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**Author:** Astraatmadja, Tri Laksmana  
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8 The point spread function and the optimization of quality cuts

In this Chapter, the analysis involving both signal and background data will be described. Rather than reproducing background data with simulations, the input is taken from ANTARES data that have been successfully reconstructed with aafit. The data used for this analysis is the data taken in 2008 with 12 lines. During this period, the lifetime of the data taking period is $6.46 \times 10^6$ s or roughly $\sim 75$ days. There are approximately $6.9 \times 10^7$ reconstructed muon events within this period.

8.1 Background data

Various maps of the muon events in local ANTARES coordinates are shown in Figure 8.1. In these maps, the celestial sphere is divided into bins of equal area using the healpix algorithm (Górski et al., 2005). It is then easy to calculate the total number events in each bins. Dividing the number of events with the total lifetime of the experiment and the size of the bin will give the background event rate in units of Hz per square degree, as indicated by the fill colour of each bin.

These maps give an overview of the background rate as a function of azimuth and zenith distance. In general the background is essentially about 1 mHz per square degree. For the unfiltered events, the background distribution is not the same in azimuthal directions. This can be attributed to the detector geometry. Events coming close to a detector line will be more likely to be reconstructed than those coming at a considerable distance from any detector line. This effect becomes less if we apply some quality cuts. At the bottom plot, only background events with $\Lambda_{\text{aafit}} \geq -6$ and $\sigma_\psi \leq 1^\circ$ are accepted. We can see that the anisotropy has diminished.

Figure 8.2 shows an azimuth-averaged muon rate as a function of zenith distance. The red line is a polynomial fit to the data, which will be useful for further calculations. We can also compare
Figure 8.1: Mollweide projections of the Altitude-Azimuth distribution of reconstructed muon events detected with the ANTARES telescope during the 12-line period in 2008 (~75 days of data taking). The top plot shows all reconstructed events without any quality cuts, while the middle and bottom plots shows all reconstructed events satisfying, respectively, loose cuts of $\Lambda \geq -6.5$ and $\sigma_{\phi} \leq 1^\circ$, and tighter cuts of $\Lambda \geq -6$ and $\sigma_{\phi} \leq 1^\circ$. For the unfiltered events, the azimuthal distribution is not really isotropic. This can be attributed to the detector geometry. This anisotropy is diminished somewhat when quality cuts are applied. These maps are made using the healpix algorithm (Górski et al., 2005) that pixelize the celestial sphere into cells of equal area and then count the number of events within each cell.
the observed rate with the expected muon rate parameterized by Gaisser (1990).

A comparison between simulated signal and observed background events in the $\sigma_{\varphi}-\Lambda_{\text{a-fit}}$ space is shown in Figure 8.3. This comparison indicates that the signal and background look rather similar, despite the different origin ($\gamma$-induced muons events versus cosmic ray induced muon events).

8.2 Weighting scheme and the point spread function

The point spread function (PSF) of downgoing muons could be determined from the simulated events. The determination of the PSF is necessary as a first step in hypothesis testing (which will be described in more detail in Chapter 9). The events that constitute the PSF, however, are results from simulations of fictitious events, which energies are generated according to simple power law. Actual GRB events, however, do not follow a simple power law. For example, at higher energy the spectrum is cut due to attenuations by extragalactic infrared background photons. The
severity of the attenuation depends on the redshift and the energy band in question.

The simulated muon events must then be weighted accordingly so that the muon spectrum reproduces the energy spectrum of a GRB event. The background events must also be weighted, since they are taken from an exposure time much longer than an actual GRB events, with a different detector configuration and varying detector conditions. The following subsection will describe the weighting scheme employed in this analysis before we move on to the modelling of the PSF.

8.2.1 Weighting scheme

For unweighted events, the normalisation of a histogram following a power law function is straightforward. For the \(j\)-th bin of the histogram, the total number of reconstructed event per unit energy would simply be

\[
\frac{dN}{dE_j} = \frac{1}{E_j \ln(10)} \frac{N_j}{\Delta \log E_j}
\]  

(8.1)
where $E_j$ is the energy at the midpoint of the $j$-th bin, $N_j$ is the total number of events in the $j$-th bin, and $\Delta \log E_j$ is the binwidth in log $E_j$.

