The handle http://hdl.handle.net/1887/20607 holds various files of this Leiden University dissertation.

**Author:** Ćubrović, Mihailo  
**Title:** Holography, Fermi surfaces and criticality  
**Issue Date:** 2013-02-27
Summary

This thesis is devoted to the physics of strongly interacting electron systems from the viewpoint of a string-theoretical paradigm known as the holographic principle. The idea is to bridge the gap between two seemingly disconnected areas: gravity and quantum fields. The arena of strongly interacting electrons is a prime example which could benefit from such a connection, for both fundamental and practical reasons. In order to understand how, let us first take a closer look at how gravity and quantum fields are related by the holographic principle.

Holography is motivated by the realization that the entropy of a black hole scales with its area. As the entropy determines the information content (and eventually the number of degrees of freedom) of a black hole, we can conclude that the information carried in a black hole can be "written" on its surface. In other words, all of its degrees of freedom are captured by a suitably defined object spanning its surface, not its volume. This has prompted 't Hooft and Susskind to conjecture that, quite generally, the dynamics in a curved spacetime, in the presence of gravity, can be equivalently thought of as a quantum field in flat spacetime with one dimension less. Finally, in 1997, Maldacena constructed an explicit example, showing that a conformal field theory (CFT – a highly symmetric field theory, invariant with respect to length rescaling at every point) is the "image" of gravity in a space with a certain specific geometry, known as Anti-de Sitter (AdS) space. The connection is in the form of a duality, meaning that the partition functions of $D$-dimensional CFT and $D+1$-dimensional AdS gravity are equal. That opens a way to calculate correlation functions, expectation values, stress tensors and other quantities on the CFT side. The list of such correspondences is known as the holographic dictionary. Importantly, AdS/CFT is a weak/strong duality, meaning that weakly coupled gravity in AdS$_{D+1}$ is dual to a strongly coupled quantum field theory. Weakly coupled gravity is just its classical limit, i.e. general
relativity, which is relatively well studied and a wealth of exactly solvable models exists. On the other hand, strongly coupled field theory is out of reach of perturbative techniques and thus poorly known.

This goes double for fermion systems to which this thesis is devoted. The core issue comes from the simple fact that fermions obey Fermi-Dirac statistics – meaning that their wave functions are antisymmetric and the density matrix of a many-fermion system will contain negative contributions. This in turn leads to the so-called ”fermion sign problem”: the partition function acquires negative contributions, which ruins its probabilistic interpretation. Therefore, the formalism of statistical mechanics (or, equivalently, Euclidean field theory) is not applicable. At weak coupling, the Landau Fermi liquid theory provides a controlled approximation scheme: the interacting system behaves as a gas of quasiparticles. A wealth of interesting systems is however outside this weakly coupled regime. The prime example is the strange metal phase of high-temperature superconductors, which shows distinctly non-Fermi liquid behavior, with its universal scaling laws such as linear resistivity scaling with temperature. It is here that we see a great opportunity to apply the AdS/CFT correspondence: it is a unique tool which provides an insight into the problem of strongly interacting fermions in a controlled way. While we are not yet able to arrive at a realistic model of any condensed-matter system, we study the universal features characterizing the holographic fermions.

We start our research by looking at the quantum-critical fermion systems. These systems have a quantum phase transition – a zero temperature transition driven by quantum, not thermal fluctuations. Quite a number of materials is conjectured to slip from a Fermi liquid to a non-Fermi liquid by passing through a quantum critical point. The gravity dual turns out to be a charged black hole in AdS space, with zero fermion density in the bulk. We study in detail the spectrum of the the system, and find gapless excitations around specific values \((E_F, k_F)\) of energy and momentum, which are clearly to be identified with the Fermi energy and Fermi momentum. We thus find holographic Fermi surfaces. Tuning the parameters of the system, we find both stable, narrow peaks corresponding to Fermi liquids, and unstable peaks with exotic and nonuniversal features such as particle-hole asymmetry, of distinctly non-Fermi liquid kind. This is in line with the expectation that the charged black hole describes a quantum critical point: it is a point from which the system might evolve either towards a Fermi or a non-Fermi liquid.
The natural step now is to see where the system flows away from the critical point, i.e. what happens when the black hole becomes unstable. The gravity picture is that of pair production in an electrostatic and gravitational field: some of the pair-produced fermions will orbit the black hole, making it unstable. The result is a novel geometry on the gravity side, and thus a novel system on the field theory side. We have dubbed this model a black hole with Dirac hair. We find that it contains only stable Fermi-liquid quasiparticles, while the unstable ones go away. After some algebra, one can obtain from the gravity side a number of results of the Fermi liquid theory. We therefore have a solid gravity dual to a Fermi liquid.

Our next goal is the exploration of the full parameter space and understanding of all possible ground states. It is found that this holographic Fermi liquid is unexpectedly robust: in the whole parameter space, the stable quasiparticles dominate the spectrum as soon as one moves away from the quantum critical (charged black hole) phase, which shows definite characteristics of a non-Fermi liquid. Somewhat unexpectedly, even in the strongly coupled setup of AdS/CFT, Fermi liquids are ubiquitous – and only disappear when quantum-critical behavior develops. It is conceivable that different, more involved gravity models would give a richer spectrum of non-Fermi liquid phases. The transition between the two phases is of the Berezhinsky-Kosterlitz-Thouless (BKT) type, i.e., of infinite order. Clearly, it has nothing to do with vortices but with a specific instability of the non-Fermi liquid (in technical terms, it manifests itself as the merger of two fixed points of the RG flow).

In conclusion, we have observed previously unknown forms of fermionic quantum criticality by employing the AdS/CFT correspondence, and obtained a proof of Fermi liquid stability from the theory of gravity. The former points to the ability of AdS/CFT to bring new developments into the field of many-body physics, while the latter is an important check, reproducing the best established result of conventional condensed-matter theory. We are still at the very beginning of holographic studies of quantum matter, but there is good reason to believe that these studies have the potential to bring entirely new results to the field.