into the realm of general relativity.

It can (and will) be argued that $\kappa = \sigma = 0$ is essential to a Hopf knot. We surely want this to be true when commencing our task of solving the 64 equations simultaneously. But what about the fact that $\gamma \in \mathbb{I}$ or $\beta = -\alpha^*$? What do they mean? Notice that in the equations (see below) the combination $\gamma + \gamma^*$ as well as $\alpha + \beta^*$ appears, often in complicated equations. Demanding these combinations to vanish would certainly help to reduce the difficulty of our task. However, it is very well possible that with these extra restrictions there will be no solution, other than the one already found: flat space-time. The more restrictions we impose, the less likely it becomes that we find a new solution. On the other hand, with very few restrictions it becomes almost impossible to solve the equations simultaneously. This might result in a delicate balance between too many and too few requirements in the definition for the knot.

10.2.1 the strategy

We must start with what we consider to be essential, try to solve as many equations as possible, and in the process decide wether we impose additional restrictions in order to relieve the difficulty of the equations encountered. We possibly then find our previous result (flat space-time) or some contradiction. In that case we know that we imposed the wrong extra condition. We return to that point, alter the restrictions and try again. If there is no progress after many repetitions, we come to a point of finally changing the assumptions in the definition of the source and start the whole process over again. There could of course be multiple repetitions of this also. It seems reasonable to assume that a non trivial solution will in the end be found. It is however uncertain how much of what we hoped to include in the definition of an electromagnetic Hopf knot in the context of curved space-time is also possible. It can only be hoped that when the sought after solution has been obtained, enough reason is left for justifying the said knowledge of the source.

We first list the equations to be solved (**10.3**), mention some equations that can be used advantageously (**10.4**), state what is to be considered essential to an electromagnetic Hopf knot (**10.5**) and then show one attempt to solve the equations in detail (**10.6.1**) and a summary of another (**10.6.2**).

10.3 the equations to be solved

From the Ricci identities (36 real or 18 complex equations):

$$D\rho - \delta^*\kappa = (\rho^2 + \sigma\sigma^*) + \rho(\epsilon + \epsilon^*) - \kappa^*\tau - \kappa(3\alpha + \beta^* - \pi) + \Phi_{00} \tag{10.1}$$

$$D\sigma - \delta\kappa = \sigma(\rho + \rho^* + 3\epsilon - \epsilon^*) - \kappa(\tau - \pi^* + \alpha^* + 3\beta) + \Psi_0 \tag{10.2}$$

$$D\tau - \Delta\kappa = \rho(\tau + \pi^*) + \sigma(\tau^* + \pi) + \tau(\epsilon - \epsilon^*) - \kappa(3\gamma + \gamma^*) + \Psi_1 + \Phi_{01} \tag{10.3}$$

$$D\alpha - \delta^*\epsilon = \alpha(\rho + \epsilon^* - 2\epsilon) + \beta\sigma^* - \beta^*\epsilon - \kappa\lambda - \kappa^*\gamma + \pi(\epsilon + \rho) + \Phi_{10} \tag{10.4}$$

$$D\beta - \delta\epsilon = \sigma(\alpha + \pi) + \beta(\rho^* - \epsilon^*) - \kappa(\mu + \gamma) - \epsilon(\alpha^* - \pi^*) + \Psi_1 \tag{10.5}$$

$$D\gamma - \Delta\epsilon = \alpha(\tau + \pi^*) + \beta(\tau * + \pi) - \gamma(\epsilon + \epsilon^*) - \epsilon(\gamma + \gamma^*) + \tau\pi - \nu\kappa + \Psi_2 + \Phi_{11} - \Lambda \tag{10.6}$$

$$D\lambda - \delta^*\pi = (\rho\lambda + \sigma^*\mu) + \pi(\pi + \alpha - \beta^*) - \nu\kappa^* - \lambda(3\epsilon - \epsilon^*) + \Phi_{20} \tag{10.7}$$

$$D\mu - \delta\pi = (\rho^*\mu + \sigma\lambda) + \pi(\pi^* - \alpha^* + \beta) - \mu(\epsilon + \epsilon^*) - \nu\kappa + \Psi_2 + 2\Lambda \tag{10.8}$$

$$D\nu - \Delta\pi = \mu(\pi + \tau^*) + \lambda(\pi^* + \tau) + \pi(\gamma - \gamma^*) - \nu(3\epsilon + \epsilon^*) + \Psi_3 + \Phi_{21} \tag{10.9}$$

$$\Delta\lambda - \delta^*\nu = -\lambda(\mu + \mu^* + 3\gamma - \gamma^*) + \nu(3\alpha + \beta^* + \pi - \tau^*) - \Psi_4 \tag{10.10}$$

$$\delta\rho - \delta^*\sigma = \rho(\alpha^* + \beta) - \sigma(3\alpha - \beta^*) + \tau(\rho - \rho^*) + \kappa(\mu - \mu^*) - \Psi_1 + \Phi_{01} \tag{10.11}$$

$$\delta\alpha - \delta^*\beta = (\mu\rho - \lambda\sigma) + \alpha\alpha^* + \beta\beta^* - 2\alpha\beta + \gamma(\rho - \rho^*) + \epsilon(\mu - \mu^*) - \Psi_2 + \Phi_{11} + \Lambda \tag{10.12}$$

$$\delta\lambda - \delta^*\mu = \nu(\rho - \rho^*) + \pi(\mu - \mu^*) + \mu(\alpha + \beta^*) + \lambda(\alpha^* - 3\beta) - \Psi_3 + \Phi_{21} \tag{10.13}$$

$$\delta\nu - \Delta\mu = (\mu^2 + \lambda\lambda^*) + \mu(\gamma + \gamma^*) - \nu^*\pi + \nu(\tau - 3\beta - \alpha^*) + \Phi_{22} \tag{10.14}$$