Equation 8.1 implies that all events have the same weight. Summing up all the weights of events within the $j$-th bin would result in the actual event rate. The number of reconstructed events in the $j$-th bin is related to the total number of generated events in the same bin by way of

$$
\frac{dN_{\text{rec}}}{dE_j}(E_j, \theta, z) = \frac{dN_{\text{can}}}{dE_j}(E_j, \theta, z) \eta_\mu(E_j, \theta) A_{\text{can}}(\theta) t_{\text{exp}},
$$

(8.2)

where $dN_{\text{can}}/dE_j(E_j, \theta, z)$ is the number of generated events at the edge of the can as a function of energy $E_j$, zenith distance $\theta$, and the redshift $z$ of the GRB source, in units of TeV$^{-1}$ cm$^{-2}$ s$^{-1}$, $\eta_\mu(E_j, \theta)$ is the muon reconstruction efficiency as a function of energy and zenith distance, $A_{\text{can}}(\theta)$ is the generated can area at the given zenith distance, and $t_{\text{exp}}$ is the time exposure. Using the prescription described in Chapters 2–3, we can now determine the number of photon-induced muons at the detector. The values used to construct the GRB spectrum are the standard values shown in Table 8.1.

We can also see that the energy spectrum of the reconstructed events also relates to the weight of individual events through

$$
\frac{dN_{\text{rec}}}{dE_j}(E_j, \theta, z) = \frac{1}{E_j \ln(10) \Delta \log E_j} \sum_{i=1}^{N_j} w_{i,j},
$$

(8.3)

By combining equation 8.3 with 8.2, the modified weight for

<table>
<thead>
<tr>
<th>Par.</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>spectral index at low energy</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>spectral index at high energy</td>
<td>2</td>
</tr>
<tr>
<td>$\epsilon_{pk}$</td>
<td>the energy at which the spectrum peaks</td>
<td>400 keV</td>
</tr>
<tr>
<td>$L_{\text{bol}}^\text{iso}$</td>
<td>isotropic bolometric luminosity</td>
<td>$8.9 \times 10^{52}$ erg</td>
</tr>
<tr>
<td>$t_{\text{exp}}$</td>
<td>rest-frame burst duration</td>
<td>10 s</td>
</tr>
<tr>
<td>$d$</td>
<td>ANTARES depth</td>
<td>$2.475 \times 10^5$ cm</td>
</tr>
</tbody>
</table>

Table 8.1: The adopted values used to calculate the muon energy spectrum at can level.
the $i$-th event in the $j$-th bin would then be

$$
w_{i,j}(E_j, \theta, z) = \frac{dN_{\text{can}}}{dE_j}(E_j, \theta, z) \eta_{\mu}(E_j, \theta) A_{\text{can}}(\theta) t_{\text{exp}} E_j \Delta \log E_j \ln(10) N_j^{-1}. \tag{8.4}$$

We can see from Equation 8.4 that for a GRB source located at given $\theta$ and redshift $z$, the individual weight $w_{i,j}$ would just be a function of the energy in the $j$-th bin. Thus all events inside the same bin will have the same weight, provided the bin width is sufficiently small.

The weighting of the background events follows a different but simpler approach. They would just simply be weighted according to ratio of the time exposure and the lifetime of the data taking:

$$w_{\text{bg}} = \frac{t_{\text{exp}}}{t_{\text{lifetime}}} \tag{8.5}$$

In this, we assume a fixed detector configuration. The muon event rate as a function energy is shown in Figure 8.4, for different values of zenith distances and redshifts. In this, a standard GRB is assumed (see Table 8.1). In this Figure, the black lines correspond to the muon energy spectrum at the edge of the can, while the blue lines correspond to the spectrum of the reconstructed events. The red lines correspond to the events satisfying the quality criteria $\sigma_{\psi} \leq 1^\circ$ and $A_{\text{afit}} \geq -6$. The spectrum of the background muons—calculated using Gaisser’s (1990) parametrization—is shown as the dashed lines. We can see that given a short exposure time and small cone angle, the background is very low.

8.2.2 The point spread function

The angular distributions of the weighted signal and background events are shown in Figure 8.5 for various values of zenith distances $\theta$ and redshifts $z$. In this, the preliminary cuts are applied. The angular distribution of the events is depicted in the Lambert azimuthal equal-area projection, which project the celestial sphere onto a disk centered at a given direction in the celestial sphere. By choosing this projection, the shape of the distribution around the center of the disk, which is chosen to be
Figure 8.4: The muon event rate as a function of energy. Each event is weighted according to the scheme in Equation 8.4. The rate is calculated for different zenith distances and redshifts.
coincident with the assumed direction of the GRB, is free from distortions and thus making shape and numerical comparison between event distributions at different zenith distance straightforward. From now on the angular distribution will be referred as the point spread function (PSF).

