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Chapter 1

Harmonic Duality

1.1 Harmony and its duality

“Music is in fact not without ambiguity – especially since the Renaissance – because it is at once the intellectual love of an order and a measure beyond the senses, and an affective pleasure that derives from bodily vibrations. Furthermore, it is at once the horizontal melody that endlessly develops all of its lines in extension, and the vertical harmony that establishes the inner spiritual unity or the summit, but it is impossible to know where the one ends and the other begins.” (Gilles Deleuze, *The Fold: Leibniz and the Baroque*).

“Music charms us, although its beauty only consists in the harmonies of numbers and in the reckoning of the beats or vibrations of sounding bodies, which meet at certain intervals, reckonings of which we are not conscious and which the soul nevertheless does make.” (Godfried W. Leibniz, *Principles of Nature and of Grace*, §17).

As the above quotes suggest (the first being a comment on the second) from a metaphysical viewpoint, musical harmony consists of two separate aspects. One is not directly perceived, ‘a measure beyond the senses’, while the other is affective and sensual. They both make up its ambiguously interwoven horizontal and vertical dimensions and this chapter will be dedicated to exploring this embroilment, which I refer to as harmonic duality.

These two aspects are normally subsumed under the term of harmony: one, its proportional facet, involves rational relationships, concerns fundamental pitches and disregards timbre and register; the other one, which we will call its timbral aspect, involves sensation and acoustic constitution. Their embroilment and interrelationships are such that none is able to subsist without the other. It is rather the perspective created by putting them musically in context what makes one of them stand out. In this measure, both facets are active in different degrees in various musics and their thresholds and contexts that produce their mixtures or separations can be composed.

The idea of a harmony polarized between two limits will be the guiding thread and principal hypothesis of this inquiry. Although this split will be shown to have been present in the debates between empirical and mathematical harmonists in ancient Greece, it has not been explicitly thematized as such. Either it is a question of one aspect dominating or substituting the other, or they are conflated without distinction. In any case, the distinction happens with some recent composers, such as James Tenney, Ben Johnston or Clarence Barlow, although only in a fleeting and unsystematic way, so that this research will take their ideas as a starting point in order to develop on their properties and consequences. This duality, with each of its poles in turn oscillating between two limits – those of consonance and dissonance in the case of the timbral pole and harmonicity and inharmonicity in the case of proportion – makes for a fourfold structure of antipodes. This will be the structuring figure in the development of further topics later in the study. It is proposed as means by which to think pitch and harmonic relations in the development of compositions and

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compositional materials. Only secondarily can it be applied to pertain or embrace other musical activities, as it is developed not only from a practice but also with an aim of understanding models and spaces that are directly relevant to a specific way of composing – that of making algorithmic environments for composition. Within a necessarily limited scope and context they are also related to other musics and to auditory perception in general, although this is to be understood only as one factor out of many that are operative when describing these more general aspects. It can nevertheless aid or serve as a specific analytical tool for some of these applications because it theorizes the relationships between quantitative and qualitative aspects of harmonic materials as well as proposing some basic computational tools that might render these ideas useful beyond composition.

Apart from compositional experience and intuition, there is evidence for this duality in various approaches for conceiving intervallic qualities. The analysis in this chapter will be based on two sources testifying to this duality: Greek harmonics and psychoacoustics. The debate between Greek harmonists around the distinction between pitch distance and proportion started by Aristoxenus traverses many centuries of music theoretical thinking. Although in a simplified way we might call Aristoxenus a ‘timbralist’ and Pythagoreans ‘proportionalists’, this too easy characterization can miss the fact that both sides include some aspects of the other; I would rather say they supplement one another and together allow us to form a wider perspective on this duality. The longstanding theoretical conflict between proportion and spectrum derives from two orientations towards intervallic qualities that emerge as different approaches to the problem of explaining consonance and dissonance or the perception of pitch and timbre. These approaches have parallels to the distinction between the continuous and discrete in mathematics, having on the one hand arithmetics and the Pythagoreans and Aristoxenus and geometry on the side of the continuum. It is through a musical perspective that correspondences are found between sounding qualities and these mathematical domains. Whole number ratios are related to periodic sounds and intervals, while a continuum of pitch and the infinitesimal gradations of timbre can be conceived as happening in a continuous line or space.

The dichotomy between harmonists is analogous to the one in pitch perception models from auditory science, namely, temporal and spatial models. The former relate to periodicity analysis of waveforms and refer to pitch-chroma, while the spatial ones involve pattern recognition in spectra obtained from the physiological filtering in the basilar membrane of the inner ear, and refer to pitch-height or distance. These models map subjective attributes to independent mechanisms taking place throughout the auditory pathway, beginning as early as the cochlea, and taking a complex and crisscrossing course through progressively higher-order auditory stages that end up projected into different cortical areas. The survey we are about to take is meant not only to describe what is known about pitch perception, but will serve to extract from the theories their mappings enough details to better characterize the duality. As we will also discuss, it not only pertains to the level of intervals and chords, or to the current sounding qualities, but is also inherited to higher levels or time frames than those of an immediate sounding present. Although the discussion of the different ‘harmonic levels’ will take place later on, we will already see in this section some ways in which properties of this harmonic duality are inherited not only to musical harmony as it is normally understood, but also, though in a limited way, to rhythm and form.
1.2 Pitch Perception

“[P]itch is held to be duplex in nature. Two pitch-like qualities are distinguished. They are given various pairs of names by various authors: tone height and tone chroma, ordinary pitch and chroma, [spectral] pitch and quality, pitch and tonality, etc.”

The two main types of pitch perception models explain in terms of time-based processes – periodicity analysis of waveforms and the counting of pulses – and spatial processes – recognition of spectra by means of patterns. Both mechanisms are active in our perception and complement each other, pitch-chroma providing a basis for presenting acoustic patterns that do not depend on the particular sound source, and pitch height providing a basis for segregation of sound objects into streams in order to separate sources.

Pitch models are approximations that explain aspects of the world, in this case that of psychological auditory experience. The models are not only rated or judged according to their capacity to explain many sometimes contradicting phenomena, but also as to how useful and compelling they are to the different approaches and disciplines that participate in their construction – be they acoustics, audiology, neurology, musicology, psychology, etc. They are also marked by their statistical and indirect approach. Some account for resulting effects rather than mechanisms, while other approaches begin with physiology and neurology, providing results which are difficult to compare and combine with the other approaches, attesting to the difficulty involved in bringing the two directions together and somehow meeting halfway. There are many gaps in the scientific comprehension of pitch recognition.

On the other hand, these models explain a lot and have been withstanding falsifiability for many decades, being constantly refined and put to work in usable applications as well as becoming stronger in their ability to account for ever more detailed and diverse classes of stimuli. The fact that they mostly translate emergent psychological effects rather than explaining all the complex steps happening along the way, gives them a economical structure that in many cases agrees better with intuition or introspection than with brute force causality.

Induction and deduction are the basis for physics and mathematics, while the study of perception deals with transduction, defined as the conversion of one form of energy to another. In physiology this means the conversion of one form of stimuli to another, wherein a physical stimulus is converted into action potentials in neurons, transmitted along axons towards the central nervous system where it is integrated. A transducer is an object that mediates between two other objects, the input carrying information that molds the energy of the medium with a specific form. Transduction resembles modulation in signal processing, where a usually high frequency carrier waveform is modulated by another signal containing the information to be transmitted through it (like in radio transmission, but also in analogy to the modulation of a note with vibrato or tremolo by a musician). The energy in the transducer is the carrier wave, the information it carries is the modulator. From the ‘point of view’ of the transducer, both the incoming energy (the specific form given to the carrier) as well as the outgoing energy is information, while from the perspective of the transduced information, the transducer becomes transparent, as it could be replaced by any other transducer. The information is independent of the medium in which it travels manifesting the physicalness of

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this information, too often taken to be more ideal than physical.

We will get acquainted with the various transductions involved in pitch perception, but what this short consideration suggests is that pitch not only relates to properties of the vibrating objects that produce the sensation, but some aspects of it are themselves created by the very process of translation, carrying with it information about this transduction. Information and medium, transducer and transduced, participate in its formation and characterization, in a double structure similar to what pitch transmits, which is both itself as a distinct perceptual quality and traces of information about the source that produced it.

1.2.1 What is pitch?

‘A regular periodic pattern dominates a pitched sound’

Pitch is a perplexing aspect of auditory perception, consisting in a distinct set of *qualia* that stand out clearly from other aspects of auditory experience. These independent but related attributes are grouped under the term ‘pitch’, but can and should be distinguished in order to characterize harmonic duality and seek strategies for traversing these aspects in composition.

Pitch is defined as the perceptual correlate of frequency. Because some of its features are induced by the perceptual process itself, this does not imply a one to one correspondence. Not all perceptions correspond to the physical stimuli and there is a fault tolerance mechanism which compensates for ‘deficient’ stimuli by completing and rounding them off. Pitch can also be understood as a psychophysical magnitude correlated closely to the periodicity of waveforms while being simultaneously blended with the timbre of the sounding source. These timbral features can be thought of as peripheral periodicities within that main periodicity, distinguishing between the main repetition period and those ‘inner’, faster ones constituting the partials.

In the 4th century B.C. Aristoxenus distinguished between pitch in general, *topos*, the space in which high and low notes are located, and *tasis*, the ‘pitching’ of this space by the melodic voice, indicating that there are two senses in which we can talk about melodic pitch, a continuous one and another in which this continuum is pierced at certain points of stability. This ‘pitching’ is a ‘steady motionlessness of the voice’, a resting place where the melodic motion stops within pitch space, therefore a *locus* within the continuum, indicating a calibration, in melody, of this space.

