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Chapter 5

A SQUID based read-out of
an MRFM cantilever
operating at milliKelvin
temperatures

We present a scheme to measure the displacement of a nanomechanical resonator at cryogenic temperatures. The technique is based on the use of a superconducting quantum interference device that detects the magnetic flux change induced by a magnetized particle attached to the end of the resonator. Unlike conventional interferometric techniques, our detection scheme does not cause direct power dissipation in the resonator. Therefore, it is particularly suitable for ultralow temperature applications. We demonstrate its potential by detecting the thermal vibrations of an ultrasoft silicon cantilever with a noise temperature of 25 mK, corresponding to a thermal force noise as low as 0.5 aN/√Hz in a 1.1 Hz bandwidth.

The results presented in this chapter have been published as:

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5.1 Introduction

Owing to its excellent sensitivity, optical interferometry is the most widely used technique to detect the motion of ultrasensitive resonators, for applications which include Magnetic Resonance Force Microscopy (MRFM) [16], the investigation of quantum effects in mechanical systems [53], and the detection of gravitational waves [54]. Unfortunately, optical detection becomes hard to implement when the size of the resonator is pushed to the nanoscale, because of the diffraction limit, and when ultralow temperatures are required, as for single spin MRFM. In the latter case, heating due to the absorption of light is found to limit the coldness of the resonator [55]. This problem can be partially circumvented by reducing the input light power, at the cost of reducing the displacement sensitivity. Recently, other techniques have been demonstrated to be more compatible with low temperature operation. In particular, both single electron transistors [56] and microwave cavities [57–59] have an outstanding sensitivity for the displacement detection of nanomechanical resonators at temperatures below 100 mK. So far, however, their implementation has been limited to systems where the detector and resonator are tightly integrated, which is not practical for scanning probe applications. Moreover, for microwave techniques the direct photon absorption still remains an issue at milliKelvin temperatures. Cantilever detection based on Quantum Point Contacts (QPC) has also been demonstrated, but has not yet been implemented at temperatures below that of liquid helium [60,61].

In this chapter, we demonstrate an alternative detection technique, based on the use of a Superconducting QUantum Interference Device (SQUID), which in principle does not require any power to be directly dissipated in the mechanical resonator. Our method involves attaching a ferromagnetic particle to the end of a cantilever (Fig. 3.3) which, when the cantilever moves, causes a change in the magnetic flux through a superconducting detection coil, positioned close to the magnetic particle (Fig. 3.5). The proportionality between the coil flux \( \Phi \) and the cantilever displacement \( x \) depends linearly on the magnetic moment \( \mu_m \) of the ferromagnetic particle and in a more complicated way on the coil geometry, the orientation of the magnetic moment, and the cantilever-coil alignment. The flux change in the detection coil is measured by a dc SQUID amplifier via a superconducting circuit of total inductance \( L_T \), which includes a calibration transformer and the SQUID input coil.

5.2 Methods

In our experiment, we use a single-crystalline Si cantilever of 100 nm thick, 5 \( \mu m \) wide, and 117 \( \mu m \) long, fabricated as reported in Ref. [31]. The estimated spring constant of the cantilever is \( k = 1.44 \cdot 10^{-4} \text{ N/m} \) for the lowest flexural mode. We attach a 4.5 \( \mu m \) diameter magnetic sphere of a NdFeB based alloy to the end of the cantilever using a nanomanipulator inside a scanning electron microscope [32] and magnetize it in a 5 T field at room temperature. The alloy has a nominal remanent field of 1.3 T [22, 23], from which we estimate that
the magnetic moment of the particle is \( \mu = 4.9 \cdot 10^{-11} \text{ Am}^2 \). The cantilever is then placed at about 10 \( \mu \text{m} \) above the inner edge of a superconducting Nb thin film detection coil (Fig. 3.6a). The Si chip that supports the cantilever is held in position and thermally anchored to a copper holder by a brass leaf spring. The detection coil has a square geometry with \( 2 \times 22 \) windings, an outer width of 670 \( \mu \text{m} \) and an estimated inductance of 0.6 \( \mu \text{H} \). The coil is connected to the input coil of a two-stage SQUID amplifier, made out of a commercial Quantum Design sensor SQUID and a custom-designed amplifier SQUID [40] (Fig. 3.11). The SQUID is operated with commercial readout electronics from Star Cryoelectronics. The SQUID flux noise scales with temperature down to about 0.5 K, leveling off to a constant value of \( \sim 0.6 \mu \Phi_0/\sqrt{\text{Hz}} \) at lower temperatures. The whole assembly is cooled in vacuum and thermally anchored to the mixing chamber of a dilution refrigerator with a base temperature of 8 mK. The cantilever can be excited in-situ via a piezo-electric actuator placed close to the cantilever chip. We determine the resonant frequencies and accompanying quality factors of the cantilever by means of ringdown measurements. At cryogenic temperatures, the fundamental mode resonant frequency is \( f_0 = 3084 \) Hz. The quality factor is \( Q_0 = 3.8 \cdot 10^4 \) at 11 mK and depends weakly on the temperature. We also measured the second vibration mode of the cantilever at \( f_1 = 49150 \) Hz, with a quality factor of \( Q_1 = 2.3 \cdot 10^4 \).

