The handle http://hdl.handle.net/1887/20680 holds various files of this Leiden University dissertation.

Author: Astraatmadja, Tri Laksmana
Title: Starlight beneath the waves : in search of TeV photon emission from Gamma-Ray Bursts with the ANTARES Neutrino Telescope
Issue Date: 2013-03-26
High-energy $\gamma$-rays produce muons when they interact with the Earth’s atmosphere. These muons will then traverse down to the bottom of the sea, producing Čerenkov light that can be detected by the detector array. This idea of detecting $\gamma$-induced showers by detecting the produced muons has been around for a long time. However, early calculations performed in the 1960s seem to indicate that $\gamma$-induced showers are muon-poor, having only less than 10% the muon content of proton-induced showers (Stanev, Gaisser & Halzen, 1985). These calculations are contradicted when muons were firmly detected at underground detectors, coming from the direction of Cygnus X-3 (e.g. Marshak et al. 1985). Despite the low rates and weak signals, these detections raised the interest to build large-area detectors that can detect high-energy muons and thus operate as $\gamma$-ray observatory. Stanev, Vankov & Halzen (1985) then identify two channels in which muons can be produced in $\gamma$ showers: photoproduction and direct muon-pair production. In photoproduction, muons are produced from the (semi)leptonic decay of pions or kaons produced by the interaction of high-energy photons with the atomic nucleus of the atmosphere. This is the most important channel to produce muons in the GeV regime. In direct muon-pair production, muons are created directly via the channel $\gamma + Z \rightarrow Z + \mu^+ + \mu^-$, in which Z is a nucleus of the atmosphere. Whereas muon production through photoproduction dies away with increasing energy, the cross section for muon-pair production increases with energy and thus muon-pair production is the dominant muon producing channel in the TeV regime.

In the following subsections we will describe the necessary formulation to calculate the muon flux generated in gamma-induced showers. For convenience, all units of length are converted into radiation lengths in the air $\lambda_{\text{rad}}$, which is taken to be 37.1 g cm$^{-2}$.
3.1 The cascade equation: Approximation A

High-energy photons interact with atoms in the atmosphere and initiate electromagnetic showers of particles that will cascade on their way through the atmosphere. Through materialization or Compton collision, pairs of electron-positron will be produced, which in turn emit additional photons by way of bremsstrahlung. At each step the number of particles increases but their average energy decreases (Rossi & Greisen, 1941). Nevertheless these secondary photons can also produce muons that can be detected by the detector array, and thus it is important to calculate the total number of photons produced in such a photon shower.

This problem of counting particles produced in electromagnetic showers can be solved if we consider only radiation phenomena and electron-pair production, which can be described by the asymptotic formula for complete screening. This solution is called Approximation A (Rossi & Greisen, 1941) and allows us to calculate the photon flux at some depth \( t \) in the atmosphere, given the initial photon energy spectrum. If the initial spectrum is in the form of a power law such as \( \gamma(e) \propto e^{-(b+1)} \), then the resulting spectrum at depth \( t \) is (Rossi & Greisen 1941; Halzen, Kappes & Ó Murchadha 2009)

\[
\gamma(e, t) = \gamma(e, t = 0) \frac{(\sigma_0 + \lambda_1)(\sigma_0 + \lambda_2)}{\lambda_2 - \lambda_1} \times \left[ \frac{\exp(\lambda_1 t)}{\sigma_0 + \lambda_1} - \frac{\exp(\lambda_2 t)}{\sigma_0 + \lambda_2} \right]
\]

In this Equation as well as the in the following calculations, \( t \) is the slant depth in units of radiation length (in the atmosphere, 1 radiation length equals 36.62 g cm\(^{-2}\)), \( \sigma_0 = 7/9 \) is the probability per radiation length that an electron pair production will take place (in a case of complete screening), and \( \lambda_{1,2} \) are the scale lengths factor of the shower growth and dissipation in the atmosphere. The formula to calculate \( \lambda_{1,2} \) as a function of spectral index \( b \), as well as its tabulation, is given in Rossi & Greisen (1941). For \( b < 1 \), \( \lambda_1 \) is positive while for \( b > 1 \), \( \lambda_1 \) is negative. This would mean that in the former case the shower would grow as it penetrates the atmosphere while in the latter it will dissipate. Thus for a general case of an arbitrary value of \( b \), the photon flux
can be decomposed into its spectrum at the top of the atmosphere and its scale factor at depth $t$, i.e

$$\gamma(\epsilon, t) = \gamma_0(\epsilon)\gamma_2(t).$$ (3.2)

