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Title: Magnetic resonance force microscopy at milliKelvin temperatures
Issue Date: 2013-09-19
Chapter 8

High sensitivity SQUID-detection and feedback-cooling of an ultrasoft microcantilever

We measure the motion of an ultrasoft cantilever carrying a ferromagnetic particle by means of a Superconducting Quantum Interference Device (SQUID) microsusceptometer. In our scheme, the cantilever motion modulates the magnetic flux coupled by the magnetic particle into the SQUID. For the cantilever fundamental mode we achieve a dimensionless coupling factor as large as 0.07 and a displacement sensitivity of 200 fm/√Hz. Simultaneously, we reach sub-attonewton force sensitivity by cooling the cantilever thermal motion below 100 mK. Finally, we demonstrate the outstanding combination of very low displacement and force sensitivity by feedback-cooling the cantilever mode to an effective mode temperature of 160 µK, the lowest value to date for a micromechanical resonator.

The results presented in this chapter have been published as:
8.1 Introduction

In recent years mechanical resonators, in particular micro and nanomechanical resonators, have been coupled to a variety of quantum devices and ultrasensitive displacement sensors, based for instance on optomechanical, microwave, electromechanical, magnetomechanical and quantum point contact detection techniques [110]. Applications of ultrasensitive mechanical resonators range from the detection of weak forces, for instance in magnetic resonance force microscopy [15] or gravitational wave detection [54], to the test of quantum mechanics in macroscopic objects [111–113].

A topic that has become increasingly popular is the quest of cooling mechanical resonators to the ground state, which is considered an enabling step in order to prepare a mechanical resonator in nonclassical states. The most remarkable achievement in this sense has been the cryogenic cooling of a 6 GHz resonator and its strong coupling to a superconducting qubit, which has enabled the first demonstration of nonclassical mechanical states [114]. On the other hand, other techniques have been proposed to cool resonators with lower frequency. Sideband cooling to the ground state has been recently demonstrated using microwave [115] and optomechanical [116] cavities. A related technique is active feedback-cooling, based on high precision measurement and control of the mechanical resonator. Feedback-cooling can be applied to a wider range of detectors and in particular it is more suitable for low frequency resonators. Indeed, very large cooling factors and extremely low temperatures have already been achieved through feedback [117–122], for resonator frequencies in the range 100 Hz - 2 MHz. Cooling ultrasoft low-frequency resonators close to the ground state might in principle allow the preparation of well-separated macroscopic quantum superpositions, and therefore enable tests of quantum mechanics at macroscopic level, including alternative wavefunction collapse models [111–113].

The efficiency of feedback-cooling can be expressed in the following way [118], in terms of a minimum achievable temperature $T_{\text{min}}$ or a minimum number of phonons $N_{\text{min}}$:

$$N_{\text{min}} = \frac{k_B T_{\text{min}}}{\hbar \omega} = \frac{1}{2\hbar} \sqrt{S_f S_x},$$  (8.1)

Here, $S_f$ and $S_x$ are one-sided power spectral densities respectively of the force noise driving the resonator and the detector displacement noise. In particular, approaching the ground state requires $\sqrt{S_f S_x} \approx \hbar$, which is achieved only when the force noise $S_f$ is dominated by the detector backaction, and the detector itself is quantum limited [123–126].

We have recently demonstrated a scheme to measure the motion of a mechanical resonator by using a superconducting pick-up coil connected to a Superconducting Quantum Interference Device (SQUID) to detect a ferromagnetic particle attached to the resonator [48]. Here, we demonstrate an improved version of this technique, which consists in directly approaching the ferromagnetic particle to the SQUID loop. This configuration allows to reach a
much stronger magnetomechanical coupling, which translates into much better displacement sensitivity and feedback-cooling efficiency.

8.2 Experimental scheme

Figure 8.1: a) Scheme of the experiment. b) Electron microscope micrograph of the cantilever with the magnet attached to its free end. (c) Layout of the SQUID microsusceptometer.

A scheme of the setup is shown in Fig. 1a. A cantilever mechanical resonator with a ferromagnetic particle with magnetic moment $\vec{\mu}$ attached to its end (from now on, the 'magnet') is approached to the superconducting loop of a SQUID. The magnet couples a magnetic flux $\Phi(x)$ in the SQUID, so that a displacement of the cantilever end $x$ will cause a flux change $\Phi_x x$. Here, $\Phi_x = \partial \Phi / \partial x$ is the magnetomechanical coupling. It can be calculated as $\Phi_x = \vec{\mu} \cdot \partial \vec{b} / \partial x$, where $\vec{b} = \vec{B}/I$ is the magnetic field generated in the dipole location by a probe current $I$ flowing in the SQUID loop. The displacement detection noise spectral density is given by $S_x = S_\Phi / \Phi_x^2$, and scales inversely with the square of the magnetomechanical coupling.