$$\delta\gamma - \Delta\beta = \gamma(\tau - \alpha^* - \beta) + \mu\tau - \sigma\nu - \epsilon\nu^* - \beta(\gamma - \gamma^* - \mu) + \alpha\lambda^* + \Phi_{12} \tag{10.15}$$

$$\delta\tau - \Delta\sigma = (\mu\sigma + \lambda^*\rho) + \tau(\tau + \beta - \alpha^*) - \sigma(3\gamma - \gamma^*) - \kappa\nu^* + \Phi_{02} \tag{10.16}$$

$$\Delta\rho - \delta^*\tau = -(\rho\mu^* + \sigma\lambda) + \tau(\beta^* - \alpha - \tau^*) + \rho(\gamma + \gamma^*) + \nu\kappa - \Psi_2 - 2\Lambda \tag{10.17}$$

$$\Delta\alpha - \delta^*\gamma = \nu(\rho + \epsilon) - \lambda(\tau + \beta) + \alpha(\gamma^* - \mu^*) + \gamma(\beta^* - \tau^*) - \Psi_3 \tag{10.18}$$

From the Bianchi identities (16 real or 8 complex equations):

$$-\delta^*\Psi_0 + D\Psi_1 + (4\alpha - \pi)\Psi_0 - 2(2\rho + \epsilon)\Psi_1 + 3\kappa\Psi_2 + R_1 = 0 \tag{10.19}$$

in which
$R_1 = -D\Phi_{01} + \delta\Phi_{00} + 2(\epsilon + \rho^*)\Phi_{01} + 2\sigma\Phi_{10} - 2\kappa\Phi_{11} - \kappa^*\Phi_{02} + (\pi^* - 2\alpha^* - 2\beta)\Phi_{00}$

$$\delta^*\Psi_1 - D\Psi_2 - \lambda\Psi_0 + 2(\pi - \alpha)\Psi_1 + 3\rho\Psi_2 - 2\kappa\Psi_3 + R_2 = 0 \tag{10.20}$$

in which
$R_2 = \delta^*\Phi_{01} - \Delta\Phi_{00} - 2(\alpha + \tau^*)\Phi_{01} + 2\rho\Phi_{11} + \sigma^*\Phi_{02} - (\mu^* - 2\gamma - 2\gamma^*)\Phi_{00} - 2\tau\Phi_{10} - 2D\Lambda$

$$-\delta^*\Psi_2 + D\Psi_3 + 2\lambda\Psi_1 - 3\pi\Psi_2 + 2(\epsilon - \rho)\Psi_3 + \kappa\Psi_4 + R_3 = 0 \tag{10.21}$$

in which
$R_3 = -D\Phi_{21} + \delta\Phi_{20} + 2(\rho^* - \epsilon)\Phi_{21} - 2\mu\Phi_{10} + 2\pi\Phi_{11} - \kappa^*\Phi_{22} - (2\alpha^* - 2\beta - \pi^*)\Phi_{20} - 2\delta^*\Lambda$

$$\delta^*\Psi_3 - D\Psi_4 - 3\lambda\Psi_2 + 2(2\pi + \alpha)\Psi_3 - (4\epsilon - \rho)\Psi_4 + R_4 = 0 \tag{10.22}$$

in which
$R_4 = -\Delta\Phi_{20} + \delta^*\Phi_{21} + 2(\alpha - \tau^*)\Phi_{21} + 2\nu\Phi_{10} + \sigma^*\Phi_{22} - 2\lambda\Phi_{11} - (\mu^* + 2\gamma - 2\gamma^*)\Phi_{20}$

$$-\Delta\Psi_0 + \delta\Psi_1 + (4\gamma - \mu)\Psi_0 - 2(2\tau + \beta)\Psi_1 + 3\sigma\Psi_2 + R_5 = 0 \qquad (10.23)$$

in which
$R_5 = -D\Phi_{02} + \delta\Phi_{01} + 2(\pi^* - \beta)\Phi_{01} - 2\kappa\Phi_{12} - \lambda^*\Phi_{00} + 2\sigma\Phi_{11} + (\rho^* + 2\epsilon - 2\epsilon^*)\Phi_{02}$

$$-\Delta\Psi_1 + \delta\Psi_2 + \nu\Psi_0 + 2(\gamma - \mu)\Psi_1 - 3\tau\Psi_2 + 2\sigma\Psi_3 + R_6 = 0 \qquad (10.24)$$

in which
$R_6 = \Delta\Phi_{01} - \delta^*\Phi_{02} + 2(\mu^* - \gamma)\Phi_{01} - 2\rho\Phi_{12} - \nu^*\Phi_{00} + 2\tau\Phi_{11} + (\tau^* - 2\beta^* + 2\alpha)\Phi_{02} + 2\delta\Lambda$

$$-\Delta\Psi_2 + \delta\Psi_3 + 2\nu\Psi_1 - 3\mu\Psi_2 + 2(\beta - \tau)\Psi_3 + \sigma\Psi_4 + R_7 = 0 \qquad (10.25)$$

in which
$R_7 = -D\Phi_{22} + \delta\Phi_{21} + 2(\pi^* + \beta)\Phi_{21} - 2\mu\Phi_{11} - \lambda^*\Phi_{20} + 2\pi\Phi_{12} + (\rho^* - 2\epsilon - 2\epsilon^*)\Phi_{22} - 2\Delta\Lambda$

$$-\Delta\Psi_3 + \delta\Psi_4 + 3\nu\Psi_2 - 2(\gamma + 2\mu)\Psi_3 - (\tau - 4\beta)\Psi_4 + R_8 = 0 \qquad (10.26)$$

in which
$R_8 = \Delta\Phi_{21} - \delta^*\Phi_{22} + 2(\mu^* + \gamma)\Phi_{21} - 2\nu\Phi_{11} - \nu^*\Phi_{20} + 2\lambda\Phi_{12} + (\tau^* - 2\alpha - 2\beta^*)\Phi_{22}$

and further, also from the Bianchi identities (2 real and 1 complex equations):