Particularly important is the case for $b = 1$ since $\lambda_1 = 0$ and $\lambda_2 < 0$, and this would make the second exponential term in Equation 3.1 essentially zero after several radiation length, making the photon spectrum independent of depth:

$$\gamma(\epsilon_\gamma, t) = 0.567\gamma(\epsilon_\gamma, t = 0),$$ (3.3)

where the photon spectrum at the top of the atmosphere $\gamma(\epsilon_\gamma, t = 0)$ is as described in Equation 2.62.

### 3.2 Pion decay

The interaction of high-energy photons with atomic nuclei in the atmosphere can produce pions through the reaction $\gamma + N \rightarrow \pi + X$ followed by leptonic decay of pions into a positive muon and a muon neutrino, or a negative muon and a muon antineutrino:

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu),$$ (3.4)

with a probability of close to 100% to occur. The formulation to calculate the muon spectrum from this channel has been calculated using the linear cascade equation and assuming a power-law photon spectrum with spectral index $b = 1$ by Drees, Halzen & Hikasa (1989), and its generalisation to an arbitrary spectral index by Halzen, Kappes & Ó Murchadha (2009).

For the case of $b \neq 1$, this paper will closely follow that of Halzen, Kappes & Ó Murchadha (2009), which begins by an ansatz that the differential pion spectrum in the atmosphere can be factorized as

$$\pi(\epsilon, t) = \gamma(\epsilon, t = 0)\pi_2(\epsilon, t),$$ (3.5)

in which $\pi_2(\epsilon, t)$ can be split in two regimes: the high energy regime where pion interactions dominate over decay, and the low
energy regime where pion interactions are neglected. The pion spectrum at high energy is

\[
\pi^\text{HE}_2(t) = \left[ \frac{\exp(\lambda_1 t) - \exp(-t/\Lambda_\pi)}{(\sigma_0 + \lambda_1)(\lambda_1 + \frac{1}{\Lambda_\pi})} - \frac{\exp(\lambda_2 t) - \exp(-t/\Lambda_\pi)}{(\sigma_0 + \lambda_2)(\lambda_2 + \frac{1}{\Lambda_\pi})} \right] \\
\times \frac{z_{\gamma\pi}}{\Lambda_\gamma A} \frac{(\sigma_0 + \lambda_1)(\sigma_0 + \lambda_2)}{\lambda_2 - \lambda_1},
\]

(3.6)

while the spectrum at low energy is

\[
\pi^\text{LE}_2(\epsilon, t) = \frac{z_{\gamma\pi}}{\Lambda_\gamma A} \frac{(\sigma_0 + \lambda_1)(\sigma_0 + \lambda_2)}{\lambda_2 - \lambda_1} \\
\times \int_0^t dt' \left( \frac{t'}{t} \right)^\delta \left[ \frac{\exp(\lambda_1 t') - \exp(\lambda_2 t')}{\sigma_0 + \lambda_1} \right] \frac{1}{\sigma_0 + \lambda_2},
\]

(3.7)

in which \( \delta = t/d_\pi \), where \( d_\pi \) is the decay length

\[
d_\pi = \frac{\epsilon t \cos \theta}{\epsilon_\pi},
\]

(3.8)

here \( \epsilon_\pi = 115 \text{ GeV} \) is the pion decay energy constant.

The integral in Equation 3.7 can be expanded into series:

\[
\int_0^t dt' \left( \frac{t'}{t} \right)^\delta \frac{\exp(\lambda_i t')}{\sigma_0 + \lambda_i} \approx \frac{1}{\sigma_0 + \lambda_i} \sum_{j=1}^{100} \frac{\lambda_i^{j-1} t_j^j}{(j-1)! (\delta + j)}.
\]

(3.9)

In Equation 3.6 and 3.7,

\[
\Lambda_\pi = 173 \text{ g cm}^{-2} = 4.66 \text{ radiation lengths}
\]

(3.10)

is the effective pion interaction length in the atmosphere,

\[
z_{\gamma\pi} = \frac{\sigma_{\gamma\pi}}{\sigma_\gamma N} = \frac{2}{3}
\]

(3.11)

is the ratio between cross sections \( \sigma_{\gamma\pi} \) and \( \sigma_\gamma N \), and

\[
\lambda_\gamma A = 446.14 \text{ radiation lengths}
\]