5.3 Results and Discussion

![Figure 5.1: a) Power spectral density of the SQUID output signal, featuring the Lorentzian-like peak due to the thermal motion of the cantilever. b) Noise temperature of the cantilever, obtained by calculating the area of the Lorentzian peaks, plotted versus the cryostat bath temperature, as measured by a calibrated thermometer. The continuous blue line represents a fit to the data with a standard saturation curve, yielding a saturation temperature of \( T_0 = (25 \pm 1) \) mK.](image)

In Fig. 5.1a, the power spectral density of the SQUID output signal\(^1\), featuring the Lorentzian-like peak due to the thermal motion of the cantilever

\(^1\)The SQUID signal is amplified using a low-noise voltage preamplifier with a gain of 100
at its fundamental vibration mode, is shown at two separate bath temperatures of 11 mK and 1014 mK. The measured spectral density is well fitted by a Lorentzian curve added to the white noise of the SQUID. The area of the peak is linearly proportional to the mean resonator energy. The proportionality constant depends on the responsivity of the detection circuit, which cannot be easily predicted. We have developed an experimental calibration procedure which enables us to directly convert the measured cantilever noise into a thermodynamic temperature. In this two-step procedure we first apply an external flux to the detection circuit through a calibration transformer. The measured current in the detection circuit as a function of frequency shows a resonant response due to the magnetic driving of the cantilever. An accurate measurement of this response allows us to infer the energy coupling $\beta^2$ between the cantilever and the detection circuit. This part of the calibration procedure is discussed in more detail in Sect. 4.1.1. For the fundamental cantilever vibration mode, we measured an energy coupling of $\beta^2 = 9.5 \cdot 10^{-6}$. Next, we measure the SQUID noise at an elevated temperature at which part of the detection circuit is in the metallic state, as described in Sect. 3.2.1. The spectral shape of this noise, combined with the energy coupling, enables us to infer the total detector responsivity. We thus obtain a direct relation between the mean squared SQUID voltage and the cantilever noise temperature (see Eq. (3.18)), which in this case is $(V^2_{\text{SQUID}})/T = 2.81 \cdot 10^{-7} \text{ V}^2/\text{K}$.

In Fig. 5.1b, the calibrated thermal energy of the cantilever, expressed as the effective noise temperature $T_N$, is plotted versus the temperature $T$ of the mixing chamber of the dilution refrigerator, as measured with a calibrated thermometer. A remarkable agreement between the cantilever noise temperature and the bath temperature is observed at temperatures from 1 K down to 30 mK. The noise temperature is then found to saturate at about 25 mK, suggesting that some residual power dissipation, combined with the exceedingly low thermal conductivity of Si at milliKelvin temperatures, is limiting further cooling of the cantilever. The blue line in the graph represents a fit to the data of the form

$$T_N = (T^n - T_0^n)^{1/n}$$

(5.1)

where $T_0$ is the saturation temperature and the exponent $n$ is determined by the temperature dependence of the limiting thermal conductivity, which scales as $T^{n-1}$ [43]. The fit yields a saturation temperature of $T_0 = (25 \pm 1)$ mK, while the exponent $n = 5 \pm 2$ is consistent with either a thermal boundary resistance ($n \sim 4$) or a phonon-mediated bulk thermal transport ($n \sim 3.5–4$) [43, 62]. In Fig. 5.2a, thermal excitation of the second vibration mode of the cantilever is shown for bath temperatures of 33 mK and 1014 mK. The noise peaks can be fitted reasonably well with a Lorentzian line shape, however the signal-to-noise ratio is lower than that of the fundamental mode. Indeed, we measure a much lower energy coupling of $\beta^2 = 5.7 \cdot 10^{-7}$, due to a sub-optimal cantilever-coil alignment for this mode, in which the magnetic particle is rotated rather than displaced. In Fig. 5.2b, the calibrated mean temperature of the second mode is plotted versus the cryostat temperature. The mode temperature follows the bath temperature from 1 K down to $\sim 30$ mK, below which we again observe a
saturation. The fit by Eq. (5.1) yields a saturation temperature of $T_0 = (18 \pm 6)$ mK, but is ill-defined in terms of the exponent $(n = 4 \pm 14)$.

Assuming that the temperature saturation is due to power dissipation in the silicon cantilever, and using a simple 3D phonon model to estimate the thermal conductivity of the beam, we find that the residual power needed to explain the observed saturation temperature is in the order of 100 aW. Such a power could be related with microwave dissipation in the ferromagnetic sphere, due to either Josephson radiation from the SQUID junctions or microwave thermal radiation from room temperature wiring. By means of careful engineering of microwave filters, thus far not implemented, it should be possible to suppress the residual power by several orders of magnitude, and to cool the cantilever to even lower temperatures.