Aristoxenus proposed an empirical account which can be considered phenomenological before its time. This is relevant in order to understand pitch as a perceptual category not reducible to natural processes. Pitch (as is also the case of many musical categories) commences as a perceptual fact that is prior to any scientific conception. Experience provides the ‘what’, psychoacoustics the ‘how’. Still, the musical priority belongs to the ‘what’.

Pitch, then, is not one dimensional magnitude but a mixture of several features. A way to tease out these other aspects is to limit the stimuli to the most elemental sounds. Sine waves are considered elemental sounds with respect to pitch, and it is only with these types of sounds that we can say that frequency and pitch actually coincide, so maybe the best and most stringent definition of pitch should limit it to the perception of sine waves and leave other aspects of it to other terms (such as *tone*).

The modern definition of pitch is similar to the one given by Arabic music theorist Safi al-Din (13th

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11 Johnston, B. Scalar Order as a Compositional Resource, 12.
a sound for which one can measure the excess of gravity or acuity with respect to another sound. The ANSI (American National Standards Institute) definition is: “that auditory attribute of sound according to which sounds can be ordered on a scale from low to high.” It could be written more specifically as ‘that auditory attribute in terms of which sine tones can be ordered on the low-high dimension’, but this is useful for research rather than musical purposes because other attributes associated with pitch are left orphaned and ‘tone’ or ‘complex sound’ become insufficient terms for sounds which don’t have a single or clear pitch but have scalable attributes.

Pitch should then be related to the repetition rate of the waveform rather than to frequency. This repetition rate is equivalent to the periodicity envelope, the recurring amplitude pattern resulting from the overall energy of all the components of a sound. This periodicity envelope, being the rate common to all or the most prominent partials components of a tone, does not necessarily coincide with the frequency of the lowest component. Only when sine and complex tones have the same repetition rate can we say that pitch and frequency are equivalent.

A broader definition of pitch is given by J. F. Schouten (1940): ‘The pitch ascribed to a complex sound is the pitch of that component to which the attention, either by virtue of its loudness or of its contrast with former sounds is strongest drawn. Therefore the pitch of a complex sound may be different depending upon the circumstances under which it is heard.’

Pitch is induced or ascribed from composite sensations as well as embedded within other sounds.

This link between pitch and periodicity will be connected further on to harmony and rhythm:

‘Nature abounds with periodic phenomena: from the motion of a swing to the oscillations of atoms, from the chirping of a grasshopper to the orbits of the heavenly bodies. And our terrestrial bodies, too, participate in this universal minuet—from the heart beat and circadian rhythms to monthly and even longer cycles.

Of course, nothing in nature is exactly periodic. All motion has a beginning and an end, so that, in the mathematical sense, strict periodicity does not exist in the real world. Nevertheless, periodicity has proved to be a supremely useful concept in elucidating underlying laws and mechanisms in many fields.’

Periodic phenomena being so ubiquitous, pitch has evolved as a mechanism for detecting periodicity in an environment. Because strict periodicity does not exist, being delimited in time and always accompanied by some sort of noise, the mechanism is tolerant of this divergence. Pitch perception consists both in the factoring and rounding off of (near) periodicity from an auditory scene as well as in the segregation of sources from this scene. If periodicity is understood as an invariance with respect to translations in time, then we can see that periodicity gives rise to discreteness, puncturing holes in the time continuum by bringing about units through repetition. From this logic, another way of describing this double function of pitch with respect to units would be: to discretize the world and to detect simultaneities. Both these functions are related to harmonic duality: periodicity detection corresponding to, and likely (partly) responsible for, the harmonic or proportional facet, while the task of stream segregation relates to the enmeshment of pitch and timbre, and many composite harmonic configurations inherit the properties of this more general function.

Pitch perception can therefore be seen as involved in two somewhat inverse functions: the segregation and the unification of sound components, both simultaneous as well as sequentially. Stream separation is a continuous process, while bundling of elements into individually identifiable impressions accounts for discreteness. This integrating aspect might condense inner modulations.

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transients or otherwise timewise separate components into unitary perceptions. Harmonic duality maps to this polarity: integration into units and differentiation of streams, the former serving the purpose of identifying proportionality (relations between units, happening in an independent dimension from that of high and low pitch distance), while the latter is of use in the identification of separate sound colors and sources (for which pitch distance is of great help).

In a sense pitch is perhaps the most distinct qualia in auditory perception – and for that matter, also with respect to other sensory modalities – in that its qualitative seemingness, the way it appears as an experience, permits a hugely gradated and finely tuned immediate apprehension that differentiates itself in a measuredly way. Pitch provides a reproducible, memorizable figure for identifying and comparing relative (but sometimes even ‘absolute’) stimuli, capturing relations in a qualitative, immediate manner. Intervallic hearing, which in a sense is equivalent to ‘pitch to the second degree’, the quality of two (or more) pitches, provides and even finer spectrum or graded scale for which to single out periodicities, this time not in the sense of high and low but as highly identifiable ‘characters’ that, with a bit of training, can be immediately apprehended.

1.2.2 Theories of pitch perception

‘Historically, theories of pitch were often theories of hearing [...] It is conceivable that pitch grew out of a mechanism that evolved for other purposes, for example to segregate sources, or to factor redundancy within an acoustic scene. The ‘wetware’ used for pitch certainly serves other functions, and thus advances in understanding pitch benefit our knowledge of hearing in general.’

The science of pitch was inaugurated by Greek Pythagoreans. Archytas (4th century B.C.) proposed the first known psychophysical account of sounds and their perception, attributing for each pitch a different speed in the propagation of vibrations, instead of what is now known to be a different frequency, the speed of propagation being independent of pitch. This issue was not settled until the seventeenth century, when the quantitative dependence between pitch and frequency was established by Marin Mersenne (1636) and Galileo Galilei (1638). Later on Joseph-Guichard DuVerney (1693) established the first resonance theory which suggested a ‘tonotopic’ (correspondence of pitch height to spatial distance) projection to the brain, followed by Joseph Sauveur’s (1701) observation that a string could vibrate simultaneously at several harmonics (coining the terms ‘fundamental’ and ‘harmonics’). Later on, theories explaining superimposed vibrations on a string were developed by various mathematicians (Taylor, Bernoulli, d’Alembert), but it was Leonhardt Euler who condensed these ideas with the concept of linearity, which implies the principle of superposition (that these partial waves propagate independently and that the total sound at each point corresponds to their sum). The thinking of vibrations as sums of more fundamental ones, with their periods at integer submultiples of the longest period lead to Joseph Fourier’s theorem (1822), which proves that the sum can be expressed as a set amplitudes and phases and that this sum is unique.

Nowadays two quantities related to pitch are distinguished: what is called spectral pitch, usually denoted as fLOCUS and periodicity pitch, or F0. August Seebeck, who first utilized acoustic sirens in auditory research, presented observations (1841) suggesting that a pitch sensation was determined not only by the fundamental, but also by other higher partials, even to the point of having acoustic signals that could elicit a pitch without possessing a fundamental partial at that frequency. He concluded that pitch corresponds to the period of the overall periodicity. A battle ensued with Georg Ohm, for whom the ear performs a Fourier analysis of signals into their partial components from...
which pitch is determined by the frequency of the spectral fundamental. Ohm’s dismissal of Seebeck was to cause many future misunderstandings, delaying the development of non spectral theories. This spectral theory was based on his law (1843) which extended the principle of linear superposition to the sensory domain. Hermann von Helmholtz (1863) refined and developed it, explaining how the process happens in the cochlea. If a complex sound is composed of sinusoids, the sensation itself can also be decomposed into simple sensory components. He also stated that: (1) only vibrations with a nonzero fundamental evoke a pitch related to that period; (2) other partials may evoke additional pitches; (3) relative partial amplitudes affect timbre but not pitch; and (4), relative phases of partials affect neither quality nor pitch. He proposed a model according to which the cochlea (and specifically the basilar membrane inside it) behaves like a bank of resonators, to be proved physiologically by Georg von Béckésy’s (1928) discovery of the process of transduction happening in the inner ear, later refined by Plomp and Levelt’s (1965) critical band model.

This history surveys the basics of what is now called place theory, also known as pattern matching. It consists in estimating pitch based on a spectral analysis of the provoking stimulus through patterns formed by the spacing and amplitudes of its partials. It is spatial for it follows the tonotopical metaphor in the way it was proposed by Helmholtz, suggesting that pitch is perceived through ‘conscious inference’ as each nerve attached to the spatially arranged resonators in the cochlea carries with it ‘specific nervous energies’, each representing a different quality of pitch.

Nowadays the cochlea has been thoroughly studied and is indeed considered a tonotopic transducer, but there are more aspects to this. The basilar membrane inside it transduces the vibrations transmitted from the tympani by moving inside a liquid spiral chamber and transmitting its vibrations to the hair cells disposed along the spiral and associated with nerve fibers that respond to specific frequencies. There is still a heated debate as to what kind of vibrations the basilar membrane undergoes. Béckésy’s model is that of a traveling wave; the problem with it is that, being a serial phenomenon, it does not allocate frequency to place along the cochlear duct. Resonance models, on the other hand, account for tonotopy through parallel vibrations. It may be that what stimulates the basilar membrane and the cells has to do with a combination of traveling waves, resonance, plus an active amplification scheme where the outer hair cells compensate and change the physical properties of the system.