$\delta^*\Phi_{01} + \delta\Phi_{10} - D(\Phi_{11} + 3\Lambda) - \Delta\Phi_{00} =$
$\kappa^*\Phi_{12} + \kappa\Phi_{21} + (2\alpha + 2\tau^* - \pi)\Phi_{01} + (2\alpha^* + 2\tau - \pi^*)\Phi_{10} - 2(\rho + \rho^*)\Phi_{11} - \sigma^*\Phi_{02} - \sigma\Phi_{20} + (\mu + \mu^* - 2(\gamma + \gamma^*))\Phi_{00}$

$\delta^*\Phi_{12} + \delta\Phi_{21} - \Delta(\Phi_{11} + 3\Lambda) - D\Phi_{22} =$
$-\nu\Phi_{01} - \nu^*\Phi_{10} + (\tau^* - 2\beta^* - 2\pi)\Phi_{12} + (\tau - 2\beta - 2\pi^*)\Phi_{21} + 2(\mu + \mu^*)\Phi_{11} - (\rho + \rho^* - 2\epsilon - 2\epsilon^*)\Phi_{22} + \lambda\Phi_{02} + \lambda^*\Phi_{20}$

$\delta(\Phi_{11} - 3\Lambda) - D\Phi_{12} - \Delta\Phi_{01} + \delta^*\Phi_{02} =$
$\kappa\Phi_{22} - \nu^*\Phi_{00} + (\tau^* - \pi + 2\alpha - 2\beta^*)\Phi_{02} - \sigma\Phi_{21} + \lambda^*\Phi_{10} + 2(\tau - \pi^*)\Phi_{11} - (2\rho + \rho^* - 2\epsilon^*)\Phi_{12} + (2\mu^* + \mu - 2\gamma)\Phi_{01}$

And finally, Maxwell's equations:

$$D\phi_1 - \delta^*\phi_0 = (\pi - 2\alpha)\phi_0 + 2\rho\phi_1 - \kappa\phi_2 \qquad (10.27)$$

$$D\phi_2 - \delta^*\phi_1 = -\lambda\phi_0 + 2\pi\phi_1 + (\rho - 2\epsilon)\phi_2 \qquad (10.28)$$

$$\delta\phi_1 - \Delta\phi_0 = (\mu - 2\gamma)\phi_0 + 2\tau\phi_1 - \sigma\phi_2 \qquad (10.29)$$

$$\delta\phi_2 - \Delta\phi_1 = -\nu\phi_0 + 2\mu\phi_1 + (\tau - 2\beta)\phi_2 \qquad (10.30)$$

10.4 useful equations

When solving the combined system of equations, use can be made of the commutation relations, which are easy to obtain from the definitions of the spin coefficients:

$$\Delta D - D\Delta = (\gamma + \gamma^*)D + (\epsilon + \epsilon^*)\Delta - (\tau^* + \pi)\delta - (\tau + \pi^*)\delta^* \qquad (10.31)$$

$$\delta D - D\delta = (\alpha^* + \beta - \pi^*)D + \kappa\Delta - (\rho^* + \epsilon - \epsilon^*)\delta - \sigma\delta^* \qquad (10.32)$$

$$\delta\Delta - \Delta\delta = -\nu^* D + (\tau - \alpha^* - \beta)\Delta + (\mu - \gamma + \gamma^*)\delta + \lambda^*\delta^* \qquad (10.33)$$

$$\delta^*\delta - \delta\delta^* = (\mu^* - \mu)D + (\rho^* - \rho)\Delta + (\alpha - \beta^*)\delta + (\beta - \alpha^*)\delta^* \qquad (10.34)$$

The following set of so called eliminant equations can be useful too. They can be derived from combinations of (10.1) - (10.18).

$$D(\rho-\rho^*)+\delta\kappa^*-\delta^*\kappa = (\rho-\rho^*)(\rho+\rho^*+\epsilon+\epsilon^*)+\kappa(\tau^*+\pi-3\alpha-\beta^*)-\kappa^*(\tau+\pi^*-3\alpha^*-\beta)$$

$$D(\mu - \mu^*) + \delta(\alpha + \beta^* - \pi) - \delta^*(\alpha^* + \beta - \pi^*) = (\gamma + \gamma^*)(\rho - \rho^*) + \alpha(\pi^* - 2\beta) - \alpha^*(\pi - 2\beta^*) + \kappa^*\nu^* - \kappa\nu + \beta\pi - \beta^*\pi^* + (\rho + \rho^*)(\mu - \mu^*)$$

$$D(\mu - \mu^* - \gamma + \gamma^*) + \Delta(\epsilon - \epsilon^*) - \delta\pi + \delta^*\pi^* = (\epsilon + \epsilon^*)(\mu^* - \mu) + \tau^*(\alpha^* + \pi^* - \beta) - \tau(\alpha + \pi - \beta^*) + \lambda\sigma - \lambda^*\sigma^* + \rho^*\mu - \rho\mu^* + 2(\epsilon\gamma - \epsilon^*\gamma^*)$$

$$\Delta(\mu^* - \mu) + \delta\nu - \delta^*\nu^* = (\mu - \mu^*)(\mu + \mu^* + \gamma + \gamma^*) + \nu(\tau - 3\beta - \alpha^* + \pi^*) - \nu^*(\tau^* + \pi - 3\beta^* - \alpha)$$

$$D(\tau - \alpha^* - \beta) - \Delta\kappa + \delta(\epsilon + \epsilon^*) = \rho(\tau + \pi^*) + \kappa^*\lambda^* + \sigma(\tau^* - \alpha - \beta^*) + \epsilon(\tau - \pi^*) - \rho^*(\beta + \alpha^* + \pi^*) + \epsilon^*(2\alpha^* + 2\beta - \tau - \pi^*) + \kappa(\mu - 2\gamma)$$