(3.12)

is the interaction length of photons in atmospheric nuclei. These values are assumed to vary little for different spectral indices and energy.
Due to the unavailability of an analytical expression for both energy regime, taking a smooth transition from one regime to another is difficult. The pion spectrum at all energy regime is then

$$\pi(\epsilon, t) = \gamma(\epsilon, t = 0) \min \left[ \pi^{\text{HE}}_2(t), \pi^{\text{LE}}_2(\epsilon, t) \right].$$  (3.13)

The muon flux at the surface of the Earth can then be obtained by using standard 2-body decay kinematics, assuming no muon decay and energy loss in the atmosphere:

$$\frac{dN_\mu}{d\epsilon_\mu} = \int_0^{t_{\max}} dt B_{\mu \pi} \int_{\epsilon'}^{\epsilon/\gamma} \frac{d\epsilon'}{(1-r)e'} \frac{\pi(\epsilon', t)}{d(\pi(t))},$$  (3.14)

in which $r = (m_\mu/m_\pi)^2$ and $B_{\mu \pi} = 1$ is the number of muons produced for each decaying pion. The maximum depth $t_{\max}$ is determined using

$$t_{\max} = \lambda_e^e - \ln \left[ \frac{\epsilon_{\max}(\langle x \rangle_{\gamma \rightarrow \mu})}{\epsilon} \right],$$  (3.15)

where $\lambda_e^e = 9/7$ is the electromagnetic cascade length and $\langle x \rangle_{\gamma \rightarrow \mu} = 0.25$ is the fraction of $\gamma$-ray energy that goes into the final muon for the case of pion decays.

For the special case of $b = 1$, we calculate the muon spectrum using the formulation by Drees, Halzen & Hikasa (1989):

$$\frac{dN_\mu}{d\epsilon_\mu} = \gamma(\epsilon_\mu, t = 0) \frac{\Lambda_{\pi}}{\lambda_{\gamma A}} z_{\gamma \pi} \frac{L_{\gamma}}{1 + (L_{\gamma}/H_{\gamma}) \epsilon_\mu \epsilon_{\pi} \cos \theta},$$  (3.16)

where

$$L_{\gamma} = \frac{1-r^2}{2(1-r)} t_{\max} \Lambda_{\pi}, \quad H_{\gamma} = \frac{1-r^3}{3(1-r)} \left[ 1 + \ln \frac{t_{\max}}{\Lambda_{\pi}} \right].$$  (3.17)

The constant terms $(\Lambda_{\pi}, z_{\gamma \pi}, \lambda_{\gamma A})$ in the Equations above are the same as in Equations 3.10–3.12

3.3 Direct muon-pair production

The Feynman diagram for direct lepton-pair production $\gamma + N \rightarrow N + l^+ + l^-$ is pictured in Figure 3.1. This reaction occurs when an impacting photon interacts with a photon within the

![Figure 3.1: Feynman diagram for lepton-pair production in the presence of a nucleus N](image)
electric field of a nucleus, producing a pair of leptons. The second photon is necessary to maintain the conservation of 4-momentum, transferring the required momentum from the nucleus. Lepton-pair production is related to bremsstrahlung by a substitution rule and the calculation of the cross section can be done if we know how to calculate bremsstrahlung by electrons (Tsai, 1974). For the interaction of a photon with nuclear electrons to produce muon-pair, the photon energy threshold must then be

\[ \epsilon_{\text{th}} = \frac{2m_\mu}{m_e} (m_\mu + m_e) \simeq 43.9 \text{ GeV}, \quad (3.18) \]

where \( m_e \) is the electron mass and \( m_\mu \) is the muon mass.

To calculate an approximate formula of muon-pair production, what is usually done is taking the Bethe-Heitler result for electron-pair production (Bethe & Heitler, 1934) and substitute the electron mass with that of muon. This generalization would not be correct, however, because the atomic form factor involved in the calculation must be integrated over the transferred momentum in which the upper limit is approximately the mass of the lepton involved (Halzen, Kappes & Ó Murchadha, 2009).

We will now discuss the necessary calculations to obtain the accurate formula for the cross section of muon-pair production.