The cochlea is an active and non-linear system. Active, because in addition to receiving acoustic energy, it also has a ‘regenerative’, ‘undamping’ mechanism which adds energy to the very signal it is trying to detect, effecting a ‘sharpening’ of the resonance required for the transduction, in order to change its tuning characteristics and resolution. The outer hair cells, through chemical, mechanical and electrical interaction with the basilar and tectorial membranes, change their and the liquid’s physical properties, producing a negative damping that injects energy into the system, under control from the central nervous system.

The cochlea is non-linear because it distorts the incoming signal, producing signals which are byproducts of its functioning, such as combination tones and spontaneous subjective pure tones.

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19 There are inner hair cells which do the actual transduction and outer hair cells which account for compensatory processes within the cochlea. Also, as an interesting fact, it happens that auditory hair cells are the only plant cells in the mammal body, or at least the only ones having the characteristic of being pressurized like plant cells. See for example, Baylor College of Medicine. Outer Hair Cell is Pressurized. Last retrieved May 21, 2011, from [http://www.bcm.edu/oto/index.cfm?pmid=15267](http://www.bcm.edu/oto/index.cfm?pmid=15267)

known as otoacoustic emissions. Another nonlinear characteristic is that it behaves differently according to frequency. Recent modeling of the basilar membrane has been achieved through a cascading filter bank, so-called gammatone filters. The proponents of this model contend that one of the main functions of the cochlea, arguably more important than frequency filtering, is to compress the enormous range of amplitude variations in sound we are capable of hearing into a more manageable dynamic range that can be transmitted through the nervous impulses transduced by the inner hair cells. This compression is frequency dependent and highly non-linear, compressing the input dynamic range by up to six orders of magnitude. Inner hair cells can only transduce around 60 dB, so to explain the 120 to 140 dB of dynamic range perception in humans (able to detect differences in amplitude of up to 14 orders of magnitude!), the outer hair cells have been shown to reduce this range by compressing another 60 dB. Given that the frequency resolution of the cochlea is somewhat broad, most of the auditory filtering occurs at higher neural processing centers in the brain, lending additional support to dynamic compression as the main function of the cochlea. These details regarding the cochlea can help understand the discussion of dissonance curves we will have in the next chapter, as they are offshoots of basilar membrane models.

For Ohm and Helmholtz, the pitch sensation was straightforwardly the lowest partial. As we know, this does not hold for many kinds of stimuli, for which other more refined algorithms have been proposed. They may estimate pitch from the spacing of the partials, from loudness patterns, or through sums of subharmonics of the partials. A same pitch can be evoked by widely varying spectra, including those without a fundamental, as in Ernst Terhardt's model (1974), where specific loudness patterns are the basis for a derived virtual pitch ‘gestalt’, inducing a pitch from partials other than the fundamental, distinguishing ‘spectral’ from ‘virtual’ partials. Virtual pitch is related to what he calls analytic listening, a learned mode of listening distinguished from the innate, synthetic listening where Ohm’s law follows. Pattern matching models thus require a learning process and the existence some sort of internally stored templates to which the input is compared. The fact that a string behaves like a pattern-matcher makes for possible mechanisms for which no learning is required, though. Furthermore, a string operates directly on the waveform, not on a spectral pattern, so the Fourier decomposition might not even be needed.

Temporal theories of pitch were brought back to life by Schouten (1938) when he showed many instances where F0 was not the fundamental partial. His said that a ‘residue’ pitch is responsible for the overall periodicity. This residue arises from the combination of high, unresolved partials – partials too high and close together to be separately distinguished and processed in the basilar membrane – and is present even when a masking noise obstructs the fundamental frequency. Later on it was found that those residues were not limited to unresolved partials but also included the resolved partials, meaning that the residue emerged from the sound as a whole.

Time models assume that the ear ‘counts’ vibrations instead of ‘guiding itself’ by the metaphor of calibrated resonators. Contrary to place models, in time based ones it is only possible to suppose that

21 “Otoacoustic emissions are small sounds caused by motion of the eardrum in response to vibrations from deep within the cochlea. The healthy cochlea creates internal vibrations whenever it processes sound. Impaired cochleae usually do not. Some healthy ears even produce sound spontaneously as internal sounds are processed and amplified.”, from Kemp, D. Understanding and Using Otoacoustic Emissions. Last retrieved May 31, 2011, from http://www.est-med.com/OAE/understanding-using_OAE_von_Kemp.pdf. This phenomenon is mostly of interest in audiology and hearing impairment research.


24 My implementation of dissonance curves includes a derivation of this model that calculates virtual pitches from a spectrum. It is used to accompany the intervals produced by the dissonance curves as it combines with them very well. More in Chapter 2.
the counting takes place in the brain, not in the cochlea. In time theory, which has its roots in the Pythagoreans and Boethius, the behavior of a string is a guiding metaphor, as it can make many sounds, one sound encompassing others but coming to the ear integrated in the unity of a single pitch. Here the elementary components of sounds are not partials (sine waves) but discrete ‘phonons’: percussions or pulses. Galileo’s account of consonance explains it in terms of *commensurability*, as the blending of two simultaneous sounds due to the proportionality of their pulse trains. This is related to the harmonic metrics we will discuss in Chapter 3.

Joseph Licklider (1951) first put forth the theory of autocorrelation, the main model behind time theories. A measure of self-similarity across time, it reveals the close relationship between periodicity and self-similarity, and as a concept it can also be used to describe and explain musical phenomena at several time scales. Autocorrelation (AC) proceeds after cochlear filtering and hair cell transduction, happening as an analysis of trains of nerve impulses. It is a two dimensional pattern with the axes of CF, characteristic frequency, and lag time, τ. The delayed and direct signals are multiplied and the result is summed up, so that for lag times corresponding to the period of the signal the sum will be maximal. There are also peaks at time intervals related to the sub periodicity of the signal, such as harmonic partials, which also contribute to the main peak. Licklider speculated that neural circuits in the lower centers of the auditory system can perform the three operations necessary for AC: delay, multiplication and temporal integration. The neural arrangement happens in two dimensions: the frequency projection from the cochlea and delayed versions of these projections. At another further network, this matrix of information is summed and integrated for the sought period to emerge. AC has affinities with pulse counting theories of consonance: “when the frequencies of two sounds, either sinusoidal or complex, bear to each other the ratio of two small integers, their autocorrelation functions have common peaks.”

AC requires accounting for the low firing rates of neurons which max out at around 300 spikes/second for which the so-called ‘volley theory’ explains firing rates higher than this limit. Time based processing has a different frequency range of operation than place mechanisms, the former limited with respect to the latter in high frequency resolution, while the latter is limited with respect to the former in the low regions, for which time processes can go below the usual cochlear pitch range at 16-30 Hz, all the way down into the infrapitch regions. Both models overlap within 30-2500 Hz, where most musical activity happens. The breakdown of time mechanisms occurs between 3.5 and 5 kHz, coinciding with the limit of melodic pitch and intervallic recognizability.

AC is a computational model, borrowed from mathematics, of which there have been many variations and refinements, some better equipped that others to predict empirical behaviors. They are usually called Auditory Image Models. The string metaphor can be seen as belonging to the AC model family as it is in essence a delay line that feeds back into itself. AC, the string and pattern matching are closely related, their difference lying in their temporal resolution. At each instant, AC reflects a relatively short interval of its input, the string metaphor reflects the past waveform over much longer time intervals. Other AC models capture regularity over longer periods and are ‘subharmonic’ counterparts to ‘harmonic’ pattern matching schemes, showing strong connections between AC and pattern matching, with the concept of the string as an analogy bridging both. Sine waves are elementary in place models, in time models they behave just like any other signal.

Both temporal and place models require a temporal integration mechanism to account for the continuity heard in sounds despite them consisting in trains of pulses or waves following each other. This integration must be limited for the perception of trills and other fast modulating articulations.

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26 Ibid., 131.
not to become smeared. It is also subject to an inherent tradeoff in wave phenomena between frequency and time, where the accuracy of frequency determination depends on a sufficiently large time window for the analysis, and the larger this window, the coarser the time resolution. Dennis Gabor’s formula \(^{28}\) (1945), derived from the uncertainty principle in quantum mechanics, says that:

\[ \Delta f \cdot \Delta t \geq k \]

The product of the uncertainties in frequency and time is always larger than a positive constant. Time and frequency cannot be simultaneously defined in an exact way, there always remains an uncertainty. Increasing the accuracy of one quantity increases the inaccuracy of the other and vice versa. This has consequences for thinking perceptual boundaries, and some topics related to harmony also relate to this inequality, its constant \( k \) determining those borderlands which, if crossed, cause sonic forms to experiment a ‘perceptual phase change’, a change in their properties by which they pass from one uniform perceptual state into another with different emergent characteristics (and this might either happen abruptly or gradually). In the case we are dealing with now, it is pertinent to the transition between pulses and pitch, or to fluctuations belonging to the ‘zone of articulation’ such as vibrato and tremolo which, when speeded up sufficiently, are conflated into the timbral modulations within a tone. This boundary can be called the rhythm-pitch boundary. Its ‘integration zone’ has a boundary beginning at around 50 ms (20 Hz) for its lower threshold, with a higher one lying around 10-15 ms (66-100 Hz), although this upper bound is more of a grey zone, much more difficult to determine. Within this region, pitch is not yet fully fused into a unit apart from its sonic components \(^{29}\). There are several layers of temporal integration simultaneously embedded on top of each other, an insight relevant also to the fact that harmony involves different time scales, something which will be dealt with during the course of this study.