From the cantilever noise temperature, the force noise spectral density can be calculated using the fluctuation-dissipation theorem [63,64]:

$$S_F = 4k_BT_Nc$$  \hspace{1cm} (5.2)

where $c = k/(2\pi f_0Q_0)$ is the cantilever damping factor. For the cantilever fundamental mode, we calculate that $c_0 = (1.7 \pm 0.1) \cdot 10^{-13}$ Ns/m. In the saturation regime, where $T_N = T_0$, we then obtain a minimum force noise of $F_{\text{min},0} = \sqrt{S_{F,0}} = (0.51 \pm 0.03)$ aN/$\sqrt{\text{Hz}}$, close to the lowest values ever reported in the literature [58]. The bandwidth in which the cantilever displacement noise is higher than or equal to the detector noise is $BW_0 = 1.1$ Hz.

Similarly, for the cantilever second vibration mode, we obtain $c_1 = (1.7 \pm 0.4) \cdot 10^{-13}$ Ns/m and $F_{\text{min},1} = (0.41 \pm 0.08)$ aN/$\sqrt{\text{Hz}}$. We cannot calculate the detection bandwidth using the same definition as for $BW_0$, since the cantilever displacement noise does not exceed the detector noise by more than a factor of 0.9. The bandwidth in which the cantilever noise is equal to half the detector noise is $BW_1 = 1$ Hz.
Next, we will discuss the displacement sensitivity of our detection scheme. The equipartition theorem [27] links the mean energy of the cantilever vibration mode to the mean squared displacement \( \langle x^2 \rangle = k_B T_N/k \). From this, we can convert the experimental SQUID white noise to an equivalent displacement noise of 9 pm/\( \sqrt{\text{Hz}} \) (see right axis of Fig. 5.1a). This value is comparable to that of other techniques used with micro- or nanomechanical resonators at cryogenic temperatures, like ultralow power interferometry [55] or QPC-based detection [60, 61]. However, the displacement sensitivity of our SQUID-based detection can still be greatly enhanced, by at least two orders of magnitude, by means of relatively straightforward modifications. On the one hand, the coupling between the magnetic particle and the detection coil can be substantially increased by reducing the size of the coil to a few microns. On the other hand, the SQUID noise can be improved by at least a factor of 2 by either using state-of-the-art conventional dc SQUIDs [65] or by implementing recently developed SQUID magnetometers based on nondissipative inductive readout, which exhibit quantum limited sensitivity [66].

Our detection scheme limits the strength of the magnetic fields used in the experiment to the critical field of the superconducting detection coil. For the Nb coils we use, this means that a maximal field of 0.2 T can be applied [67]. This does not preclude magnetic resonance imaging experiments, since the Boltzmann polarization of a spin sample at milliKelvin temperatures still exceeds that of a conventional 7 T room temperature MRI by three orders of magnitude. Also, when the sample size is decreased to the nanometer scale, statistical polarization becomes dominant, which is independent of the magnetic field [28, 30]. We note that the SQUID itself is spatially separated and shielded from the detection coil and is not affected by any externally applied magnetic field.

5.4 Conclusions and Outlook

In conclusion, we have developed a SQUID-based technique suitable for detecting the displacement of a nanomechanical resonator at ultralow temperatures. Using this technique, we have measured the thermal vibrations of a silicon cantilever at temperatures as low as 25 mK, achieving a force noise spectral density of \((0.51 \pm 0.03)\) aN/\( \sqrt{\text{Hz}} \). We believe that we can reduce the cantilever temperature further, possibly to sub-milliKelvin temperatures, by implementing straightforward modifications, in particular an improved electromagnetic shielding. The only fundamental limit appears to be the back-action noise of the SQUID amplifier. However, we estimate that this will only start playing a role at temperatures below 100 µK.

The described technique is also naturally suitable for detecting the motion of nanowire resonators, which pose even larger problems for interferometric detection, due to their low reflectivity [68]. Force sensors based on ultrathin nanowires can in principle reach an unprecedented sensitivity. Single-crystalline SiC nanowires, for example, can have damping factors as low as 4 fN\( \mu \)m/s at
room temperature [69], which would translate to a force noise spectral density lower than 0.1 aN/√Hz at 10 mK.

To illustrate the potential of such noise figures, for example in an MRFM experiment, we calculate that a force of 0.02 aN is generated by a single proton spin flipping in a field gradient of 1.7 MT/m (which is the maximum gradient achievable with a NdFeB sphere of 3 µm in diameter). With a force noise of 0.1 aN/√Hz, the single spin would be detectable in an averaging time less than 25 s. We note, however, that the force sensitivities in current MRFM experiments are limited by surface interaction fluctuations, also called non-contact friction [70–72]. We hope that the fact that our detector allows for operation at much lower temperatures can help find ways to address this issue.