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4 Muon event rate from single GRBs

Once we know how to produce gamma ray-induced muons in the atmosphere and how they lose their energy in seawater, we are now in the position to calculate the muon yield both on the surface of the sea and at detector level.

4.1 Muon flux from a fictive test source

I first calculate muons produced from a fictive, unattenuated test source with fluence \( f_\gamma = 10^{-1} \text{ TeV}^{-1} \text{ km}^{-2} \text{ s}^{-1} \) at 1 TeV. The source is a point source with negligible diameter, assumed to be located at zenith distance \( \theta = 30^\circ \). The muon flux is calculated for three spectral indices \( b = (0.6, 1, 1.6) \) and cutoff energy at \( \epsilon_{\text{max}} = 300 \text{ TeV} \). For the background estimation, the opening angle of the search cone is taken to be \( \theta_{\text{cone}} = 1^\circ \). The results are shown in Figure 4.1 and compared to a background of cosmic ray-induced muons flux for the same zenith distance.

These results are reasonably consistent with the results of Halzen, Kappes & Ó Murchadha (2009). We can see that the dominant channel of muon-production at low energies is by pion decay. However the number of muons that can be created from this way goes down with photon energy. At high energies, because the cross-section of the muon-pair production goes up with photon energy before reaching saturation point at \( \epsilon_\gamma \gtrsim 10 \text{ TeV} \), the dominant muon production mechanism is direct-pair production.

A comparison is also made using CORSIKA. Simulations are performed for fictive sources with the same zenith distances and spectral indices as in the previous calculation, but with cutoff energy at \( \epsilon_{\text{max}} = 1000 \text{ TeV} \). The photon spectrum is normalized so that the fluence will be \( f_\gamma = 10^{-1} \text{ TeV}^{-1} \text{ km}^{-2} \text{ s}^{-1} \) at 1 TeV. The results of the simulations and the analytical calculations for the same parameters are shown in Figure 4.2.

For \( b = 1 \), there is a good agreement between CORSIKA and the analytical calculation. However for \( b = 0.6 \), the CORSIKA results are systematically lower while the shape of the muon spectrum is
consistent. For $b = 1.6$, the difference at lower muon energy is even more pronounced.

These systematic differences however appear only at sub-TeV energies. At TeV energies the shape of the muon spectrum is reasonably in agreement, barring the fluctuations caused by low statistics at very-high energy energies.

4.2 Muon flux from single GRB

Confident with the consistency of the calculation, I proceed by calculating the muon flux for single GRB events located at different redshifts. Using Equation 2.62, the photon flux arriving at the top of the atmosphere from GRBs with spectral indices $b = (0.5, 1, 1.25, 1.5)$, redshifts $z = (0.05, 0.1, 0.2, 0.5)$, and zenith distances $\cos \theta = (0.5, 1)$ can be determined. A typical GRB power spectrum measured by BATSE is $b \simeq 1.25$ (Preece et al., 2000), however measurement inconsistencies have been reported and thus the shape of the spectral index at high energy is still debatable and might not be in the form of a simple power law (see e.g. Kaneko et al. 2008 and González et al. 2003). Until this debate is clarified, it is reasonable to assume a soft spectrum with index $b \simeq 1$. The other spectral indices, $b = 0.5$ and $b = 1.5$ which corresponds respectively to a harder and softer spectrum, while not entirely impossible nevertheless have a small possibility

![Figure 4.1: The $\nu_f$ spectrum of a fictive, unattenuated test source with fluence $f_\gamma = 10^{-1}$ TeV$^{-1}$ km$^{-2}$ s$^{-1}$ at 1 TeV, for photon spectral indices $b = (0.6, 1, 1.6)$, photon energy cutoff $\epsilon_{\text{max}} = 300$ TeV, and zenith distance $\theta = 30^\circ$. The spectrum is decomposed into its major contributing components: Pion decay and direct pair production. For a comparison, the spectrum of cosmic ray-induced muons for the same zenith distance is also shown, (see Equation 3.30). The search cone has an opening angle of 1°. The result is largely consistent with that of Halzen, Kappes & Ó Mur Chadha (2009).](image-url)
of occurring and is thus also considered to study their possibility of observing the muon signal.