The impacting photon energy will be fully shared by the resulting muon-pair according to

\[ \epsilon_\gamma = \epsilon^+_{\mu} + \epsilon^-_{\mu}, \quad (3.19) \]

or in terms of fraction of photon energy:

\[ x_+ = \frac{\epsilon^+_{\mu}}{\epsilon_\gamma}, \quad x_- = \frac{\epsilon^-_{\mu}}{\epsilon_\gamma}, \quad x_+ + x_- = 1. \quad (3.20) \]

To take into account the atomic and nuclear form factors, we need the differential cross section equation as a function of the momentum transfer. Since this work concerns very high-energy photons, we can use the ultrarelativistic approximation written as (Bethe & Heitler, 1934)

\[ \frac{d\sigma}{dx_+} = 4\alpha Z^2 \left( r_0 \frac{m_e}{m_\mu} \right)^2 \left[ \left( x_+^2 + x_-^2 \right) \Phi_1(\delta) + \frac{2}{3} x_+ x_- \Phi_2(\delta) \right], \]
where $\alpha$ is the fine-structure constant, $Z$ is the charge of the nucleus—
for the Earth’s atmosphere $Z = 7.37$ (Rossi, 1952), $r_0$ is the classical
electron radius, and $\delta$ is the screening parameter equal to the
necessary minimum momentum transfer from the nucleus:

$$\delta \simeq q_{\text{min}} = \frac{m_\mu^2}{2\epsilon_\gamma x_+ x_-}.$$  \hfill (3.22)

The functions $\Phi_{1,2}$ are integrals of form factors over transferred
momentum $q$. Whereas electron-pair production involves only
the atomic form factors, in the case of muon-pair production it is
also necessary to consider the nuclear form factors since the
momentum involved is much larger than the inverse square of the
atomic radius (Tsai, 1974). The functions $\Phi_{1,2}$ would then be

$$
\Phi_{1,2}(\delta) = \int_{\delta}^{q_{\text{max}}} dq \frac{dq}{q^2} [F_n(q) - F_a(q)]^2 \psi_{1,2}(q, \delta),
$$  \hfill (3.23)

where $F_n$ and $F_a$ are respectively the nuclear and atomic form
factors and $\psi_{1,2}$ are the wave functions of the nucleus.

Equation 3.23 has been solved with several assumptions. We take
the solution of Kelner, Kokoulin & Petrukhin (1995) in which
a single function $\Phi(\delta) = \Phi_1 = \Phi_2$ is used for the case of complete
screening. By taking the effects of complete screening into
account we consider the fact that atoms are essentially neutral at
large distance. This is because the electric charge of the nucleus get “screened”
by the atomic electrons, i.e. their field are canceled by opposite electric charge of the atomic electrons, reducing
the effective charge according to distance and thus limiting the maximum
distance at which photons can still interact.

The contribution from inelastic form factors is also considered.
This must also be taken into account since muon bremsstrahlung
occurs on electrons bound in the atom and not on free electrons
(Kelner, Kokoulin & Petrukhin, 1995).

Having considered both elastic and inelastic form factors, Equation
3.21 then becomes

$$
\frac{d\sigma}{dx} (x, \epsilon_\gamma) = 4\alpha Z^2 \left( \frac{m_e}{m_\mu} \right)^2 \left[ 1 - \frac{4}{3} x(1 - x) \right] \left[ \Phi_{\text{el}}(\delta) + \frac{1}{Z} \Phi_{\text{in}}(\delta) \right].
$$
The elastic contribution $\Phi_{el}(\delta)$ is in the form of

$$\Phi_{el}(\delta) = \ln \left[ \Phi_{\infty} \frac{1 + (D_n e^{1/2} - 2)\delta/m_\mu}{1 + BZ^{-1/3}e^{1/2}\delta/m_e} \right],$$  \hspace{1cm} (3.25)$$

where

$$\Phi_{\infty} = \frac{BZ^{-1/3} m_\mu}{m_e}, \quad \delta = \frac{m_\mu^2}{2\epsilon_\gamma x(1-x)}, \quad e^{1/2} = 1.6187\ldots$$

$$B = 202.4 \quad D_n = 1.49 \quad \text{for Hydrogen, and}$$
$$B = 183 \quad D_n = 1.54A^{0.27} \quad \text{otherwise.} \hspace{1cm} (3.26)$$

Here $A$ is the atomic number of the nuclei involved. For our case of the Earth’s atmosphere, $A = 14.78$ (Rossi, 1952).