Several kinds of temporal integration have been incorporated into AC models. They involve two kinds of frequencies: those arriving from the cochlea, which are subject to the uncertainty relation, and those produced by the synchronization of neural circuits to the periodicity of incoming waves from the cochlea (‘phase locking’), determined with arbitrary accuracy. They allow transients to reset the integration process, avoiding integrating for longer than necessary. These facts suggest a combination of time and place models.

There has been no way to eliminate either time or place models as they both explain, in their most current versions, many kinds of inputs and predict most kinds of behaviors. To a great extent they are found to be complementary. Licklider proposed a ‘duplex’ theory which included a learned neural network to integrate them. There is still strong support for a two mechanism hypothesis, its main drawback being the lack of parsimony in conceiving two mechanisms instead of one (plus a possible third mechanism to integrate the two). Place processing adapts well to resolved harmonics, while time processing handles unresolved ones. Despite the evidence for both mechanisms and the fact that they provide a better fit to empirical behaviors, a two mechanism hypothesis also compounds the difficulties of both.

As for this research and my position as composer, evaluating this double mechanism hypothesis from the standpoint of music and harmony shows that the problem should not be limited to pitch perception on its own, but that it may be revealed more clearly in musical and particularly harmonic situations. A concept of a ‘single’ pitch is already a reduction, as pitch always happens beyond laboratory conditions within an auditory environment, rarely unaccompanied by other pitched and non pitches sounds. Nevertheless, a compositional hypothesis such as harmonic duality, derived by awareness, introspection and experience gathered by exposure and handling of musical materials,


can provide support for the fact that two parallel, supplementary processes are active because they have corresponding affinities with each of the harmonic facets and are likewise entangled with each other. The perspective of composition can’t directly intervene in advancing these multidisciplinary models, although it could suggest targets and situations on which the models can be experimentally tested, helping to challenge and refine them through the elucidation of the qualities and properties of harmonic duality. In this way, harmonic duality not only takes some of its features from pitch perception models, but can also give and suggest some ideas back to auditory science.

1.2.3 Neurobiological studies of pitch from the bottom-up

Parallel pathways are followed by the signal transduced by the cochlea in its way from the auditory nerve to the auditory cortex, each matching the properties of time and place based processes. Piecewise neurological descriptions, where each step increases the complexity and time frame of the perceptual process is what cognitive psychologists call a ‘bottom-up’ description, experiences understood in terms of smaller processes that accumulate into emergent contents of consciousness. They are ‘data-driven’, proceeding from the sensory towards the cognitive. In contrast, ‘top-down’ strategies refer to approaches that explain in terms of higher level qualia or learned ‘schemas’, expectations or concepts which are analyzed into subsystems of ‘black-box’ components (where it is foremost to know their effects rather than their functioning) interacting all the way down towards physiological sense mechanisms.

Bottom-up and top-down processes are simultaneously involved in perception, though their terminating points are hard to conciliate. High level concepts or expectations mediate upon the data acquisition process. Also, most lower level features have been quite well studied, but as the descriptions progress up through the auditory pathway from the brain stem towards the auditory cortex, the gaps in understanding become larger and less detailed. As the processing ascends through the auditory pathway, it involves more subsystems and larger time windows, becoming more contextualized and imbricated with memory and learning. The information in the brain is not processed sequentially but works in a massively parallel way, accounting for the fact that the coordination between relatively slow data processing units can produce relatively fast reactions for complex stimuli.

From the top-down perspective, psychometric studies have shown the mathematical spaces that describe subjective qualities related to pitch, intervals and tonality. These models are a sort of ‘average’ or overall outcome of the many interactions taking place physio- and psychologically. The evidence for a combined temporal/place operation given by bottom-up descriptions is complemented by the top-down methods with the insight that pitch requires more than one dimension to be accounted for in subjective terms. Both methods provide a wider picture for the duality of pitch perception and its consequent supervention on harmonic situations.

The bottom-up description of auditory processing in the brain goes, in a simplified way, as follows: once the auditory information has been transduced into the auditory nerve, it proceeds to several auditory centers in the brain stem. The first one is the Cochlear Nucleus, where several kinds of neurons specialize in detecting onsets of tones, patterns of peaks or the ending segments of stimuli, information which will be further processed when projected higher up in the auditory pathway. Other neural circuits perform more complex roles, such as decodification of amplitude modulations in the signal. Various types of amplitude modulations are relevant for pitch processing, the periodicity envelope being an amplitude variation resulting from the overall effect of many interacting partial components.

Temporal autocorrelation has been neurologically explained through delayed and undelayed
responses of neural networks to the periodicity carrying envelopes of a signal. Two neural circuits, an oscillator and an integrator, are synchronized to the signal envelope, producing an output signal when they coincide. This signal corresponds to integer multiples of the carrier waveform in the amplitude modulated signal. This model (supplemented with ‘volley principle’ mentioned earlier) can account for the temporal codification of periodicities up to 1 kHz and even higher (up to 3-5 kHz). Many of these circuits running in parallel are necessary to cover the known spectral and temporal ranges of hearing.

The next station where axons from the Cochlear Nucleus end up is a crucial auditory structure called the Inferior Colliculus (IC), the main center of tone perception. Its central nucleus has an arrangement of neural nets where the temporal information concerning periodic signals (synchronized spikes in nerve fibers) is transformed into a spatial representation (a neural map). This is the station where the ‘high’ and ‘low’ spatial attributes of pitch arise out of temporal periodicity information. It is also the processing center where the constitution of the auditory filters and critical bandwidths is achieved. Its tonotopic structure enhances and alters the cochlear tuning curves through a further filtering performed by neurons tuned to specific frequencies.

This three dimensional matrix consists of around 30-40 laminae of around 30x50 neurons each. One axis (the one with 30 neurons) codifies tonotopy, the frequencies of the spectral content of the signal arriving from the basilar membrane. The second axis (of around 50 neurons) corresponds to periodotopy, arriving from the temporal analysis performed in the Cochlear Nucleus. Periodotopy lies orthogonal to tonotopy and has a range of variation of around five octaves for each column of constant characteristic frequency CF; in some cases going all the way down to 10 Hz, and going up to a fourth of the CF. Periodotopy is also known as ‘best modulation frequency’, indicating its derivation from the amplitude demodulation having been performed lower down the pathway. Both characteristic and modulation frequencies can be thought in terms of amplitude modulation, as carrier and modulator. Perception mechanisms concurrently codify both periodic and timbral features of sounds and these modulations cover a span that goes from the rate at which musical articulations happen (such as tremolo and other changes in the amplitude envelope related to onsets and release portions of sounds) all the way up to the ‘inner’ modulations in the timbre (roughness and timbral effects related to AM such as spectral brightness – as in the electronic music technique of ring modulation).

A 500 Hz tone with an amplitude modulation of 50 Hz will activate not only the neuron tuned to detect 20 ms, but also a neuron tuned to 100 Hz (10 ms) because 20 ms is a multiple of 10 ms, as well as one tuned to 200 Hz (5 ms). A neuron tuned to 6-2/3 ms (150 Hz) will also be activated, but to a lesser extent because dividing by 3 may result in a weaker coincidence as, belonging to a prime number power other than 2, it might involve a longer neural path. These responses behave more like comb filters than band pass filters in that one period provokes not one but a whole series of firings. Of all these firings that project higher up into the auditory cortex, their sum and integral will be maximal at the point of highest coincidences, thereby concluding the process of autocorrelation through neural networks with a robust pitch estimation.

Higher up from the IC, information processing in the cortex is linked to short and long term memory. Schemas arise in connection with long term memory: functional organizations of

31 La., small posterior hill. Most functions in the auditory system pass through this important and relatively large nucleus, which also includes multi sensory connections related to visual, tactile and olfactory pathways.
32 “Cognitive psychologists attempt to specify, through the interpretation of statistical data obtained from experiments, how the mind works. And they often express that working in terms of “mental structures” and “mental processes.” In his book on memory, Sir Frederic Bartlett (1886–1969) had introduced this distinction to explain how the memory of a story is first encoded (a process) into a schema (a structure), and then subsequently decoded (another process) as a recollection that may depart in significant ways from the original experience.” Gjerdingen, R. (2002). The
neurons that codify structured information, developing characteristic responses to environmental stimuli. They arise through three types of mechanisms: projection, self-organization and association. Projection, as with tonotopy, is the result of a strict order in the wiring between neural centers, maintaining these structures from the periphery up to the highest cortical areas. Self-organization has been the focus of computer simulations on how higher level response functions arise out of the activation patterns of simple neural networks exposed to musical stimuli. This is how structures resembling torus-like spaces have been found in simulations regarding pitch and tonality. Association is less close to perception and more an attribute of higher cognitive functions, linking, relating and unifying diverse modes of information processing (such as sense modalities) into higher level structures, also involving the imagination.

With respect to music, brain scanning techniques have shown some general functions of the auditory cortex. The primary auditory cortex identifies basic musical featured such as pitch and intensity, the secondary cortex deals with melodic, harmonic and rhythmic patterns, while the tertiary auditory cortex is thought to integrate these patterns into an overall perception of the music. These three functions correspond to the three main time scales in music (sound materials, mid size phrases/textures and large scale forms).