Throughout the calculation, the values $\Delta t_* = 10$ s, $\epsilon_{\text{bk}*} = (b - a)\epsilon_{\text{p}*}/(1 - a) = (b - 1)400$ keV, and $L_{\text{bol}*}^{\text{iso}} = 8.9 \times 10^{52}$ erg are used. These values are the average values determined from Swift observations (Butler et al., 2007; Butler, Bloom & Poznanski, 2010). After calculating the number of photons at the top of the atmosphere, the muon flux at the surface of the sea is then determined by means of Equation 3.14 or 3.16—depending on the spectral index considered—and Equation 3.28. The muon flux at the surface is then transformed to the muon flux at detector level by way of Equation 3.36, and the corresponding energy at detector level is calculated by solving Equation 3.35.

The results of this series of calculations are shown in Figure 4.3 using the attenuation model by Finke, Razzaque & Dermer (2010). One panel in each of these Figures plot the muon flux of GRBs for one spectral index. For each spectral index, the muon flux from GRBs at different redshifts is also shown and indicated with the colour scheme shown in the legend. For each redshift, an area is drawn to show their dependence on zenith distance. The the borders of the area drawn for each redshifts are the the muon flux at zenith distance $\theta = 0$ (solid lines) and at $\theta = 60^\circ$ (dashed lines). Anything in between those two lines are then the amount of sig-
nals from any zenith distance between the borders. A background flux due to cosmic ray induced-muons calculated from Equation 3.30 is also shown for the same limit of zenith distances, indicated by the black area. The search cone (or the opening angle) is taken to be 1°. The same calculations for other attenuation models were made, but upon inspection of the numbers, results indicate that the magnitude of attenuation does not differ much for nearby universe, i.e. $z \lesssim 0.2$. Hence here only results calculated using the calculation by Finke, Razzaque & Dermer (2010) are shown.

The results shown in Figure 4.3 indicate that the number of muons reaching the detector depends heavily on the GRB’s distance from us and its power spectrum. The redshift is an important factor because it determines the number of photons that survives all the way from the GRB to the top of the atmosphere, and the power spectrum determines the number of photons produced in
the GRB.

The muon spectrum is then integrated to obtain a muon event rate with energies higher than $\epsilon_{\mu,\text{detector}}$:

$$N_\mu(>\epsilon_{\mu,\text{detector}}) = \int_{\epsilon_{\mu,\text{detector}}}^{\infty} d\epsilon_{\mu} \frac{dN_\mu}{d\epsilon_{\mu}}$$  \hspace{1cm} (4.1)

The result of this integration is shown in Figure 4.4, using the attenuation model by Finke, Razzaque & Dermer (2010). This result can give us an idea of how many muon events per unit area per unit time we can expect from any GRB event with the given power spectrum, redshift, and zenith distance.

To explore further the effect of distance on the muon event rate at the detector, in Figure 4.5 the event rate of muons with energies higher than 0.1 TeV per unit area per unit time, $N_\mu(\epsilon_{\mu,\text{detector}} > 0.1 \text{ TeV})$, is plotted as a function of redshift. The black horizontal
lines are the background rate from cosmic ray-induced muons at zenith distances $\theta = [0^\circ, 60^\circ]$.

Figure 4.5 tells us the minimum redshift and maximum zenith distance to observe, for example, at least one muon event per kilometer square per second. For example, a GRB event with power spectrum $b = 1.25$ that occurs at the zenith must have a redshift of $z \lesssim 0.07$ if we want to observe at least one muon per kilometer square per seconds. The number of muons produced from a photon spectrum with $b = 0.5$ and those from $b = 1.5$ exhibit a large deviation, ranging from $N_\mu \sim 1$ to $N_\mu \sim 10^4$ km$^{-2}$ s$^{-1}$. This is because a photon flux with a hard spectrum can produce electromagnetic showers that grow in the atmosphere, while fluxes with softer spectrum produce showers that instead dissipate in the atmosphere.

The number of detectable muons depends also on the size of
the detector. ANTARES is projected to have an effective muon area of $A_{\text{eff}}^\mu \sim 10^{-2}$ km$^2$ while IceCube is expected to have an area of $A_{\text{eff}}^\mu \sim 1$ km$^2$ (Halzen, Kappes & Ó Murchadha, 2009). In Figure 4.6, I calculate the total number of detectable muons during the whole duration of the burst for four different detector sizes. The downgoing muon effective areas considered are $A_{\text{eff}}^\mu = (10^{-3},10^{-2},0.1,1)$ km$^2$, which are assumed to be constant with respect to the muon energy.

With a larger detector we can see farther GRBs, up to $z \sim 0.3$ for $b = 0.5$. Using a detector with the size of ANTARES, however, one can only detect at least one muon from GRBs at a redshift up to $z \sim 0.2$ for the same spectral index.