The inelastic contribution $\Phi_{in}(\delta)$ is

$$\Phi_{in} = \ln \left[ \frac{m_\mu/\delta}{m_\mu/\delta + e^{1/2}} \right] - \ln \left[ 1 + \frac{1}{B'Z^{-2/3}e^{1/2}\delta/m_e} \right], \hspace{1cm} (3.27)$$

where $B' = 1429$. We can see that the differential cross section is symmetric in $x_+$ and $x_-$, thus we can write

$$x_+x_- = x - x^2,$$

where $x$ substitutes either $x_+$ or $x_-$ and the other becomes $(1-x)$.

In Figure 3.2 Equation 3.24 for various values of photon energy $\epsilon_\gamma$ is shown. We can see that due to the “screening” effect the cross section does not increase indefinitely but saturates as $\epsilon_\gamma$ increases. I integrate the differential cross section over $x$ to obtain the total cross section as a function of photon energy and the result is shown in Figure 3.3. In the figure it is shown that saturation of the cross section occurs when the impacting photon energy $\epsilon_\gamma \approx 10$ TeV.

Using the cascade equation, we can calculate the muon-pair flux at sea level:

$$\frac{dN_\mu}{d\epsilon_\mu} = 2\lambda_{rad} \frac{N_A}{A} \gamma_0(\epsilon_\mu) \int_0^1 dx x^5 d\sigma \left( x, \frac{\epsilon_\mu}{x} \right) \int_0^{t_{max}} dt \gamma_2(t, b),$$  \hspace{1cm} (3.28)$$

where $N_A$ is the Avogadro number.
Figure 3.2: Differential cross section of muon-pair production (Equation 3.24) in the Earth’s atmosphere for various values of impacting photon energy $\epsilon_\gamma$, as a function of $x = \epsilon_\mu/\epsilon_\gamma$ which is the ratio between the resulting muon energy and the photon energy. The atomic and mass number of the atmosphere is taken to be $(A, Z) = (14.78, 7.37)$. 

\[ d\sigma/dx \]
3.4 Other channels of muon production

A \( \gamma \)-shower can also produce kaons and the hadronic decay of kaons can produce a positive muon and a muon neutrino or a negative muon and a muon antineutrino:

\[
K^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu).
\]  

(3.29)

This reaction has only \( \sim 63.5\% \) chance of occurring (Gaisser, 1990). Furthermore, results from Halzen, Kappes & Ó Murchadha (2009) showed that the muon yield from kaon decays and other channels involving kaons can be neglected.

Positrons produced in \( \gamma \)-showers can also produce pairs of muon by interaction with an atomic electron through reaction \( e^+e^- \rightarrow \mu^+ + \mu^- \). However, cross section for this reaction is very small and peaked at \( \sim 61 \text{ GeV} \) and falls rapidly with energy and is essentially zero for \( \epsilon_\mu \gtrsim 700 \text{ GeV} \) (Halzen, Kappes & Ó Murchadha, 2009). Thus this production channel can also be neglected altogether.

3.5 Cosmic ray-induced muon background

In order to calculate the detection significance of photon-induced muons, we need to know the amount of the background in our observation. In our case of photon-induced muons detection, the
background consists of cosmic-ray induced muons. These muons are produced mainly through leptonic decay of pions, which is essentially the same channel discussed in Section 3.2. Leptonic decay of Kaons is also another channel of muon production albeit it is less important.

The energy spectrum of cosmic-ray induced muons, as a function energy and zenith distance, has already been parametrized by Gaisser (1990) as

\[
\frac{dN_{\mu}}{d\varepsilon_{\mu}} \approx 0.14 \varepsilon_{\mu}^{-2.7} \left[ 1 + \frac{1}{1.1 \varepsilon_{\mu} \cos \theta / 115 \text{GeV}} + \frac{0.054}{1 + \varepsilon_{\mu} / 850 \text{GeV}} \right] \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \tag{3.30}
\]

This parametrization overestimates the actual measured muon flux for energies below 10 GeV because at that energy regime muon decay and muon energy loss become important factors (see Figure 6.1 in Gaisser 1990). However, this will not be our concern since this is far below the energy regime we are interested in, and Equation 3.30 fits perfectly well for high-energy regime. This equation estimates the muon flux at sea level, thus if we want to estimate the muon background at detector we have to apply the appropriate muon energy loss formula for seawater. We will discuss this later in Section 3.6.