$\delta(\rho - \epsilon + \epsilon^*) - \delta^*\sigma + D(\beta - \alpha^*) = \rho(\alpha^* + \beta + \tau) - \rho^*(\tau - \beta + \alpha^* + \pi^*) + (\epsilon^* - \epsilon)(2\alpha^* - \pi^*) + \sigma(\pi - 2\alpha) + \kappa(\gamma^* - \gamma - \mu^*) + \kappa^*\lambda^*$

$D\lambda + \Delta\sigma^* - \delta^*(\tau^* + \pi) = \sigma^*(3\gamma^* - \gamma + \mu - \mu^*) + (\pi + \tau^*)(\pi - \tau^* + \alpha) + \lambda(\rho - \rho^* - 3\epsilon + \epsilon^*) - \beta\pi - \tau^*\beta^*$

$D\nu + \Delta(\alpha + \beta^* - \pi) - \delta^*(\gamma + \gamma^*) = \nu(\rho - 2\epsilon) + \lambda(\pi^* - \alpha^* - \beta) + \mu(\pi + \tau^*) - \mu^*(\alpha + \beta^* + \tau^*) + \gamma(\pi - \tau^*) + \gamma^*(2\alpha + 2\beta^* - \pi - \tau^*) + \sigma^*\nu^*$

$\Delta(\beta^* - \alpha) + \delta\lambda + \delta^*(\gamma - \gamma^* - \mu) = \nu(\epsilon^* - \epsilon - \rho^*) + \lambda(\tau - 2\beta) + \alpha(\mu + \mu^*) - \mu^*(\pi + \tau^* + \beta^*) + \mu(\pi + \beta^*) + (\gamma - \gamma^*)(\tau^* - 2\beta^*) + \sigma^*\nu^*$

$D\mu + \Delta\rho - \delta\pi - \delta^*\tau = \rho^*\mu - \rho\mu^* + \pi(\pi^* - \alpha^* + \beta) + \tau(\beta^* - \alpha - \tau^*) + \rho(\gamma + \gamma^*) - \mu(\epsilon + \epsilon^*)$

10.5 essentials of a Hopf knot

We must define the source now. For every constant real a the vector (compare (4.1))

$$\mathbf{F}(t, \mathbf{r}) = \frac{a}{((t-i)^2 - r^2)^3} \begin{pmatrix} (x - iy)^2 - (t - i - z)^2 \\ i(x - iy)^2 + i(t - i - z)^2 \\ -2(x - iy)(t - i - z) \end{pmatrix}$$

defines the Riemann-Silberstein vector of an electromagnetic knot. In the limit of very small a, the energy(density) is accordingly small and the curvature of space-time due to this field can be safely neglected. This is the situation described in the introduction, except that the constant a should now appear in the representation of the Faraday tensor. In this case there is a direct relation between the first tetrad vector field, **l**, and the electromagnetic field. We expect this relation to hold also in the case of stronger fields, so that stating properties for the first of tetrads is tantamount to having corresponding properties for the electromagnetic field.

An important property of an electromagnetic Hopf knot is that the field is null. From section **3.4** we know that the principal null directions of the field coincide: in spinor-language we have $\phi_{AB} \propto \vartheta_A\vartheta_B$ for some spinor ϑ. In the notation of section **1.2.1**, let $\mathbf{l} \leftrightarrow \vartheta\bar{\vartheta}$ and $\mathbf{n} \leftrightarrow \iota\bar{\iota}$ for some suitable but unspecified ι. The spinor form of the defining equations for the tetrad representation of the faraday tensor (section **7.4.3**) can be seen to be $\phi_0 = \phi_{AB}\vartheta^A\vartheta^B, \phi_1 = \phi_{AB}\vartheta^A\iota^B$ and $\phi_2 = \phi_{AB}\iota^A\iota^B$. With present choice for **l** we therefore have ϕ_2 as the only nonzero component. From section **3.4** we also know that Maxwell equations in flat space-time implies that $\mathbf{l}$ defines a geodesic and shear-free null congruence.

This remains true in a general curved space-time (see below), and we therefore proceed as follows.

We choose $\mathbf{l}$ to be the eigenvector field of the Faraday tensor. This implies that the Faraday tensor is represented by ϕ_2 alone, the other components vanish. Maxwell's equations (10.27) and (10.29) now give $\kappa = 0 = \sigma$.

In M^4 the Robinson congruence is not proportional to the gradient of a scalar field. Having in mind the converse of the limiting procedure just described, it seems implausible that at one stage in the process we suddenly do have this proportionality. We therefore assume $\rho \neq \rho^*$, in particular $\rho \neq 0$. Only the latter property will be of importance in the sequel.

The choices that we will make next represent only one possible set. Others have been considered, some of which could afterwards be rejected. With one particular set of choices we finally obtained few difficult equations that still have to be solved. However, we would like to present an example that has been brought to completion. Besides, the choices discussed in the following section allow for a better survey of the method used.
We would also like to mention that changes in the choice of the form of $\mathbf{l}$ (see below) have been considered, with no prospect of obtaining a solution quickly.

