Stellingen

Propositions belonging to the thesis

Uniform Infinite and Gibbs Causal Triangulations

by Stefan Zohren

1. For random planar rooted trees where the measure condenses on trees with a unique infinite spine one has for the spectral dimension $d_s$ the following bound, $\frac{2d_h}{1+d_h} \leq d_s \leq \frac{2d_h}{1+d_h}$, where $d_h$ is the fractal dimension and $\bar{d}_h$ is what we call the hull dimension (Chapter 2).

2. In the latter case, if the outgrowths from different vertices on the spine are i.i.d., the size $|A^i|$ of the outgrowths from a vertex $i$ on the spine has a probability generating function $\langle z^{|A^i|} \rangle = 1 - (1 - z)^{\alpha l(1 - z)}$, where $l(x)$ varies slowly at zero, and the probability that the height of the outgrowths from a vertex on the spine exceeds $n$ falls off as the inverse of $n$, then one has $d_h = 1/\alpha$ and $d_s = 2/(1 + \alpha)$ (Chapter 2).

3. The uniform measure on infinite causal triangulations (UICT) is equivalent to a size-biased critical Galton-Watson process with offspring distribution $p(n) = 2^{-n-1}$, that is the Galton-Watson process conditioned on survival at infinity (Chapter 3).

4. One can show convergence of the joint rescaled boundary length and area process of the UICT to a diffusion process and from this one can extract the quantum Hamiltonian of CDT (Chapter 3).

5. Sections of UICT can be constructed from a growth process which adds triangles to the existing triangulation according to two possible moves with certain probabilities (Chapter 4).

6. One can prove both using the branching as well as the growth process that the fractal dimension of UICT is almost surely 2 (Chapter 4).

7. One can apply the Krein-Rutman theory of positivity preserving operators to study several properties of the transfer matrix for a Ising-type model coupled to CDT to determine regions in the quadrant of parameters where the infinite-volume free energy converges (Chapter 5).

8. Probably the nerdiest way of declaring love to your fiancée is to put it into some weird propositions of a thesis.