### 3.6 Passage of muons through seawater

Upon traversing a medium, energetic muons lose their energy through ionization and radiative processes. This energy loss can be treated by taking the standard formula to calculate the average energy loss (Barrett et al., 1952)

\[
- \frac{d\varepsilon}{dx} = a(\varepsilon) + b(\varepsilon) \varepsilon, \tag{3.31}
\]

in which \(a(\varepsilon)\) is the ionization contribution of the energy loss, while \(b(\varepsilon) = b_p(\varepsilon) + b_t(\varepsilon) + b_n(\varepsilon)\) is the radiative contribution consisting of \(e^+e^-\) pair production \(b_p\), bremsstrahlung \(b_b\), and photonuclear interaction \(b_n\).
Here I take the approach of Klimushin, Bugaev & Sokalski (2001) by splitting \( a(\epsilon) \) into two separate processes, \( a(\epsilon) = a_c(\epsilon) + a_e(\epsilon) \), where \( a_c \) is the classical ionization process sufficiently described by the “Bethe” equation (Nakamura & Particle Data Group, 2010) and \( a_e \) is the \( \epsilon \) diagrams for bremsstrahlung treated as part of an ionization process. \( a_c \) can thus approximated by

\[
a_c(\epsilon) = a_{c0} + a_{c1} \ln \left( \frac{W_{\text{max}}}{m_{\mu}} \right), \quad W_{\text{max}} = \frac{\epsilon}{1 + \frac{m_{\mu}^2}{2m_{\epsilon}}} \quad (3.32)
\]

in which \( W_{\text{max}} \) is the maximum transferable energy to the electron and \( m_{\mu,\epsilon} \) are respectively the masses of muon and electron. The coefficients, in units of \( (10^{-6} \text{ TeV cm}^2 \text{ g}^{-1}) \), are \((a_{c0}, a_{c1}) = (2.106, 0.0950)\) for \( \epsilon \leq 45 \text{ GeV} \) and \((a_{c0}, a_{c1}) = (2.163, 0.0853)\) for \( \epsilon > 45 \text{ GeV} \). For \( a_e \), a polynomial approximation is used:

\[
a_e(\epsilon) = 3.54 + 3.785 \ln \epsilon + 1.15 \ln^2 \epsilon + 0.0615 \ln^3 \epsilon \times 10^{-9} \text{ TeV cm}^2 \text{ g}^{-1}, \quad (3.33)
\]

where \( \epsilon \) is in units of GeV.

The terms of \( b \) are parametrized in a polynomial function in the form

\[
b_i(\epsilon) = \sum_{j=0}^{4} b_{ij} \ln^i \epsilon, \quad \text{where } i = p, b, n. \quad (3.34)
\]

Here the energy input \( \epsilon \) is also in units of GeV. The values of coefficients for \( b_{ij} \) is already calculated by Klimushin, Bugaev & Sokalski (2001) and is tabulated in their Table II. These formulations of energy losses are expected to still valid for \( \epsilon_{\text{detector}} = 30 \text{ GeV} - 5 \text{ TeV} \) and slant depth \((3 - 12) \text{ km}\) with errors up to \(\pm (6 - 8)\% \) (Klimushin, Bugaev & Sokalski, 2001).

Taking into account these contributions, the total muon energy loss in seawater as a function of energy is shown in Figure 3.4. In this figure we can see that at high energies radiative processes are more important than ionization. The critical energy at which the energy loss from ionization and radiative processes are equal can be calculated by solving \( \epsilon_{\mu c} = a(\epsilon_{\mu c}) / b(\epsilon_{\mu c}) \). In the case of seawater this is \( \epsilon_{\mu c} \sim 590 \text{ GeV} \). Below this critical energy the dominant process is ionization while above this limit the radiative processes starts to dominate.
Figure 3.4: The muon energy loss in seawater as a function of energy, calculated from Equations 3.32 to 3.34. The total energy loss (solid line) is decomposed into contributions from different processes, indicated in the legend. This Figure is made using the values of Klimushin, Bugaev & Sokalski (2001).