We can define a dimensionless coupling factor $\beta$ by the expression $\beta^2 = \Phi_x^2 / kL$, where $k$ is the cantilever spring constant and $L$ is the SQUID inductance. $\beta^2$ can be thought as the ratio between the magnetic energy $\Phi_x^2 x^2/2L$ coupled into the SQUID loop inductance, and the total mechanical resonator energy $kx^2/2$. In a quantum mechanical picture, if $L$ were part of a quantum $LC$ resonator coupled to the mechanical resonator, then $\lambda = \beta \hbar \sqrt{\omega_1 \omega_2}$ would
be the energy coupling in the interaction hamiltonian. Here, $\omega_1$ and $\omega_2$ are the frequencies of mechanical and electrical systems.

As a general rule, the coupling $\beta$ is maximized by making the SQUID loop as small as possible, with thin linewidth, and placing the magnet at an optimal position, which is normally at a distance comparable with the loop size. The precise dependence of the coupling with position depends in a non-trivial way on the orientation of $\vec{\mu}$ with respect to the SQUID loop and the cantilever motion.

### 8.3 Experimental setup

In our experiment the resonator is an ultrasoft micromachined silicon cantilever, of the type developed for MRFM [31], shown in Fig. 1b. It has a very low spring constant, $k = 90 \mu N/m$. A NdFeB alloy magnetic particle with diameter 0.3 $\mu$m is attached to the cantilever and magnetized as described in Ref. [48]. The magnetic dipole $\vec{\mu}$ is oriented along the easy axis of the cantilever. The SQUID, shown in Fig. 1c, is a gradiometric microsusceptometer based on Nb/AlOx/Nb technology. The diameter of each loop is 30 $\mu$m, the linewidth is 4 $\mu$m and the total SQUID inductance has been experimentally estimated as $L = 250$ pH. A feedback coil and a field coil are integrated in the circuit. The gradiometric design allows large rejection of external field, and the Josephson junctions are located far from the SQUID loop, so that approaching a magnetized particle to the loop does not affect the junction critical current. The SQUID is operated with a commercial SQUID electronics in two-stage mode with a SQUID array as second stage. The measured SQUID flux noise during the experiment was $\sqrt{S_{\Phi}} = 1.0 \mu \Phi_0/\sqrt{\text{Hz}}$, which includes a non-negligible contribution from 1/f noise.

The SQUID is mounted on a custom made three-dimensional piezo fine-stage with a range of 2 $\mu$m at cryogenic temperature. The cantilever is oriented perpendicular to the SQUID chip surface, in order to avoid snap-to-contact, and is mounted on a custom made three-dimensional coarse approach based on piezo rotators $^1$, with a range up to 1 mm. The combined use of both stages allows for an easy alignment of the magnet above the SQUID loop in order to vary and optimize the magneto-mechanical coupling. The alignment can be performed at low temperature, starting with an initial misalignment as large as 300 $\mu$m, using the magnetic flux coupled into the SQUID by the magnet as a guide. A small piezoelectric actuator placed underneath the cantilever chip allows both to drive the cantilever for mechanical characterization and to apply a feedback force. The assembly is mounted on a mechanical suspension cooled in a cryogen-free dilution refrigerator. During the experiment reported here, the base temperature of the suspended mass was about 28 mK.

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8.4 Results

We have characterized the cantilever fundamental mode by means of ringdown measurements. Far from the surface, the frequency is \( f_0 = 4163 \) Hz and the quality factor is \( Q = 4 \times 10^4 \). When the cantilever is close to the surface we observe a position dependent frequency shift and additional damping, in part due to magnetic coupling to the insulator surface spins [106] and in part due to the diamagnetic shielding of the SQUID superconducting lines. We have experimentally determined a position with relatively large coupling at a distance of about 5 \( \mu \)m from the SQUID loop line, where surface-induced nonlinearities are not an issue. Here, the frequency was \( f_0 = 4450 \) Hz, while the \( Q \) factor was slightly temperature dependent, about \( Q = 4 \times 10^4 \) at 1 K and \( Q = 2.8 \times 10^4 \) at \( T < 100 \) mK.

Subsequently, we have characterized the cantilever brownian motion. Inset of Fig. 2 shows two spectra of the SQUID output signal acquired at two different bath temperatures, \( T = 28 \) mK and \( T = 470 \) mK. The noise is remarkably clean from vibrational noise, despite the operation in a pulse-tube dilution refrigerator, showing that the attenuation provided by the mechanical suspensions is sufficient. Measurements of the area under the Lorentzian noise peak at several bath temperatures show a linear behavior for temperatures higher than 200 mK, demonstrating that cantilever motion is thermal and allowing for an absolute calibration. For bath temperatures below 150 mK the cantilever is no more well thermalized to the thermal bath, and its effective noise temperature saturates at approximately \( T_0 \approx (90 \pm 10) \) mK. This saturation temperature is higher than that measured with a previous setup involving a pick-up coil and a remote SQUID (\( T_0 = 25 \) mK) [48]. We have checked that the saturation temperature does not depend significantly on the magnet-SQUID distance and coupling and on SQUID working point, so we don’t attribute the overheating to SQUID Josephson radiation dissipated in the magnet or in the cantilever. Instead, we observe a very slow trend to further cooling, with time constant of the order of several hours, so we believe that the saturation temperature is rather limited by a poor thermalization of the coarse approach stage which supports the cantilever chip. An optimized thermal design of the latter might lead to a further reduction of \( T_0 \).