1.2.4 Pitch height and chroma

Pitch height has ‘low’ and ‘high’ as its main attributes, carrying also within itself timbral aspects such as spectral envelope (defining source, instrumental family and register). Pitch chroma, also called ‘pitch tonality’ or ‘tonal quality’, refers to periodicity pitch as well as to the musical categories of pitch classes, which are independent of register and timbre. It is specified as the quality that makes a pitch different from others inside an octave, such as the fact that all C’s, C#’s, D’s, etc., in different octaves possess a *qualia* of their own. The fact that melodies can be transposed to any arbitrary pitch shows that chroma happens not in relation to absolute, but to *relative* pitch, hence concerning intervallic qualities (‘octave’, ‘fifth’ or ‘seventh’) rather than absolute pitches. In relative hearing it is meaningless to refer to a ‘G-sharpness’, but nevertheless chroma is usually understood as the ‘colors’ through which pitch perception traverses a ‘registerless’ octave, a conception which is very different from the usual experience of listening to melodic ‘characters’ as functions within a harmonic or scalar context (such as tonic, mediant, dominant, leading note, etc). These aspects are not distinguished by the concept of chroma, rendering this concept less useful for harmony as one might have expected.

Both pitch dimensions are represented by a helical image where chromas spiral around a circle which is stretched upward like a spring by the height dimension, each circle completing a cycle at the octaves. Compounding the problem with the definition of chroma is the fact that most researchers assume only 12 chromas per octave, picturing the circle as equally spaced, conflating a proportional with a logarithmic conception. It’s as if at each semitone a new chroma would appear, which is not the way musical intervals behave. This mistake is linked to an uncritical inheritance of a much simplified functional music theory. The loci in the pitch distance line (or octave equivalent circle for that matter) where proportions arise corresponds to an irregularly spaced grid related to the stacking of divisions by whole numbers. Moreover, it is known since antiquity that there exist...
much more than 12 intervals in an octave, and even in the case of twelve note equal temperament the same interval can have different functions depending on its context, each function related to a different intervallic identity.

Of interest for our discussion are studies whose aim has been to map the subjective dimensions of pitch height and chroma in the auditory cortex. Each feature is independently varied by means of synthetic probe tones. These variations are made in order to perform functional magnetic resonance imaging mappings of the brain responses of test subjects. The analyses of these images, both for the group of subjects as a whole and for individuals, have shown that temporal and spatial extraction processes map to distinct regions of the secondary auditory cortex, reflecting the psychological findings of the two component subjective pitch. There is a region in Heschl’s gyrus in the secondary auditory cortex which deals with both height and chroma; sections anterior to the gyrus were activated by changes in chroma-only test tones, while sections posterior to the gyrus corresponded to height-only tones. This latter area is known to be specialized in the segregation of multiple sound objects from an auditory scene (timbral identification of sources), while the areas activated by chroma have been shown to relate to melodic processing as well as to the extraction of prosody in speech. Chroma processing relates to the extraction of “coherent information streams that can be analyzed independently of the specific sound source,” alluding to the abstract nature of pure pitch patterns devoid of timbre from which melodies are constituted, generally proceeding from a single source. This is a more useful conception of chroma, one which alludes to the ‘pitching’ of melodic space more than to colors. The results also suggest a connection between prosody and melody.

By encompassing its material and perceptual principles, the theorization of harmony can benefit from these traits of tensions and entanglements. Here we see that both harmonic dimensions are related to dualities such as multiplicity/unity, source/pattern, value/character, and so on. They play themselves out in different but parallel ways, and to greater or lesser degrees at several levels of organization, ranging from the levels of timbre, pitch, melody, chords, tonal and metric fields, and even higher up to formal levels such as sections, movements and pieces as a whole.

1.2.5 Top-down psychological studies of pitch

Psychometrics and other empirical techniques from music psychology have provided the main source of knowledge for top-down descriptions. The literature on the subject is large and outside the topic of our present research, so we’ll focus only on aspects that pertain to the multi dimensional character of pitch perception in order to link them to harmonic duality and harmonic space.

Apart from the aforementioned chroma and height spiral proposed by Shepard, Carol Krumhans1 has arrived at so-called probe tone ratings of contextual pitch. Listeners were asked to rate how well a single tone fitted against a fixed tonal background (scales, chord cadences or small pieces), varying the tone over the whole chromatic range for each rating. The statistics agreed quite well with the ratings of consonance having been proposed by some music theories, especially in the case of listeners with a musical background. Intervallic sizes were seen to be less important than their tonal function, implying that context and tonal hierarchies have influence beyond the perception of intervals in isolation, and that invariant structures are abstracted at the level of tonal centers or keys.

36 Ibid., 10042.
These probe-tone profiles were measured for major and minor tonalities and then compared by correlating all major-major, major-minor and minor-major key pairs to arrive at an ‘interkey’ distance matrix. This matrix was subjected to a multidimensional scaling analysis, a statistical technique for interpreting psychological data which produces spatial representations based on distance metrics. What obtained was a four dimensional solution construed as a toroidal map depicted as two circles, one of fifths and the other of thirds, together with corresponding relative and parallel minor and major tonalities, which are represented by the variables $\varphi$ and $\theta$ as the angular rotation along each circle in the torus (see Figure 1, B). This map, obtained independently from music theory, is similar to the harmonic space that we will discuss in Chapter 3, as well as to Hugo Riemann’s tone maps or Arnold Schoenberg’s regions chart. As has already been delineated previously, my criticism here is that these kinds of studies want to prove aspects of functional tonality and twelve tone equal temperament from a psychological point of view by way of studies which secretly assume the very theories they want to prove, especially because the simple examples given to the test subjects are derived from these theories. In this case though, the researchers are aware of this limitation and do not make claims outside cadential equal-tempered music and what is interesting for us is that these visualizations show to how a more generalized kind of harmonic space can arise as a mental schema.

Through ‘cognitive modeling’, Leman & Carreras\textsuperscript{38} computer simulated a perceptual learning system whose input is acoustic data. It implements inner ear filtering and dynamic range compression, neural firing patterns and periodicity analysis by autocorrelation, together with a second cognition module based on a self-organizing network which, after learning by self-organization, yields a schema of tone center perception as is thought to be carried by neural networks in the brain. Afterwords, it is tested with musical sequences in a similar way to the probe tone ratings, but as a computer simulation, producing results that fit very well with the psychological studies (see Figure 1, D & E).

Probe tones relate to the Aristoxenian concept of dynamis, to be dealt with in the next section. They also relate to the statistical weights of pitch sets that are part of the development of stochastic harmonic fields (elucidated in Chapter 3). Other memory and patterning processes in pitch sets, such as Markov chains, which add orderings to the probabilities of each interval, can be layered on top and understood in relation to probe tone ratings: as potentials or probabilities that structure pitch at a level beyond the immediate intervallic time frame, a level lying between the note, the motive and reaching out towards the phrase.

1.2.6 Pitch research in relation to harmony, melody and timbre

In the past, research into pitch was guided mostly by musical considerations but nowadays it encompasses a much wider field of sonic phenomena. After having run through the workings of pitch perception models, now it is of interest to see how they relate back to musical topics. We will do this in relation with harmonic duality and also to sum up some of the distinctions and findings that have been made up to this point.

No pitch model explains nor predicts octave equivalence. It is neither an assumption nor an emergent property of the models. Why, then, is it one of the most striking features of pitch perception? Moreover, it seems to be an exclusively auditory phenomenon having no equivalent in other sense modalities – to consider musical tone color as relating to visual color and frequency leads to the conflicting fact there are no visual octave relations. There is physiological evidence of laminae with octave architecture in the Inferior Colliculus and in the next station in the auditory pathway, the Mediate Geniculate Body in the thalamus, which seem to process pitch classes within each lamina. Both the basilar membrane and the tonotopic/periodotopic maps in the IC behave as approximately logarithmic transducers, especially for higher frequencies. Ernst Weber's law states

39 Visual perception does occur within a span of slightly less than an octave (from 390 to 770 trillion Hz), although the main difference between the two senses has to do with superposition. In audio, superposed frequencies are perceived as increasing in quantity from single to multiple, whereas visually, color frequency superposition results in different hues of single colors. The analogous visual percept of pitch would more likely be the direction of incident light rays, as they strike different regions of the retina in a transduction similar to that of the basilar membrane. See Roederer, J. (2008). Physics and Psychophysics of Music, Berlin: Springer, 174 and its footnote.
that most senses transduce physical stimuli logarithmically, meaning in this case that a double amount of physical intensity would correspond to a single step increase in sensation. Putting aside the fact that transducers follow the law only approximately, physically there would be no reason to expect its exponent to be exactly the integer 2. Many sensory transductions have been measured, none of them having an integer constant\(^{40}\). Its prominence could arise not because octaves are special to perception, but conversely, that is, because the acoustic structure of octaves produces singular perceptual effects. Being the only interval where the partials of the upper tone match all the frequencies of the lower tone could mean that octaves require less operations for the pitch processor, and hence stand out from other intervals. It seems that both motivations must be operative and that some perceptual mechanisms might have evolved as adaptations to the specific makeup of octaves.