Since we know the number of signal and noise events in our detector, we can now calculate the expected detection significance of each individual GRB as a function of redshift. The significance

Figure 4.6: The total number of muons with energies $E_{\mu, \text{detector}} > 0.1$ TeV for GRBs from different redshifts and different spectral index as indicated by the colour code in the legend. The intrinsic burst duration $\Delta t_i$ is assumed to be 10 sec, thus making $t_{90} = (1+z)\Delta t_i$. The total muon count is calculated by assuming different detector sizes, which are assumed to be independent of energy.
$S$ is calculated according to the procedure outlined by Li & Ma (1983). The total signal $N_{\text{on}}$ is the number of muon events within a $1^\circ$ search cone and during the $t_{\text{on}} = t_90$ time interval, while the total number of background $N_{\text{off}}$ is the number of muons within the same search cone but some amount of time $t_{\text{off}}$ before the GRB took place. The statistical significance $S$ (the number of standard deviation above background) is determined using the likelihood ratio method:

$$S = \sqrt{2} \left\{ N_{\text{on}} \ln \left[ \frac{1 + \alpha}{\alpha} \left( \frac{N_{\text{on}}}{N_{\text{on}} + N_{\text{off}}} \right) \right] + N_{\text{off}} \ln \left[ (1 + \alpha) \left( \frac{N_{\text{off}}}{N_{\text{on}} + N_{\text{off}}} \right) \right] \right\}^{1/2},$$

where $\alpha$ is the ratio $\alpha \equiv t_{\text{on}}/t_{\text{off}}$. The time $t_{\text{off}}$ to measure the background rate is taken to be 2 hours, i.e. $t_{\text{off}} = 7200$ s, thus making $\alpha$ very low. The results of these calculation is shown in Figure 4.7, again for four different detector sizes.

These results correspond to the detection significance of observing GRBs with a certain power spectrum, zenith distance, and redshift. We can also use this result to determine the maximum redshift where a GRB has to occur if we want to have at least 3$\sigma$ or 5$\sigma$ detection significance. As an example, for an ANTARES-sized detector to detect a GRB signal with 5$\sigma$ significance, a GRB event at zenith must be closer than $z \lesssim 0.05$ if its power spectrum is $b = 1$.

### 4.3 Conclusions

The most important factors in detecting a possible TeV component of a GRB are the redshift, the spectral index, and the effective area of the detector. The redshift determines the number of photons that survive to the top of the atmosphere, while the hardness of the spectrum determines whether the electromagnetic spectrum grows or dissipate in the atmosphere. The dependence of these two quantities is presented in Figure 4.8. A typical GRB has a spectral index $b = 1$–1.25 (Preece et al., 2000; Kaneko et al., 2008). For an ANTARES-type telescope, a typical GRB must then be located at redshift $z \lesssim 0.05$, while a larger telescope with a muon collecting area of $A_{\mu}^{\text{eff}} = 1$ km$^2$ can see up to $z \lesssim 0.1$. 
A recent analysis of Fermi GRB data by Zhang et al. (2011) suggests that the peak of the distribution in $b$ has shifted to $b \sim 1.6$, a much steeper slope than what was suggested by previous observations. Consequently, the maximum redshift that permits a $3\sigma$ detection is lower: Redshift $z \lesssim 0.005$ for an ANTARES-type telescope and $z \lesssim 0.01$ for a $\text{km}^3$ neutrino telescope. In the analysis of Zhang et al. (2011), the peak distribution of integral index $a$ is $a \sim -0.1$, which is not significantly different with previous results.

The limitation pertaining to distance proves to be a great hindrance to the detection of TeV $\gamma$-rays from GRBs, as there are not many GRBs with known redshift that took place at so close distance. Recent analysis of 425 Swift GRBs suggests that the redshift distribution of GRBs is peaked at $z \sim 1$ (Butler, Bloom & Poznanski, 2010). Within this data set, there are 144 GRBs with known...
redshift and 3 of them have $z \leq 0.15$. This corresponds roughly to a fraction of $P(z \leq 0.15) \sim 7 \times 10^{-3}$.

From these results we can conclude that a role of neutrino-telescopes as a gamma-ray telescope can only be played-out restrictively to the nearest GRB sources. As nearby GRBs tend to belong to a different population (i.e. short GRB) than the ones farther away, other considerations must also be taken in view of the different luminosity and burst duration of this population.

The rate of muon signals calculated in this Chapter does not yet include the detection efficiency of the detector. To understand this effect a Monte Carlo simulation of the detector response to the muon signals must be performed. This will be discussed in Chapter 7.