Figure 3.5: The muon energy loss by passing a layer of seawater with vertical depth $d = 2475$ m is pictured here in the form of muon energy at the surface of the sea $\epsilon_{\text{surface}}$ as a function of muon energy at the detector level $\epsilon_{\text{detector}}$. We plot the energy loss for different zenith distance $\theta$, thus the path length is $R = d / \cos \theta$. 

\[
d\epsilon/dx_{\mu} \left[\text{TeV cm}^2\text{ g}^{-1}\right] \\
\begin{array}{c}
\text{Ionization} \\
\text{Pair production} \\
\text{Bremsstrahlung} \\
\text{Photonuclear} \\
\text{Total energy loss}
\end{array}
\] 

\[
\begin{array}{c}
10^{-2} \\
10^{-3} \\
10^{-4} \\
10^{-5} \\
10^{-6} \\
10^{-7}
\end{array}
\]

\[
\begin{array}{c}
10^{-3} \\
10^{-2} \\
10^{-1} \\
10 \\
10^2 \\
10^3
\end{array}
\]

\begin{align*}
\text{Energy [TeV]} \\
\epsilon_{\text{surface}} [\text{TeV}] \\
\epsilon_{\text{detector}} [\text{TeV}]
\end{align*}

$\theta = 84.96^\circ$ 
$\theta = 80^\circ$ 
$\theta = 75^\circ$ 
$\theta = 70^\circ$ 
$\theta = 60^\circ$ 
$\theta = 45^\circ$ 
$\theta = 30^\circ$ 
$\theta = 0^\circ$
If we integrate Equation 3.31 we can obtain the integral equation
\[
\int_{\epsilon_{\text{surface}}}^{\epsilon_{\text{detector}}} \frac{d\epsilon}{a(\epsilon) + b(\epsilon)\epsilon} + R = 0, \tag{3.35}
\]
in which \(\epsilon_{\text{surface}}\) is the energy at the surface of the sea and \(\epsilon_{\text{detector}}\) is the energy at detector level, located at slant depth \(R = d / \cos \theta\) where \(d\) is the vertical distance of the detector and \(\theta\) is the zenith distance from which the source came. The slant depth formula assumes a plane-parallel layers of the sea which does not take into account the curvature of the Earth. This is however a good approximation for zenith distances less than \(\sim 85^\circ\), which is the range of zenith distances we are interested in.

Solving Equation 3.35, we can obtain \(\epsilon_{\text{surface}}\) if \(\epsilon_{\text{detector}}\) is the input and vice versa. I solve Equation 3.35 to obtain \(\epsilon_{\text{surface}}\) as a function of \(\epsilon_{\text{detector}}\). The result for ANTARES depth of \(d = 2475\) m below sea level is shown in Figure 3.5 for several slant depths.

The relation between \(\epsilon_{\text{surface}}\) as a function of \(\epsilon_{\text{detector}}\) is particularly useful to obtain the muon flux at detector level:
\[
\frac{dN}{d\epsilon_{\text{det}}}(\epsilon_{\text{det}}, R) = \frac{dN}{d\epsilon_{\text{sur}}} \left(\epsilon_{\text{sur}}\right) \frac{d\epsilon_{\text{sur}}}{d\epsilon_{\text{det}}} \bigg|_{\epsilon_{\text{det}}, R} \tag{3.36}
\]

With these in mind, we can now proceed to calculate the muon spectrum of a GRB based on its observed photon spectrum at the top of the atmosphere.

3.7 On the multiplicity of downgoing muons

The calculations of muon production developed in this Chapter is a time-averaged model and thus is incapable of predicting the rate of muon bundles due to the occurrence of several muons produced in a \(\gamma\)-induced shower. It is important, however, to quantify accurately the rate of downgoing muon bundles, as they can be misidentified as signals expected from \(\gamma\)-induced muons.

To this end, simulations of muon production from \(\gamma\) showers have been performed with CORSIKA (Heck et al., 1998), a program built to simulate in detail extensive air showers initiated by cosmic-ray particles, including high-energy photons. A number of showers with primary photons ranging from 1 TeV to 100 TeV
is produced (see Table 3.1 for details on the number of showers produced for each energy of the primary photon). Hadronic interactions in the atmosphere are simulated with the QGSJET model while the electromagnetic interactions are simulated with the EGS4 package. The photon source is fixed to an assumed position in the sky, with an azimuth angle of 0° (toward the North) and zenith distance of 30°.

The result of the simulation can be seen in Table 3.2 and Figure 3.6. Table 3.2 shows the rate of single muon events produced in each photon shower with given photon energy $\epsilon_\gamma$. For each primary energies, single muon rates are shown for three different muon energy threshold: No threshold at all, $\epsilon_\mu \geq 0.7$ TeV, and $\epsilon_\mu \geq 0.9$ TeV. For each threshold, two quantities are shown: The number of showers that produce at least one muon passing the energy threshold and the number of shower producing only single muons passing the energy threshold.