Similar relations of equivalence can also exist within other intervallic spans, the most fundamental being those with prime factors other than 2, such as 3, 5, 7, etc., (quintal, tertial, septimal equivalences), but it’s possible to conceive other intervals than those. There are many problems, however, in making compositional materials out of these new equivalences, firstly, because octave equivalence is stronger and will mask and interfere with the ‘chromas’ within the other fundamental intervals, so appropriate musical contexts have to be devised in order for these different relationships to stand out; secondly, because many of these ‘chromas’, say, within a twelfth (3) or tenth plus octave (5), happen to be the same than those within an octave, making the differentiation more difficult; thirdly, obtaining these chromas is not a trivial question, they don’t arise unavoidably, but have to be generated by some process of division (by proportional – harmonic or arithmetic – or continuous – geometric – means)\(^{41}\), and this does not in any way guarantee that these ‘chromas’ will be experienced as belonging in any way to their generator. The case of 7 as a generator seems to be promising, and I will discuss in due course some strategies for making some kind of perceptible septimal equivalence pitch sets.

Another topic of pitch perception of interest to harmony is that of dynamic pitch, as it happens in speech as well as with glissandos. It poses big challenges to pitch models, whose design is based on stationary pitch. It can conceivably arise through different mechanisms from static pitch, and it has been hypothesized that stable pitch could even arise from dynamic pitch mechanisms. It also involves the question of how much frequency modulation the pitch mechanism is capable of tracking and to what extent frequency modulation might be transformed into amplitude modulation in the extraction process. Regarding harmonic duality, dynamic pitch belongs to the timbral dimension as it lacks a fixed periodicity. Whatever the pitch gradient detection mechanism might consist in, it most likely interacts with periodicity processors, so depending on the situation, a constantly moving pitch might make the entwinement of the two aspects give way to the domination of the spectral facet or provide space for a combination of both\(^{42}\).

\(^{40}\) E. H. Weber’s psychophysical law, ‘the size of a just noticeable difference is proportional to the stimulus intensity’, 1834, was furthered by G. T. Fechner, ‘\(S = k \log I + C\)’; \(S\) = sensory magnitude, \(I\) = stimulus intensity, \(k\) and \(C\) = constants’, 1860, later refined by J. A. F. Plateau, ‘equal stimulus ratios produce equal sensation ratios’, \(S = k I^n\); sensory magnitude = a constant times a stimulus intensity to the power of \(n\), 1872. This began to be measurable in practice until the 1930’s, giving way to the first subjective scales (Fletcher-Mundson curves for loudness, mel scale of pitch, and many others). Stanley Stevens later generalized the law, measuring many sensory conditions (heat, color, intensity, taste, smell, cold, tactility, etc, as well as auditory pitch and loudness, 1950’s). He does not encounter any integer proportionality constants. (Information gathered from Warren, _op. cit_, 108-109 and Weber-Fechner Law. In Wikipedia. Last retrieved November 16, 2012, from [http://en.wikipedia.org/wiki/Weber-Fechner_law](http://en.wikipedia.org/wiki/Weber-Fechner_law)

\(^{41}\) To be discussed in the following section on Greek harmonics. The best known example of a tuning and scale which has a different module than an octave is the Bohlen-Pierce scale, which divides the ‘tritave’ (twelfth) geometrically into 13 equal divisions.

\(^{42}\) James Tenney’s glissando pieces, starting from _For Ann (rising) _ (1969), pertain to the ambiguity between chroma and pitch height as embodied in Shepard tones. In _Koan_ for String Quartet (1984), discrete periodicities are made to stand out from within the continuum.
A further issue related to pitch, even more associated to musical composition, is the blurring of timbre and pitch, or more precisely, the situations that bring out multiple pitches within timbre, in connection to what is called ‘multiple pitch’ in auditory research. It is interesting to conceive timbral variations in terms of pitch, contemplating a territory that lies between a steady single pitch (a sine tone) and very fast frequency fluctuations of that pitch producing different kinds of noise, depending on the speed and span of the variations. The realms in this continuum extend from simple to more or less complex overtone structures, transmuting into ever more inharmonic dispositions of partials, reaching out all the way up to saturated and unstable overtones, of which there are a large amount of varieties and densities culminating in the full range fluctuations of white noise\textsuperscript{43}. The territory within these poles where the perception of pitch transitions from single to multiple lies more or less where partials become inharmonic and begin to fluctuate, before their density and speed surpasses certain perceptible complexity. Perceptually, this threshold is dependent on loudness, involves context, requires time and usually implies an analytic mode of listening. When it does happen (for instance after a sustained and somewhat loud beating of a tam-tam or a triangle\textsuperscript{44}), recognition can easily transmute between a timbre with pitch and multiple pitches within timbre, depending on various factors, some physical (related to the properties of the sound emitting materials as well as the movement of waves within the acoustic space), some psychoacoustic (involving thresholds of fusion/separation of partials) and some mental (dependent on modes of attention). The zone that hovers above single timbre and multiple pitch is interesting for composition inasmuch as its delineates regions of ambiguity and separation between periodicity and spectrum. Compositional mediation can amplify, zoom-in, break/attract, slow down/accelerate, or accentuate/soften these timbral components.

We should finally also mention a distinction made between a source of excitation and a filtering-resonance system. Timbre has several facets in relation to pitch: up-down height (register) and spectral constitution. They are named pitch-height and tone-height, and pertain to the spectral fine structure (the relative amplitudes and distances between the partials) and the spectral envelope of a sound (its global spectral profile), respectively. Both can be independently varied as when a vowel is changed in pitch, changing the fine structure (the excitation pattern) but not the resonance regions (the formants); inversely, one can change the vowel but not the pitch, varying only the tone-height. This source-filter model shows two dimensions operative within timbre, discerning between source recognition (related both to excitation and filter), variations of timbre within a single source (related mostly to filter), differences between families of sources (wind, string, percussion, etc) and internal range of tessituras (reliant on the excitation mechanism)\textsuperscript{45}.

### 1.2.7 Infrapitch and rhythm

Even though this larger section is mainly concerned with pitch in relation to harmony, an important aim in this study is to show how harmony can be conceived at several time scales and what properties and differences it has in each domain. We will open up the discussion of the rhythmic realm in its relation to harmony by pursuing some of these links.


\textsuperscript{44} I can think of some compositions which take as a point of departure the phenomenon of multiple pitch in its relation to timbre, beginning with LaMonte Young's Composition 1960 #1 (a very long drone of a perfect fifth), through Tenney's Having never written a note for percussion (1971, a very long tam-tam crescendo-diminuendo) and Alvin Lucier's Silver Streetcar for the Orchestra (1988, for triangle, also producing moving hyperbolic interference patterns in space).

Some features of pitch spill outside its normal range of operation, mapping aspects of harmonic duality into other perceptual registers. There are studies which have thrown insights into the ranges of operation of the two aspects and mechanisms of pitch perception, charting their registers and showing how periodicity analysis works well below the range of hearing in the cochlea and is connected to the zone of rhythm and articulation\textsuperscript{46}. ‘Repeating Frozen Noise’ (RFN) tones, consisting of iterated repetitions of white noise segments, have been used to probe into the infrapitch regions. They are generalized periodic stimuli: their periodicity can lie below the audible range but still have audible overtones; deriving from noise, their partials have uncorrelated random amplitudes and phases, not restricted in their waveform or spectrum; they also possess a rich timbral quality. At infrapitch periods, the close spacing of these partials means that they are always unresolved by the basilar membrane, mapping their pitch periodotopically while projecting a ‘pitchless’ timbre tonotopically. Each different noise sample will have a different but rich timbral quality.

These noise segments can be heard as global percepts of iterance over a range of around 15 octaves, from 0.5 Hz to around 8 kHz (see Figure 2). Above that, all harmonics lie beyond the range of hearing, so the tones become indistinguishable from sine waves. Below that they have a timbre which, due to the spacing of partials becomes especially rich below 1000 Hz. Below 100 Hz, it begins to have a hissing component, being a continuous percept from 100 to 70 Hz and a pulsed one from 70 to 20 Hz. According to their features and qualities, the 5 octaves in the infrapitch range (20 to 0.5 Hz) have been divided into high and low ranges. Periodicities from 4 to 20 Hz are reminiscent of machines, lacking discrete component events, their quality described as ‘motoboating’; the low range (4 to 0.5 Hz) have been described as ‘whooshing’. High range periods correspond in time scale to phonemes, the low range corresponds to syllables and words. Periods higher than 2 s do produce iterance percepts, although they are broken into limited portions of the waveform. Even so, periods can be detected for up to 20 s. They apply well into the musical range of melodic phrases and small scale form.

For most part of the hearing range, place and time mechanisms overlap, precisely where music happens: between 20-40 and 3-5,000 Hz. In this region proportion and timbre are entangled. Place perception spans from the limit of low pitch up to the end of the hearing range, the highest octave belonging exclusively to timbre (labelled ‘amelodic’). Time perception comprises periodicity below the auditory range of cochlear perception, encompassing both rhythm and pitch and involving two mechanisms within its operation. One, pertaining to unresolved harmonics, spans from infrapitch up to around 2.5 kHz, the limit of unresolved harmonics, using the mechanism for periodicity detection (‘complex pattern’) of periodotopic activation of neurons in the inferior colliculus. The second mechanism, based upon resolved harmonics, involves phase locking in the cochlear nucleus lower along the pathway.

Beyond the laboratory, extra cochlear aspects of periodicity perception also incorporate other sense modalities belonging more to the body than to the head and ear, such as proprioception and kinesthesia\textsuperscript{47}, as it happens in a situation of a collective of musicians synchronizing to a common rhythm (the fine tuning of their articulations to some groove), realized more through the body than the ear, by means of low frequency resonance.