A glance at the PSF in Figure 8.5 seems to indicate that the most interesting signal events are well-reconstructed to better than \( \sim 0.5^\circ \) from the supposed direction of the GRB. We can also see that for any GRB with redshift closer than \( z \lesssim 0.01 \), an excess of signal over background will be immediately apparent in the PSF distribution, except when the zenith distance of the source is very close to the horizon (e.g. \( \theta = 75^\circ \)).

The PSF depicted here are shown for events satisfying the preliminary quality cuts, which were chosen rather arbitrarily. In the following section we will discuss the optimization of cuts that maximize the discovery potential.

### 8.3 Optimization for discovery: The model discovery potential

Within the context of frequentist statistics, one can claim a discovery when the probability that the effect will be mistaken as a background fluctuation is very small. What is agreed as small is generally in the order of \( \alpha = 5.73 \times 10^{-7} \) (an area in a two-sided 5\( \sigma \) Gaussian distribution tails). Thus in a counting experiment using Poisson statistics, for a given \( \alpha \), we can calculate the critical number of events \( n_{\text{crit}} \) in which

\[
P(\geq n_{\text{crit}}|\mu_b) \leq \alpha = 5.73 \times 10^{-7},
\]

which is the minimum number of events that has to be detected, for a given background rate \( \mu_b \), so that the probability that the observed number of events are caused by random background fluctuation is equal of less than \( 5.73 \times 10^{-7} \). Should we also expect to detect signal events with rate \( \mu_s \), we could then calculate the probability to observe a number of events greater than \( n_{\text{crit}} \) given the expected rate \( \mu_s + \mu_b \):

\[
P(\geq n_{\text{crit}}|\mu_s + \mu_b) = 1 - \beta.
\]
Figure 8.5: The angular distribution of signal and background events for different zenith distances and redshifts. Different rows correspond to different zenith distances $\theta$ and different columns correspond to different redshifts $z$. Each plot is centered on the supposed direction of the GRB. The blue lines indicate an equally-spaced grids at $1^\circ \times 1^\circ$. The events are mapped in Lambert azimuthal equal-area projection.
The value $1 - \beta$ is the usually called the *discovery potential*. If we fix $1 - \beta$ to a certain value, e.g. $1 - \beta = 0.5$, we could then calculate the minimum signal required such that Equation 8.7 is satisfied. The value of $\mu_s$ satisfying Equation 8.7 could then be interpreted as the *least detectable signal* $\mu_{\text{ids}}$.

The ratio between this least detectable signal $\mu_{\text{ids}}$ and the expected number of signal $\mu_s$ is called the Model Discovery Potential or MDP. If we choose the cut criteria that minimize the MDP, we would then minimize the required signal that will give a probability $1 - \beta$ it would yield an observation at significance level $\alpha$.

For this minimization of the MDP, I choose two cut criteria: the maximum opening radius $\Psi$ around the supposed direction of the

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Figure 8.6: The model discovery potential (MDP) for 50% chance of making a $5\sigma$ discovery, plotted as a function of opening radius $\Psi$ and $\Lambda_{\text{safe}}$. For each point in the plot, the MDP is determined from the background contained within the radius $\Psi$ from the supposed direction of the GRB and with a minimum reconstruction likelihood $\Lambda_{\text{safe}}$. The top, middle, and bottom rows correspond respectively to zenith distances $0^\circ$, $15^\circ$, and $30^\circ$. The left, middle, and right columns correspond respectively to redshifts $0.05$, $0.02$, $0.01$, $0.005$. 

GRB and the minimal \( \Lambda_{\text{aafit}} \) value.

The result of this minimization is given in Figures 8.6 and 8.7 for \( \alpha = 5\sigma \) and \( 1 - \beta = 0.5 \). For various fictitious GRB events at 4 different redshifts and 6 different zenith distances, the MDP is calculated as a function of the opening radius \( \Psi \) and the minimum likelihood \( \Lambda_{\text{aafit}} \). At each point in the \( \Psi - \Lambda_{\text{aafit}} \) space, all events satisfying the cuts are admitted to calculate the MDP, and the point that give the minimum MDP is then marked in the Figures as a white cross.

We can see in these Figures that the position of the minimum MDP does not change significantly with redshift and changes only slightly with zenith distance. This is a good result because this
means that MDP is an unbiased estimator, i.e. the minimized true signal rate $\mu_{\text{ids}}$ is independent of the expected signal rate $\mu_s$. Table 8.2 summarizes the combination of cut values that minimizes the MDP for all zenith distances.