Figure 3.6 shows the distribution of the muon multiplicity. For each shower with given photon energy $\epsilon_\gamma$ the distribution of the

<table>
<thead>
<tr>
<th>$\epsilon_\gamma$ [TeV]</th>
<th>No threshold</th>
<th>$\epsilon_\mu \geq 0.7$ TeV</th>
<th>$\epsilon_\mu \geq 0.9$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>792740</td>
<td>529689</td>
<td>(66.82%)</td>
</tr>
<tr>
<td></td>
<td>945618</td>
<td>604308</td>
<td>(63.91%)</td>
</tr>
<tr>
<td></td>
<td>2181219</td>
<td>1203441</td>
<td>(55.17%)</td>
</tr>
<tr>
<td></td>
<td>1218344</td>
<td>311458</td>
<td>(41.98%)</td>
</tr>
<tr>
<td></td>
<td>599949</td>
<td>21334</td>
<td>(2.15%)</td>
</tr>
<tr>
<td>10</td>
<td>599983</td>
<td>121</td>
<td>(0.02%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\epsilon_\gamma$ [TeV]</th>
<th>$\epsilon_\mu \geq 0.7$ TeV</th>
<th>$\epsilon_\mu \geq 0.9$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>208</td>
<td>208</td>
</tr>
<tr>
<td>2</td>
<td>699</td>
<td>630</td>
</tr>
<tr>
<td>5</td>
<td>2135</td>
<td>1914</td>
</tr>
<tr>
<td>10</td>
<td>1602</td>
<td>1409</td>
</tr>
<tr>
<td>50</td>
<td>3990</td>
<td>3207</td>
</tr>
<tr>
<td>100</td>
<td>4698</td>
<td>3652</td>
</tr>
</tbody>
</table>
Figure 3.6: The distribution of muon multiplicity $N_µ$ at the surface of the sea. Each curve shows the fraction of $N_µ$ produced from showers with given photon-primary with energy $ε_γ$. Photons with energy $ε_γ ≳ 10$ TeV can produce large muon bundles. However, if a certain muon energy threshold is applied (middle and bottom plots), we can see that the majority of the events are single muons.
number of muons \( N_\mu \) produced in the shower, at the surface of the sea, is shown. The top plot shows the distribution of \( N_\mu \) for muons with any energy. We can see that for photon primaries with energy \( \epsilon_\gamma \lesssim 10 \text{ TeV} \), the majority of the showers produce no muons at all, with a probability of \( \sim 20\% \) producing at least one muon. At higher primary energies, there is a higher chance to produce multiple muons within a shower. However, the muons must penetrate the depth of the sea in order to be detected by the ANTARES telescope. Thus only muons with sufficiently high energy are detected. If we only count muons with energy larger than \( 700 \text{ GeV} \) (middle plot of Figure 3.6) or \( 900 \text{ GeV} \) (bottom plot of Figure 3.6), it is clear that the majority of events contain a single muon and that high-energy muon bundles are rare. Table 3.2 shows that in the photon energy range of \( 2 \text{ TeV} \leq \epsilon_\gamma \leq 10 \text{ TeV} \), at most \( \sim 11\% \) of the muons with \( \epsilon_\mu \geq 0.9 \text{ TeV} \) arrive in bundles. The rate of muon bundles is thus rather low.

If the very high energy muon bundles pass through the detector, it is still possible to reconstruct a track. At this energy, the muons will travel essentially at parallel angles and could therefore be reconstructed as a single muon track. This is due to the limited two-track resolution of the detector that hinders the abil-
ity to distinguish multiple muon tracks coming at approximately the same time (Halzen, Kappes & Ó Murchadha, 2009). From the CORSIKA simulation, we could calculate the angular separation $\psi$ of the tracks with respect to the original photon directions. In Figure 3.7 the distribution of $\psi$ is plotted for muons with any energy as well as for muons passing a certain energy threshold. Six energy thresholds are considered, ranging from 590 GeV to 10 TeV. We can see that for TeV muons, the distribution of $\psi$ is peaked at around $\sim 0.001^\circ$, which is much smaller than the angular resolution of the ANTARES detector.

Consequently, the simulations of ANTARES' sensitivity to down-going muons can then be performed by generating single muon tracks. We will discuss this simulation in Chapter 7.