\textsuperscript{47} Proprioception is “the sense that indicates whether the body is moving with the required effort, as well as where the various parts of the body are located in relation to each other.” Proprioception. In \textit{Wikipedia}. Last retrieved September 12, 2012, from \url{http://en.wikipedia.org/wiki/Proprioception}. It involves muscle memory, equilibrium, balance and position and is also conveyed by internal organs such as bowels. Kinesthesia refers more exclusively to bodily perception of (self) movement. Many internal organs resonate to low frequency sound waves and serve as sensors by transmitting their movements to the cerebellum, where the signals are integrated.
Rhythm entails periodicities of acoustic patterns that correspond to slowed down versions of intervallic proportions, meaning that there is a quality of consonance or dissonance to rhythmic patterns. Understood as infrapitch analogues of pitch-range phenomena, harmonic concepts can pertain to the rhythmic domain, though the parallelism is not straightforward since each domain has its own specificities. Rhythmic consonance should be more sensitive to the complexities of periodicities involved, as is the case with quintuplets, which are already quite more difficult to ‘digest’ (to use a term from Clarence Barlow) than their analogues in the pitch range (corresponding to thirds). Another difference is that phase phenomena are crucial and inherent to rhythm, while in pitch they are mostly inaudible (or blend as part of the timbre but are not relevant proportionally). Phase plays an important role in metric accents and other kinds of rhythmic displacements. Rhythm is made of pulses, concerning onsets more than continuations.

With the aid of mathematical graph theory, Justin London\textsuperscript{48} analyses the psychological spaces corresponding to pitch and rhythm, showing them to be topologically different and arguing that pitch and rhythm phenomena do not correspond isomorphically to each other. Perceptual phase transitions ensue different emergent properties for objects in each realm, so more than trying to ensure isomorphic relations, what these spaces reveal are key differences between the two domains. Pitch properties have structures similar to Riemann Tonnetz and the psychological spaces we have reviewed, while ‘metric spaces’ have disconnected graphs that are non-planar (their edges cross) as well as highly dependent on tempo, changing the limits and perceptibility properties of each metric graph as a function of tempo. Tonal spaces, on the other hand, are planar (no crossings between nodes and edges) and independent of the tonic, showing how, for pitch, there is no ‘tempo’; all pitches behave in an ‘absolute’ way with respect to their inner rhythmic constituents (see Figure 3). Perceptual phase transitions are categorical borderlands that register changes belonging to different

qualitative realms, each with its own properties, despite there being a continuity in the physical processes grounding them. The same types of phenomena are recognized differently at both sides of the borders, making for autonomous attributes in each realm. This continuum generates a perceptual discontinuum that all the same inherits, transforms and disregards some of these properties.

At the pitch/pulse boundary of around 16-20 Hz temporal integration breaks down, so around it lies the zone of disconnection between sound elements. These discrete sound elements become pulses, acoustically different from the integrated sound they were part of. I have the impression that there could be a perceptual analogue to the physical phenomena of acoustic impedance. As is the case in the resonance of tubes in the physics of wind instruments, there happen two types of impedance: bipolar and unipolar. Crossing a boundary results in a change from a bipolar towards a unipolar condition, from alternate current impedance (positive-negative air displacement) in pitch-integrated waves, to direct current impedance (from no displacement to maximum displacement) in the differentiated individual pulses of the rhythmic realm. Each rhythmic element might contain a timbre inside each pulse, but in another sense it is just a DC pulse, an on-off switch. Rhythm and meter are composed of patterns of these on-off elements, beyond the actual sounds that fill them (which are themselves AC: bipolar and integrated). This AC/DC boundary thus marks a distinction within melody between its horizontal (DC) patterns and the vertical (AC) timbres that make up its notes. More research needs to be done with respect to this topic because even if this transition is not happening acoustically but only perceptually, it provides an interesting way in which to conceive a crucial difference between pitch and rhythm.

Notwithstanding their differences, there are some important similarities between pitch and rhythm,

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49 “Acoustic impedance \( Z \) is the ratio of the acoustic pressure \( p \), measured in Pascals, to the acoustic volume flow, measured in cubic metres per second.” *Specific acoustic impedance* \( z \) “is an intensive property of a medium. We can specify the \( z \) of air or of water. The acoustic impedance \( Z \) is the property of a particular area and medium: we can discuss for example the \( Z \) of a particular duct. \( z \) usually varies strongly when you change the frequency. The acoustic impedance at a particular frequency indicates how much sound pressure is generated by a given acoustic flow at that frequency.” Additionally, “DC (direct current) means constant or slowly varying current. AC (alternating current) means any current in which the movement is alternating backwards and forwards (oscillating) with no overall motion. AC is more interesting because the impedance can vary with the frequency of oscillation of the current.”. Taken from Wolfe, J. (2010). What is acoustic impedance and why is it important? Last retrieved June 12, 2011, from http://www.phys.unsw.edu.au/jw/z.html.
conducive to an analysis of rhythm in terms of harmony and pitch as well as the other way around. Rhythm shares the same perceptual mechanism as pitch periodicity, but lacks the spectral one. We can nevertheless speak of the timbral and proportional aspects of rhythm, although ‘timbral’ in this sense would have a different connotation than with pitch. By omitting or binding (slurring) some of its constituent pulses, simple rhythmic relationships can become fluid, blurring their periodicities and appearing more complex and ‘floating’, incorporating elements of noise within rhythmic intervallic proportions.50 The ‘irrational’, slurred rhythms and cloud-like formations archetypal of atonal modernism, are perceived as unit-free, and can be therefore be considered ‘timbral’ in that they seem more like a flux than a pattern.

Another sense in which there is a timbral or continuous facet to proportional rhythms lies in the way metric patterns are articulated. The proportional facet deals with rhythmic layers in terms of relationships and numeric ratios, even if these ratios are not exact in performance: there is tolerance with respect to rhythmic phenomena, even more sensitive in its effects than in the pitch domain, maybe because of the higher allowance given by the slower speed at which this happens. ‘Groove’, the systematic displacement of rhythmic positions, can be understood in a harmonic light as a timbral tolerance acting on proportional patterns, as they are still inferred as being exact, even if conveyed through this distorted medium. This special, sensitive and not easily describable musical feature can provide varied ‘colorings’ or ‘enhancements’ to the same pattern, a crucial aspect of many musical styles. Once a groove fails to convey the rhythmic pattern it belongs to by transgressing its tolerance limits, it stops being a groove to a pattern, becoming instead another pattern.

Rhythm relates to multiples and divisors of its pulses much like pitch, and analogies such as tempo octaves and other rhythmic partials (subdivision or multiplication by 2, 3, 5 etc.) make a lot of sense, with augmentation/diminution of patterns being an octave equivalence of sorts. This concerns rhythm’s vertical, multiplicative dimension, in contrast to its horizontal, additive dimension (its metric sense). There are also pitch range analogues of rhythm, as with early consonance/dissonance theories, which were stated in rhythmic terms. Accents and other phase phenomena in rhythm also relate to spectral features when accelerated up to pitch speed. The concept that best bridges the similarities between the two domains is also the string and resonance is the term that best applies to both harmony and rhythm. We will also see further on that some of these aspects also inherit to higher levels of composition, such as larger sequences, sections, and forms, as there are formal analogues to rhythmic and harmonic phenomena, although their appearance is also altered with respect to their original realms, though sharing properties which are still related to harmonic duality.

1.2.8 Concluding remarks

The pitch related qualias which we have been surveying, such as chroma, timbre, and their supervenient properties of harmonicity and consonance are established principally on a subcortical level of auditory processing (from the ear to the brain stem). Notwithstanding the role of cognitive top-down processing, as well as the various modes of attention, it is psychoacoustic constraints that for the most part determine the harmonic properties of compositional materials. These constraints provide the starting points for a perceptually informed (and computationally assisted) harmonic research from which the potentials and possible functionalities arising out of these materials are experimented with, in order to build and tryout, in practice, the methods or ‘logics’ that are the consequences and extensions of these harmonic properties into the wider scale of musical forms. This is done in rapport with a a theorization that picks up from these findings in order to generalize and systematize them, serving as a platform for the formulation and speculation of strategies to be put again to the test, leading again back into the first stage. This cycle of musical experimentaton

50 As mentioned in Stockhausen, K. (1957) ...How Time Passes... (C. Cardew, Trans., 1959). Die Riehe, 3, 29.
consists of trials and examinations, reflections and modifications, jumping back and forth between the levels of praxis and theory, intermingled with evaluations, analysis, intuitions and serendipities that lead as much to interesting discoveries as to dead ends. Its repeating form resembles more a spiral than a circle, opening up with each new turn in ways that are quite similar to intervallic spirals, never returning to the same interval twice.

As can be seen after laying the ground in this section for establishing the psychoacoustic aspects of pitch, the crux of this research hinges on compositional materials, although the main focus will revolve around methods for incorporating these materials into a musical structure, discovering and inventing the ‘logics’ that can set these ‘onto-logics’ into motion. Around the second half of the twentieth century arose what is now a more or less established line of thought that considers three levels of composition, namely material, method and form, as set out by some important experimental composers. We could now say that with respect to our harmonic research, sonic materials function, as it were, ontologically, in the sense that they define basic musical entities. This is not ontology in the strict philosophical sense, as the term is used to denote the basic constituents or objects used for composition as seen from a perceptual perspective; not as the inherent being of any generalized musical material whatsoever, but only in relation to their ‘perceptual being’, and it is in this sense that this project subscribes to a ‘perceptual ontology’ of sorts. These materials are also ontological in the sense that they lie ‘outside time’ (Xenakis), as their properties are considered in isolation from their musical disposition, as is the case of intervals, scales, probabilities, dissonance metrics, rhythmic patterns, etc.