10.6 an attempt to solve the equations

10.6.1 first attempt

We assume that there exists coordinates $\{t, x, y, z\}$ such that

$\mathbf{l} = \mathcal{A}(1, \frac{\mathbf{S}}{bS})$

$T_{(2)(2)} = -bS$, other components vanishing

$bS = |\mathbf{S}|$, $\mathbf{S} = \frac{(1+x^2+y^2+(t-z)^2)}{((t^2-r^2)^2+1+2(t^2+r^2))^3} \begin{pmatrix} 2(x(t-z)+y) \\ 2(y(t-z)-x) \\ x^2+y^2-(t-z)^2-1 \end{pmatrix}$,

$\mathcal{A} = \mathcal{A}(t, x, y, z)$ and $b = b(t, x, y, z)$ undetermined

and $\phi_0 = 0 = \phi_1$; $\kappa = 0 = \sigma$, $\rho \neq 0$.

The space-time under consideration contains a geodesic shear-free null congruence, **l**. In addition, since $\phi_0 = 0 = \phi_1$, we have $\Phi_{00} = \Phi_{01} = \Phi_{02} = 0$. All the requirements for the Goldberg-Sachs theorem (section **7.6**) to be valid are therefore satisfied. We conclude that the space-time is algebraically special and:

$\Psi_0 = 0 = \Psi_1$.

Now apply a type 3 transformation to the tetrad (see section **7.4.4**) such that $\frac{\epsilon}{A} - \frac{DA}{2A^2} + \frac{iD\theta}{2A} = 0$. The function A will be absorbed in $\mathcal{A}$. From now we also have:

$\epsilon = 0$.

Next, consider a type 1 transformation with $a = -\frac{\tau}{\rho}$. From the Ricci identities, (10.1)-(10.18), and what we know so far, it can be inferred that $Da = -\pi^*$ and $\delta a = -\lambda^* + 2\alpha^* \frac{\tau}{\rho} - \rho^* \frac{\tau^2}{\rho^2}$. These relations for a, Da and δa imply that we now have in addition:

$\tau = \pi = \lambda = 0$.

Plugging the known variables into the equations listed above and skipping the ones that have become trivial, we see that, only due to $\phi_0 = \phi_1 = \kappa = \sigma = 0, \quad \rho \neq 0$ and suitable tetrad transformations, a great simplification has occurred:

$D\rho = \rho^2$
$D\alpha = \alpha\rho$
$D\beta = \beta\rho^*$
$D\gamma = \Psi_2$
$D\mu = \rho^*\mu + \Psi_2$
$D\nu = \Psi_3$
$-\delta^*\nu = \nu(3\alpha + \beta^*) - \Psi_4$
$\delta\rho = \rho(\alpha^* + \beta)$
$\delta\alpha - \delta^*\beta = \mu\rho + \alpha\alpha^* + \beta\beta^* - 2\alpha\beta + \gamma(\rho - \rho^*) - \Psi_2$
$-\delta^*\mu = \nu(\rho - \rho^*) + \mu(\alpha + \beta^*) - \Psi_3$
$\delta\nu - \Delta\mu = \mu^2 + \mu(\gamma + \gamma^*) - \nu(3\beta + \alpha^*) + \Phi_{22}$
$\delta\gamma - \Delta\beta = \gamma(-\alpha^* - \beta) - \beta(\gamma - \gamma^* - \mu)$
$\Delta\rho = -\rho\mu^* + \rho(\gamma + \gamma^*) - \Psi_2$
$\Delta\alpha - \delta^*\gamma = \nu\rho + \alpha(\gamma^* - \mu^*) + \gamma\beta^* - \Psi_3$

$D\Psi_2 = 3\rho\Psi_2$
$-\delta^*\Psi_2 + D\Psi_3 - 2\rho\Psi_3 = 0$
$\delta^*\Psi_3 - D\Psi_4 + 2\alpha\Psi_3 + \rho\Psi_4 = 0$
$\delta\Psi_2 = 0$

$$-\Delta\Psi_2 + \delta\Psi_3 - 3\mu\Psi_2 + 2\beta\Psi_3 - D\Phi_{22} + \rho^*\Phi_{22} = 0$$
$$-\Delta\Psi_3 + \delta\Psi_4 + 3\nu\Psi_2 - 2(\gamma + 2\mu)\Psi_3 + 4\beta\Psi_4 - \delta^*\Phi_{22} - 2(\alpha + \beta^*)\Phi_{22} = 0$$

$$D\Phi_{22} = (\rho + \rho^*)\Phi_{22}$$

$$D\phi_2 = \rho\phi_2$$
$$\delta\phi_2 = -2\beta\phi_2$$

Since $\Phi_{22} \propto \phi_2\phi_2^*$, $D\phi_2 = \rho\phi_2$ implies $D\Phi_{22} = (\rho + \rho^*)\Phi_{22}$, which can therefore be omitted from the list.
From $T_{(2)(2)} = -bS$ it follows that $\phi_2 = e^{i\Omega}\sqrt{\frac{bS}{2}}$ for some real function Ω. $D\phi_2 = \rho\phi_2$ then results in

$$\rho = iD\Omega - \mathcal{A}\frac{2(t-z)}{1+x^2+y^2+(t-z)^2} + \frac{1}{2b}Db.$$

Since $\rho, \alpha, \beta^*, \phi_2$ and $(\Psi_2)^{\frac{1}{3}}$ all satisfy the same equation $D(...) = \rho(...)$, it is tempting to express all these variables in terms of one of them:

$\alpha = f^{\alpha}\rho$, $\beta = f^{\beta}\rho^*$, $\phi_2 = f^{\phi}\rho$, $\Psi_2 = f^{\Psi}\rho^3$ in which all the $f^{\cdots}$ satisfy $Df^{\cdots} = 0$.
In the same spirit $D\gamma = \Psi_2$ then implies $\gamma = \frac{1}{2}f^{\Psi}\rho^2 + f^{\gamma}$, and $D\mu = \rho^*\mu + \Psi_2$ suggests the form $\mu = f_1\rho^* + f_2\rho + f_3\rho^2$. Substitution of this expression into the equation leads to $f_3 = \frac{1}{2}f^{\Psi}$ and $f_2 = f^{\Psi}\frac{\rho\rho^*}{2(\rho-\rho^*)}$. Note that indeed $Df_2 = 0$. To summarize:

$$\alpha = f^{\alpha}\rho$$
$$\beta = f^{\beta}\rho^*$$
$$\phi_2 = f^{\phi}\rho$$
$$\Psi_2 = f^{\Psi}\rho^3$$
$$\gamma = \frac{1}{2}f^{\Psi}\rho^2 + f^{\gamma}$$
$$\mu = f^{\mu}\rho^* + f^{\Psi}\frac{\rho^3}{2(\rho-\rho^*)}$$
$$\Phi_{22} = 2f^{\phi}(f^{\phi})^*\rho\rho^*$$

$$Df^{\cdots} = 0.$$

$$D\rho = \rho^2$$
$$D\nu = \Psi_3$$
$$-\delta^*\nu = \nu(3\alpha + \beta^*) - \Psi_4$$
$$\delta\rho = \rho(\alpha^* + \beta)$$

$$\delta\alpha - \delta^*\beta = \mu\rho + \alpha\alpha^* + \beta\beta^* - 2\alpha\beta + \gamma(\rho - \rho^*) - \Psi_2$$
$$-\delta^*\mu = \nu(\rho - \rho^*) + \mu(\alpha + \beta^*) - \Psi_3$$
$$\delta\nu - \Delta\mu = \mu^2 + \mu(\gamma + \gamma^*) - \nu(3\beta + \alpha^*) + \Phi_{22}$$
$$\delta\gamma - \Delta\beta = \gamma(-\alpha^* - \beta) - \beta(\gamma - \gamma^* - \mu)$$
$$\Delta\rho = -\rho\mu^* + \rho(\gamma + \gamma^*) - \Psi_2$$
$$\Delta\alpha - \delta^*\gamma = \nu\rho + \alpha(\gamma^* - \mu^*) + \gamma\beta^* - \Psi_3$$

$$-\delta^*\Psi_2 + D\Psi_3 - 2\rho\Psi_3 = 0$$
$$\delta^*\Psi_3 - D\Psi_4 + 2\alpha\Psi_3 + \rho\Psi_4 = 0$$
$$\delta\Psi_2 = 0$$
$$-\Delta\Psi_2 + \delta\Psi_3 - 3\mu\Psi_2 + 2\beta\Psi_3 - \rho\Phi_{22} = 0$$
$$-\Delta\Psi_3 + \delta\Psi_4 + 3\nu\Psi_2 - 2(\gamma + 2\mu)\Psi_3 + 4\beta\Psi_4 - \delta^*\Phi_{22} - 2(\alpha + \beta^*)\Phi_{22} = 0$$

$$\delta\phi_2 = -2\beta\phi_2$$

From the equations for $\delta\rho, \delta\Psi_2$ and $\delta\phi_2$ we deduce:

$$\delta f^\Psi = -3f^\Psi(f^\beta + f^{\alpha*})\rho^*, \quad \delta f^\phi = -f^\phi(3f^\beta + f^{\alpha*})\rho^*, \quad \delta\rho = \rho\rho^*(f^\beta + f^{\alpha*}).$$

This is a good point for an extra assumption that will lead to a welcome further simplification.
If we assume $\beta = -\alpha^*$, valid in the flat space-time case, we get:

$$D\rho = \rho^2$$
$$D\nu = \Psi_3$$
$$-\delta^*\nu = 2\nu\alpha - \Psi_4$$
$$\delta\rho = 0$$
$$\delta\alpha + \delta^*\alpha^* = \mu\rho + 4\alpha\alpha^* + \gamma(\rho - \rho^*) - \Psi_2$$
$$-\delta^*\mu = \nu(\rho - \rho^*) - \Psi_3$$
$$\delta\nu - \Delta\mu = \mu^2 + \mu(\gamma + \gamma^*) + 2\nu\alpha^* + \Phi_{22}$$
$$\delta\gamma + \Delta\alpha^* = \alpha^*(\gamma - \gamma^* - \mu)$$
$$\Delta\rho = -\rho\mu^* + \rho(\gamma + \gamma^*) - \Psi_2$$
$$\Delta\alpha - \delta^*\gamma = \nu\rho + \alpha(\gamma^* - \mu^*) - \gamma\alpha - \Psi_3$$

$$-\delta^*\Psi_2 + D\Psi_3 - 2\rho\Psi_3 = 0$$
$$\delta^*\Psi_3 - D\Psi_4 + 2\alpha\Psi_3 + \rho\Psi_4 = 0$$
$$-\Delta\Psi_2 + \delta\Psi_3 - 3\mu\Psi_2 - 2\alpha^*\Psi_3 - \rho\Phi_{22} = 0$$
$$-\Delta\Psi_3 + \delta\Psi_4 + 3\nu\Psi_2 - 2(\gamma + 2\mu)\Psi_3 - 4\alpha^*\Psi_4 - \delta^*\Phi_{22} = 0$$

$$\delta f^\Psi = 0$$
$$\delta f^\phi = 2f^\phi f^{\alpha*}\rho^*$$

Except for ν, we have expressed all spin coefficients in terms of ρ. From $D\nu = \Psi_3$ and $-\delta^*\Psi_2 + D\Psi_3 - 2\rho\Psi_3 = 0$, we see that if $\delta^*\Psi_2 = 0$, we can express Ψ_3 and ν in terms of ρ.
Alternatively, and less restrictive, we can use the commutation relations, which now read:

$$\Delta D - D\Delta = (\gamma + \gamma^*)D$$
$$\delta D - D\delta = -\rho^*\delta$$
$$\delta\Delta - \Delta\delta = -\nu^* D + (\mu - \gamma + \gamma^*)\delta$$
$$\delta^*\delta - \delta\delta^* = (\mu^* - \mu)D + (\rho^* - \rho)\Delta + 2\alpha\delta - 2\alpha^*\delta^*,$$

the second of which gives $(\delta^* D - D\delta^*)\rho = -\rho\delta^*\rho$, or $D(\delta^*\rho) = 3\rho(\delta^*\rho)$. We therefore put

$$\delta^*\rho = f^\rho\rho^3, \quad Df^\rho = 0.$$

Also, $(\delta^* D - D\delta^*)f^\Psi = -\rho\delta^* f^\Psi$, thus $\delta^* f^\Psi = f^{ps}\rho, \quad Df^{ps} = 0$.