### 8.4 Optimization for sensitivity: The model rejection factor

A method commonly used to set an unbiased sensitivity limit is the Model Rejection Factor (MRF) technique (Hill & Rawlins, 2003). With this method the selection cut that minimizes the expected upper limit of the experiment—assuming that no true signal is present—can be determined.

Suppose we choose events satisfying a certain selection cuts, Figure 8.8: The model discovery factor (MRF) for 90% confidence limit as a function of opening radius $\Psi$ and $\Lambda_{\text{safe}}$. For each point in the plot, the MRF is determined from background events contained within the radius $\Psi$ from the supposed direction of the GRB and with a minimum reconstruction likelihood $\Lambda_{\text{safe}}$. The top, middle, and bottom rows correspond respectively to zenith distances $0^\circ$, $15^\circ$, and $30^\circ$. The left, middle, and right columns correspond respectively to redshifts $0.05$, $0.02$, $0.01$, $0.005$. 

\[ \theta = 0^\circ, z = 0.05 \quad \theta = 0^\circ, z = 0.02 \quad \theta = 0^\circ, z = 0.01 \quad \theta = 0^\circ, z = 0.005 \]

\[ \theta = 15^\circ, z = 0.05 \quad \theta = 15^\circ, z = 0.02 \quad \theta = 15^\circ, z = 0.01 \quad \theta = 15^\circ, z = 0.005 \]

\[ \theta = 30^\circ, z = 0.05 \quad \theta = 30^\circ, z = 0.02 \quad \theta = 30^\circ, z = 0.01 \quad \theta = 30^\circ, z = 0.005 \]
we then obtain an expected background rate $\mu_b$ and an expected signal rate $\mu_s$. We can then simulate the experiment and obtain a number of observed events $n_{\text{obs}}$. Given $\mu_b$ and $n_{\text{obs}}$, we can then calculate the confidence interval $\mu_r = (\mu_1, \mu_2)$ using the Feldman–Cousins unified approach (Feldman & Cousins, 1998).

Since we do not know the actual upper limit until we are looking at the data, we can determine the average upper limit that would be observed after we performed a hypothetical repetition of the experiment (assuming that the background rate is $\mu_b$ and there is no true signal) and obtain all possible outcomes of the observed number of events $n_{\text{obs}}$. This average upper limit would then be the sum of all upper limits for the possible numbers of
\[ n_{\text{obs}} \text{ weighted by their Poisson probability of occuring:} \]
\[ \bar{\mu}_a(\mu_b) = \sum_{n_{\text{obs}}=0}^{\infty} \mu_a(n_{\text{obs}}, \mu_b) \frac{n_{\text{obs}}^{n_{\text{obs}}}}{n_{\text{obs}}} \exp(-n_{\text{obs}}) \]  

(8.8)

By calculating \( \bar{\mu}_a \) after the cut are applied, we can then determine the MRF which is the quantity \( \bar{\mu}_a / \mu_s \). We could optimize the cuts by finding the combination of cut values that would minimize the MRF.

The result of the MRF minimization is shown in Figures 8.8 and 8.9 for a 90% confidence interval. For various fictitious GRB events at 4 different redshifts and 6 different zenith distances, the MRF is shown as a function of the opening radius \( \Psi \) and the minimum likelihood \( \Lambda_{\text{aafit}} \). The point that give the minimum MRF is marked in the Figures as a white cross. Table 8.2 summarizes the combination of cut values that minimizes the MRF.

The cut that minimize the MRF is looser than that of the MDP minimization. This is because we aim to optimize for sensitivity. The short exposure time practically reduces the background rate to zero. Consequently, the cuts should be looser. This is apparent especially at angles close to the horizon. At these angles the expected background rates is very low hence the search cone can be very large and the \( \Lambda_{\text{aafit}} \) cut very loose. As a result, one will accept admit more events.

<table>
<thead>
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<th>( \theta ) [( ^\circ )]</th>
<th>MDP</th>
<th>MRF</th>
<th>MRF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Psi )</td>
<td>( \Lambda_{\text{aafit}} )</td>
<td>( \Psi )</td>
</tr>
<tr>
<td>0</td>
<td>1.50</td>
<td>-6.08</td>
<td>2.66</td>
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<td>3.61</td>
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<tr>
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<td>1.34</td>
<td>-6.21</td>
<td>5.58</td>
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<tr>
<td>75</td>
<td>1.93</td>
<td>-6.25</td>
<td>12.39</td>
</tr>
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Table 8.2: The combination of cut values that minimizes the MDP and the MRF.