From the calibrated cantilever temperature, and the estimated value of \( k \) and \( Q \), we can infer, using the fluctuation-dissipation formula, the thermal force noise \( \sqrt{S_f} = (0.8 \pm 0.1) \) aN/\( \sqrt{\text{Hz}} \). Furthermore, we can infer the absolute cantilever mean displacement fluctuation \( \langle x^2 \rangle = k_B T / k \) and from this the magnetomechanical coupling \( \Phi_x = (5.3 \pm 0.5) \times 10^6 \Phi_0 / m \), the dimensionless coupling \( \beta = 0.07 \pm 0.01 \) and the displacement noise floor \( \sqrt{S_x} = (200 \pm 20) \) fm/\( \sqrt{\text{Hz}} \). This is about 4 orders of magnitude in energy better than our previous experiments with an intermediate pick-up coil [48], and about 2 orders of magnitude better than interferometric detection of ultrasoft cantilever at subkelvin temperature [118]. Despite the relatively large coupling factor, we estimate that the backaction force noise of the SQUID is still negligible, about 30 times lower than the thermal force noise. This is largely due to the relatively
Figure 8.2: Inset: SQUID output voltage noise, in V/√Hz, showing cantilever thermal noise, acquired at two different bath temperatures, 470 mK (top curve) and 28 mK (bottom curve). Main panel: feedback-cooling of the cantilever starting from an initial effective temperature $T_0 = 90$ mK, for different gain $g$. The noise spectra are calibrated in cantilever displacement and refer, from top to bottom, to $g = 0, 54, 109, 543, 1086, 2172$ respectively. For the first 4 datasets, the best fit with a Lorentzian noise superimposed to white noise is shown, while for the last 2 datasets the Lorentzian features are completely suppressed.

Feedback-cooling is performed by using the SQUID signal to apply a feedback force to the cantilever through the piezo actuator. We apply a viscous feedback force by passing the feedback signal through a low-pass filter which allows for variable gain and nearly $-90$ deg phase shift. Under purely viscous feedback the quality factor is reduced from the intrinsic value $Q_0$ to an effective value $Q = Q_0/(1 + g)$, where $g$ is a normalized gain factor [127]. The mean energy of the cantilever mode is then reduced to an effective value $k_BT \approx k_BT_0/(1 + g)$. The latter relation holds only in the high $Q$ limit. Fig. 2 shows the power spectral density of the cantilever thermal motion measured by the SQUID for different values of $g$. The gain $g$ has been increased beyond the noise squashing limit, where noise correlations introduced by the feedback modify the usual Lorentzian peak superimposed to measurement noise into a Lorentzian dip. As the undamped noise is well described by a Lorentzian peak superimposed on top of the SQUID white noise, it is easy to model the system and determine the effective energy of the resonator even in the high gain limit [118]. In this way, we find that the maximum cooling factor is achieved.
when the Lorentzian spectrum is completely whitened \((g = 1086)\), and we determine the corresponding temperature as \(T_{\text{min}} = (160 \pm 10) \, \mu\text{K}\). This is equivalent to a mean number of phonons \(N_{\text{min}} \approx 760\). For even higher gain the effective resonator temperature increases, due to the injection of displacement detection noise by the feedback, which generates an additional driving force.

### 8.5 Discussion and prospects

Our result represents the lowest temperature achieved to date by feedback-cooling of a soft micromechanical resonator, improving by a factor of 20 over previous results \[118\]. This is a consequence of the combination of ultralow force noise and displacement noise, the latter being a consequence of the high magnetomechanical coupling factor achieved in this experiment. Eq. (8.1) states that a further progress will necessarily require a significant reduction both in \(S_f\) and \(S_x\). In our scheme \(S_x\) can be reduced in two ways. The first is to further increase the magnetomechanical coupling. This can be easily done in our scheme by optimizing the geometrical parameters. For instance, a factor of 8 can be gained by doubling the magnet diameter, as \(\Phi_x\) scales with the magnetic moment \(\mu\) and thus with the magnet volume. The second is to replace the SQUID with an even better magnetic flux sensor, like the recently demonstrated Josephson Parametric Amplifier (JPA), which is expected to be quantum limited \[66\]. On the other hand \(S_f\) can be slightly improved by a better thermalization of the cantilever holder, but a more significant reduction will eventually require a radically different mechanical resonator, possibly with much higher \(Q\). In this case, back-action from the detector can become dominant on the thermal noise, and the resonator noise will be completely determined by the detector. An interesting possibility, which can be naturally compatible with SQUID detection, is the recently proposed magnetomechanical resonator consisting of a \(\mu\text{m}\) size superconducting particle levitated in a trapping magnetic field \[128\]. The combination of an ultrahigh \(Q\) levitated resonator with a quantum limited amplifier, may eventually allow ground state cooling of the center of mass of a micron size particle, enabling the creation of a quantum superposition of spatially separated states of a macroscopic object and test of wavefunction collapse models \[111–113,128\].