The same commutation relation also gives rise to $D(\delta^*\Psi_2) = 4\rho(\delta^*\Psi_2) + 3f^\Psi f^\rho\rho^6$, which suggests $\delta^*\Psi_2 = f_4\rho^4 + f_5\rho^5$. Substitution of this expression leads to $\delta^*\Psi_2 = f^p\rho^4 + 3f^\rho f^\Psi\rho^6$ and $Df^p = 0$. The equation that contains $D\Psi_3$ now becomes:

$$D\Psi_3 = 2\rho\Psi_3 + f^p\rho^4 + 3f^\rho f^\Psi\rho^6.$$

This again leads to a suggestion for Ψ_3 that, when substituted, results in:

$$\Psi_3 = f^p\rho^3 + f^\rho f^\Psi\rho^5.$$

Alternatively, use $\Delta\alpha - \delta^*\gamma = \nu\rho + \alpha(\gamma^* - \mu^*) - \gamma\alpha - \Psi_3$ and $\delta\gamma + \Delta\alpha^* = \alpha^*(\gamma - \gamma^* - \mu)$ to get $\Psi_3 = \nu\rho + \delta^*(\gamma^* + \gamma)$. Substitute γ in terms of ρ and use $\delta D - D\delta = -\rho^*\delta$ several times in order to get:

$$\Psi_3 = (\nu + f^g)\rho + \tfrac{1}{2}f^{ps}\rho^3 + f^\rho f^\Psi\rho^4, \quad Df^g = 0.$$

Comparing these results gives an expression for ν in terms of ρ:

$$\nu = -f^g + (f^p - \tfrac{1}{2}f^{ps})\rho^2 - f^\rho f^\Psi(\rho^3 - \rho^4).$$

$D\nu = \Psi_3$ can now be used to arrive at $f^p = f^{ps}$ and $f^\rho = 0$ or $f^\Psi = 0$.

Time to pause and recapitulate what the result is up till now.

$\alpha = f^{\alpha}\rho = -\beta^*$
$\gamma = \frac{1}{2}f^{\Psi}\rho^2 + f^{\gamma}$
$\mu = f^{\mu}\rho^* + f^{\Psi}\frac{\rho^3}{2(\rho-\rho^*)}$
$\nu = -f^g + \frac{1}{2}f^p\rho^2$

$\phi_2 = f^{\phi}\rho$
$\Phi_{22} = 2f^{\phi}(f^{\phi})^*\rho\rho^*$

$\Psi_2 = f^{\Psi}\rho^3$
$\Psi_3 = f^p\rho^3$
$\Psi_4 = -(f^h + 2f^g f^{\alpha})\rho + (f^p f^{\alpha} + \frac{1}{2}f^q)\rho^3 + f^p f^{\rho}\rho^4$

$D\rho = \rho^2$
$\delta\rho = 0$
$\delta^*\rho = f^{\rho}\rho^3$
$\Delta\rho = (f^{\gamma} + f^{\gamma *})\rho - f^{\mu *}\rho^2 - \frac{1}{2}f^{\Psi}\rho^3 + \frac{1}{2}f^{\Psi *}\rho(\rho^*)^2 + f^{\Psi *}\frac{\rho(\rho^*)^3}{2(\rho-\rho^*)}$

$\delta f^{\Psi} = 0$
$\delta f^{\phi} = 2f^{\phi}f^{\alpha *}\rho^*$
in general $\delta f^a = f^b\rho^*$

$\delta^* f^{\Psi} = f^p\rho$
in general $\delta^* f^a = f^b\rho: \quad \delta^* f^g = f^h\rho, \quad \delta^* f^p = f^q\rho, \quad \delta^* f^{\mu} = f^m\rho$

$\delta^*\Psi_2 = f^p\rho^4$

$Df^{\cdots} = 0$
$f^{\rho} = 0$ or $f^{\Psi} = 0$

$\delta\alpha + \delta^*\alpha^* = \mu\rho + 4\alpha\alpha^* + \gamma(\rho - \rho^*) - \Psi_2$
$-\delta^*\mu = \nu(\rho - \rho^*) - \Psi_3$
$\delta\nu - \Delta\mu = \mu^2 + \mu(\gamma + \gamma^*) + 2\nu\alpha^* + \Phi_{22}$
$\delta\gamma + \Delta\alpha^* = \alpha^*(\gamma - \gamma^* - \mu)$
$\Delta\alpha - \delta^*\gamma = \nu\rho + \alpha(\gamma^* - \mu^*) - \gamma\alpha - \Psi_3$

$\delta^*\Psi_3 - D\Psi_4 + 2\alpha\Psi_3 + \rho\Psi_4 = 0$
$-\Delta\Psi_2 + \delta\Psi_3 - 3\mu\Psi_2 - 2\alpha^*\Psi_3 - \rho\Phi_{22} = 0$
$-\Delta\Psi_3 + \delta\Psi_4 + 3\nu\Psi_2 - 2(\gamma + 2\mu)\Psi_3 - 4\alpha^*\Psi_4 - \delta^*\Phi_{22} = 0$

The equation $-\delta^*\mu = \nu(\rho - \rho^*) - \Psi_3$ can be used to show that $\frac{1}{2}\frac{1}{\rho\rho^*}f^p[\rho^3 - \frac{\rho^4}{\rho-\rho^*} + \rho^*\rho^2] = f^g\frac{(\rho^*-\rho)}{\rho\rho^*} + f^m$. Since $D(f^g\frac{(\rho^*-\rho)}{\rho\rho^*} + f^m) = 0$ we should also have $D(\frac{1}{\rho\rho^*}[\rho^3 - \frac{\rho^4}{\rho-\rho^*} + \rho^*\rho^2]) = 0$. It is easy to show that this implies:

$\rho = \rho^*$.

