

Cover Page



Universiteit Leiden



The handle <http://hdl.handle.net/1887/21743> holds various files of this Leiden University dissertation.

Author: Pannekoek, Rene

Title: Topological aspects of rational points on K3 surfaces

Issue Date: 2013-09-17

Topological aspects of rational points on K3 surfaces

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van Rector Magnificus prof. mr. C. J. J. M. Stolker,
volgens besluit van het College voor Promoties
te verdedigen op dinsdag 17 september 2013
klokke 13:45 uur
door

Rene Pannekoek

geboren te Apeldoorn
in 1981

Samenstelling van de promotiecommissie:

Promotor: prof. dr. P. Steenhagen

Copromotor: dr. R. M. van Luijk

Overige leden: prof. dr. S. J. Edixhoven

prof. dr. A. N. Skorobogatov (Imperial College)

prof. dr. J. Top (Rijksuniversiteit Groningen)

dr. R. S. de Jong

dr. A. Várilly-Alvarado (Rice University)



UNIVERSITEIT LEIDEN

What Song the Syrens sang, or what name
Achilles assumed when he hid himself among
women, although puzzling Questions are not
beyond all conjecture.
SIR THOMAS BROWNE

Contents

Introduction	ix
0.1 Diophantine geometry	ix
0.2 Topological aspects of rational points	x
0.2.1 Completions of a number field	x
0.2.2 The Hasse principle	xi
0.2.3 Density of rational points	xii
0.3 Obstructions to rational points	xiii
0.4 Rational points on surfaces	xv
0.5 Geometrically rational surfaces	xv
0.6 K3 surfaces	xvi
0.6.1 Existence of rational points	xvii
0.6.2 Brauer group and density questions	xvii
0.6.3 Elliptic fibrations on K3 surfaces	xvii
0.6.4 Failure of weak approximation on K3 surfaces	xviii
0.7 An open question about K3 surfaces	xix
0.8 Contents of this thesis	xix
1 Elliptic curves over p-adic fields	1
1.1 Introduction	1
1.2 Preliminaries on Weierstrass curves	2
1.3 Extensions of topological abelian groups	5
1.3.1 The profinite topology	6
1.3.2 The extension problem	7
1.4 Weierstrass curves with additive reduction	12
1.5 Proof of the main theorem	15
1.5.1 The case $p = 2$	16
1.5.2 The case $p = 3$	17
1.5.3 The case $p = 5$	18
1.5.4 The case $p = 7$	19

1.5.5	The proof	19
1.6	Examples	20
2	Density results for quartic surfaces	23
2.1	Some open subsets	24
2.1.1	Outline of the rest of the chapter	25
2.2	Elliptic fibrations	26
2.2.1	The level of a point on a Weierstrass curve	27
2.3	Weierstrass models for the fibres	27
2.3.1	The group structure on the fibres	30
2.3.2	The bad fibres	32
2.4	Using elliptic fibrations to prove density	35
2.4.1	One elliptic fibration	35
2.4.2	Two elliptic fibrations	36
2.5	Density in $\mathcal{C}_{c,1}$	37
2.6	Density in \mathcal{A}_c	40
2.7	Density in $\mathcal{B}_{c,n}$ for all n and in $\mathcal{C}'_{c,n}$ for $n \geq 2$	43
2.8	Proof of the main theorem	47
3	Density results for Kummer surfaces	49
3.1	Birational invariance of density results	50
3.2	Procylic and topologically cyclic groups	52
3.3	Elliptic curves with good twists	55
3.3.1	Notation and definitions	55
3.3.2	Partition of the rational points of a Kummer surface	56
3.3.3	Elliptic curves with good twists	57
3.3.4	From good twists to density results	57
3.3.5	A partial converse to Theorem 3.20	58
3.4	Density results for Kummer surfaces	59
3.4.1	Topologically cyclic groups and density results	60
3.4.2	Proof of Theorems 3.1–3.2	63
3.5	Large product topologies	66
3.6	Proof of Theorem 3.4	71
3.7	Proof of Theorem 3.5	72
4	Refinements and computations	75
4.1	Introduction	75
4.1.1	Goal of this chapter	75
4.1.2	Computer calculations	76

4.2	Definitions	76
4.2.1	Mestre's construction	77
4.2.2	An affine model for C	78
4.3	Creating good twists	79
4.4	Properties of the curve C	80
4.5	Existence criteria for Mestre points	86
4.5.1	Assumptions and definitions	86
4.5.2	The case where p does not divide $\#\mathcal{E}(\mathbb{F}_p)$	87
4.5.3	The case of anomalous reduction	88
4.5.4	Good points over ramified twists	94
4.6	Existence criteria for good twists	96
4.6.1	Unramified twists	96
4.6.2	Ramified twists	99
4.7	A computer experiment	99
4.7.1	Results of the experiment	101
4.8	sage code	102
4.8.1	Looking for two-element sets of generators	103
4.8.2	Finding pairs in the image of $\mathcal{C}(\mathbb{F}_p)$	104
4.8.3	The criteria involving anomalous reduction	105
4.8.4	Wrapper code	107
5	Descent on superelliptic curves	111
5.1	Definitions and statement of results	111
5.2	Properties of C and J	112
5.3	Relating certain divisors on C	113
5.4	The homomorphism $(x - T)$	114
5.4.1	Descent	115
5.4.2	Some values of $(x - T)$	116
5.5	The image of $(x - T)$	118
5.6	An algebraic lemma	119
5.7	Proof of the main theorem	120
	Bibliography	123
	Samenvatting	127
	Dankwoord	137
	Curriculum vitæ	139