Although we did not expect this to happen, in fact we only used $\rho \neq 0$. Again we get a pleasant reduction in the complexity of the remaining equations. The reality of ρ implies $f^\rho = 0$, since $(\delta^*\rho)^* = \delta\rho^*$. It also implies $f^\Psi = 0$, as can be seen from the derivation of the expression for μ in terms of ρ. This in turn implies $f^p = 0$, and therefore $f^q = 0$ also. Now, the only nonzero Ψ is Ψ_4. The equation $-\Delta\Psi_2 + \delta\Psi_3 - 3\mu\Psi_2 - 2\alpha^*\Psi_3 - \rho\Phi_{22} = 0$ however tells us that we ran into a contradiction, since $\Phi_{22} \neq 0$.

10.6.2 second attempt

In the previous attempt we imposed $\beta = -\alpha^*$, that was also valid in the flat space-time in case we use the tetrad (9.1). However, in section **9.2.1** we did not fix $\mathbf{n}$ in such a way as to obtain $\tau = 0$, whereas in **10.6.1** we did have $\tau = 0$. This does not mean that the attempt in the previous section *must* fail, but we might have better prospects when we use relations valid in the flat space-time case related to a tetrad that is more akin to the one in the curved space-time. Therefore, here we present a summary of an attempt to solve the equations, in which we assume a relation that is valid in the flat space-time case when we use the tetrad (9.3): $\beta = -\alpha^* + a\rho^*$. This exercise has not been brought to completion, but is presented here to show to what kind of task it leads.

Like before, we start with $\phi_0 = 0 = \phi_1$, $\kappa = 0 = \sigma$ and $\rho \neq 0$. Again, this implies $\Psi_0 = 0 = \Psi_1$. We now choose a such that $\tau + a\rho = 0$, and apply a tetrad type 1 transformation with this a. This gives $\tau = 0$, which in combination with (10.3) and (10.16) leads to $\pi = 0$ and $\lambda = 0$ in addition. All this leads to equations that still appear difficult. We therefore use the freedom of a type 3 transformation to get $\epsilon = 0$ like before. (We could go a different direction here, since this is just a choice, motivated only by the reduction of the complexity. Other choices also reduce the complexity. It is difficult to come up with a physically preferred choice. At least, present choice does admit an interpretation: the use of an affine parameter.) It is clear that we now have exactly the same set of equations as the first set in the previous subsection. But now, as a second choice, we make a different one from before: $\beta = -\alpha^* + a\rho^*$, and combine this with the *assumption* that ρ is real. We should therefore refrain from using the previous expression for μ in terms of ρ, and have instead $\mu = f^\mu\rho + f^\Psi\rho^2$. It is easy to find $Da = 0$. From the equations for $\delta\rho, \delta\Psi_2$ and $\delta\phi_2$ we deduce $\delta f^\Psi = -3af^\Psi\rho$ and $\delta f^\phi = -f^\phi(3f^\beta + (f^\alpha)^*)\rho$. Applying the commutation relation for δ^* and D (now different from before) to Ψ_2 leads to

$\delta^*\Psi_2 = f^p\rho^4$, which in turn leads to $\Psi_3 = f^\rho\rho^2 + f^p\rho^3$. $D\nu = \Psi_3$ then gives $\nu = f^\rho\rho + \frac{1}{2}f^p\rho^2 + f^\nu$. Continuing in this spirit, it is perhaps not too difficult to find few more expressions for the tetrad representatives in terms of ρ and functions $f^{\cdot\cdot}$ that satisfy $Df^{\cdot\cdot} = 0 = D\Delta f^{\cdot\cdot}, \delta f^{\cdot\cdot} = f^{'\cdot\cdot}\rho$ and $\delta^* f^{\cdot\cdot} = f^{''\cdot\cdot}\rho$. We end up with only a few of the original equations that are not identically satisfied, and many unknown functions $f^{\cdot\cdot}$ that all satisfy the same relatively simple equations. What is left from the original set can be used to relate some of the $f^{\cdot\cdot}$'s. Additional assumptions lead to simpler equations, but could also lead to contradictions.

10.7 final remarks

The contradiction in the first attempt is the result of the collection of assumptions made, at least one of which should not have been made. It seems reasonable to assume that it is possible, changing assumptions every time when needed, to bring this exercise to a satisfactory end. In practise however this turns out to be very laborious. The Geroch-Held-Penrose [93] formalism is an alternative for the Newman-Penrose formalism that might be better suited to our problem.

Suppose we succeeded in our task and would have an expression for all tetrad components of the Weyl tensor in terms of t, x, y, z. We would then have a formidable new task of showing that this solution really is a new one or that it is a known one in disguise. Diffeomorphism invariance in general relativity leads to an egalitarian principle among all coordinate systems. The same solution expressed in weird but equally valid coordinates could look very different from the solution in the usual coordinates. The problem of proving or disproving that two solutions are the same, expressed in different coordinate systems, is called the equivalence problem and is notorious. If it would lead to a solution already known, the present context could possibly clarify questions about its source.