The next compositional level, that of method or logic, focuses on the coherence and consistency of these sonic entities, their relations and motions ‘inside time’, that is to say, in a musical context. This is where most of the fun happens in compositional research, where theorization is put to use. If this chapter serves to set the stage for the characterization of harmonic materials, then it will be in the other chapters that the development of relations and logics between them shall take place.

Regarding the third level of what Tenney calls ‘aesthetic experience’, we could mention that it revolves around the musical dramaturgy of a work. It involves relations of relations, or structures and forms, and we will try to see how far some harmonic concepts can be extended to this level of scale. It also involves extra-musical considerations, be they the kind of experience that is intended for a piece, or, in more contemporary terms, the kinds of narratives, connections, causalities (or lack thereof) that string together the musical forms contained in a work, also in sonic situations that lack directionality, as with non-narrative forms. They are also extra-musical in the sense of pertaining to ideas beyond music itself, be they inspired by other artistic disciplines, political or social issues, myths, maths, specific circumstances of a piece or installation, and many other etc’s. This is the level in which thought invests matter, being in close rapport with the level of compositional logics, as it defines, selects and alters these logics according to an overriding aesthetic design, concept or dramaturgy. The three levels influence each other in different and complex ways (in a manner reminiscent of ‘bottom-up’ and ‘top-down’ processes), and behind these considerations lies the intention to keep the theorization of materials as indifferent as possible to their aesthetic uses in order to take full advantage of their behaviors, leaving open the prospect of materials influencing and suggesting dramaturgies or, conversely, knowing when aesthetic decisions require us to ignore or modify the way the qualities of these materials are put to use.

We can escape the danger of falling prey to the dispute between ‘nature’ and ‘culture’, or between ‘cultural conventions’ and ‘biological determinism’ by having a clear idea of where and how the

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51 These concepts begin with Arnold Schoenberg, but I’m thinking more on the formulations by John Cage, who focused more on the first two categories (and divided form into structure and form), and also others such as Iannis Xenakis and Karlheinz Stockhausen. This approach is quite independent of aesthetic school, though they are all share an interest in systematic composition.
levels of imbrication of these aspects stand and by keeping a healthy dose of skepticism for those approaches that might reduce musical entities to either of the two extremes. It is important to be able to ‘modulate’ all the given and inherited materials and arbitrarily bend them into whatever direction is required by the compositional decisions. It is also as important to be able to listen and pay attention to the materials, letting them kick back and have their say, not serving only to be meddled with. The neurobiological basis of harmonic structures assists by delineating some of their material characteristics, with the intention that they remain aesthetically neutral. However their characteristics are used, their main requirement is for them to open up paths for compositional speculation and experimentation.

Having said all this, it may sound paradoxical to say that the psychological aspect of pitch and harmonic materials is not even be that important after all. I do not claim that harmonic materials arise exclusively in the human mind. Beyond psychology or biology, some of these properties are inherent to the sound emitting objects themselves, to the acoustic waves or to the numbers lying behind the definition of their patterns. Some proportional aspects of these materials lie beyond psychology, and involving, as we will see in the next section regarding aisthesis, not only sensation but also intellection. The cognitive approach to the study of consciousness focuses on efficient causes, when, as we will see, it is the other kinds of Aristotelian causes that are as or even more important for music, precisely the causes that lie outside temporality (formal and material cause).

Our survey of the psychological and cognitive processes responsible for pitch and lying behind harmony has been done not to reduce harmonic materials to biological processes, but to find in all these loosely connected bunches of facts evidence for and a characterization of harmonic duality in order to map out possibilities to be exploited in algorithmic composition. As we will see in the following section, harmonic duality can be characterized from a very different point of view, that of Greek harmonics, and both readings, the biological and the historical-metaphysical help to delineate and underpin a harmony much more robust and interesting than if it came from a single perspective. What’s more, and I see it as an advantage, these accounts are supplementary and do not correspond easily to each other, each involving a quite different register of thought.
1.3 Greek Harmonics

The duality of harmony can be traced back to the Greek harmonists by journeying through some of their ideas in the interest of differentiating and characterizing its attributes. The excursion also serves to introduce and discuss many elements and concepts that a renewed and present day harmony needs to incorporate. A rereading of Greek theory can help unmuddle, restore and reinterpret ideas masked or ignored by conservatory harmony as a result of layers accumulated over the centuries. Being historical, the review is nonetheless focused from a present perspective with a view on opening up new compositional possibilities. A reevaluation of some implications of Pythagoreanism, much underestimated in our times, concludes the section.

The science of harmonics \(^\text{52}\) (harmonké) was a companion to the sciences of rhythmics and metrics whose task was together to classify and describe the regular and repeated patterns of form and structure underlying the diversity of melodic, rhythmic or metric sequences in music. Metrics deals with patterns formed by lengths of syllables in verse, rhythmics with patterns within which sequences of long and short syllables are divided and grouped into repeated structures, and harmonics with the structures underlying melody (melos). The harmonists set out to identify the varieties of scales and tuning systems which could be reckoned as musical, a task that implied finding quantitative representations for intervals and melodies, classifying scales and their transformations, and seeking underlying fundamental principles behind these structures. Questions such as their rooting in human culture or in something independent of humans, or in mathematics, as well as the status of their applicability beyond the musical sphere were the kind of issues raised and discussed by harmonists. As such, it was a full blown science in the sense of a discipline to discover and demonstrate a body of truths, regardless of whether they could be assimilated to mathematical sciences or to the ‘sciences of nature’ (physiologia). Needless to say, this is very much in sync with our project.

1.3.1 The two schools of harmonics

The two main doctrines within harmonics are the mathematical and empirical. They are fundamentally and irreconcilably opposed in their premises, methods, and aims. The first group is epitomized by Pythagoras, even though there are only second hand accounts of him. Instead there are many Pythagorean scholars, some more mathematical, more philosophical or metaphysical than others, some more practical and linked to musical practice. Their basic premise is that there is a strong connection between pitch intervals and whole numbers, the account ranging from straightforward correlations up to outright cosmological accounts. The earliest mathematical harmonist of whom there are surviving fragments of text is Philolalus, whose ideas fall into the metaphysical kind, identifying the structure of the cosmos with the proportions of the main concords inside the octave and describing the world as a harmony (harmozein, ‘to fit together’) between the unlimited (continua) and the limited (which set limits through shapes and other discrete structures). Beyond its philosophical aims, it already gives us an initial and generalized definition of

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harmony: the fitting together of disparate elements as well the tension between continua and discontinua: a musical scale is the limiting of the continuum of sound by ratios. Plato follows this line of reasoning in the *Timaeus*, portraying the Demiurge and the creation of the World Soul from principles stemming from harmonic science, though they are put to use in a purely metaphysical way, beyond any musical reality. Devoid of sensory considerations, concords and discords were properties of numbers, having no relation to actual sounding tones.

Of the other Pythagoreans, Archytas is far more interesting for us as he takes a route that correlates numbers, interval ratios, sounds as heard, music as practiced, and physical accounts of sounds. He provides what is perhaps the first example of psychophysics, inaugurating musical science. Later harmonists, the most significant living between the fifth and the third centuries B.C., included Aristotle, who concentrated on metaphysics, and later on Euclid, who tried to systematize earlier systems to the greatest extent possible. Ptolemy’s approach, in the second century A.D., summed up the most relevant developments of mathematical harmonics along with empirical backing while furthering the music of the spheres in connection with his astronomical interests.

On the side of the empirical harmonists the important figure is Aristoxenus, whose treatise *Elementa Harmonica* is the largest and most complete extant treatise on empirical harmonics (and of any kind of harmonics for that matter) of that era, providing much information regarding his school and the critiques leveled against mathematical (and other empirical) harmonists. His purpose is to make of empirical harmonics a science as rigorous and systematic in method as mathematical harmonics (he was a student of Aristotle and took cue from his account of what good science should be), despite the fact that their aims and procedures were incompatible.

The main characteristic of empirical harmonics is its emphasis on music as heard and the ear as the ultimate judge of musical materials. Instead of measuring intervals with discrete ratios, Aristoxenus measures them in terms of distances in a continuous linear space. Instead of associating the consonance of an interval to the arithmetic properties of ratios, he took their consonances and magnitudes as given facts. Since intervals could be slightly mistuned but still perceived as belonging to the same intervallic category, this was taken to mean that even the principal concords of the scale had a narrow range of variation. Notes exist as points along the continuum enclosed within tolerance ranges. This is the first instance of the crucial concept of harmonic tolerance.

Instead of taking sides with one point of view or the other, what we want is to sort out various themes that traverse harmonic duality because they stand out from the approaches and theories of both schools – even though each doctrine has certain dependencies on the other and does not map cleanly into each side of the duality. What I have referred to as the proportional aspect of harmony is connected with mathematical, while the timbral attribute is connected with empirical harmonics. Furthermore, the opposite poles of the discrete and the continuous are associated, respectively, with ratios and pitch distances, and each connected with a mathematical science, arithmetics and geometry.

Harmonic duality may be seen in the light of these couples. I call *timbral* the continuous aspect because it has to do with sensation, with intervals perceived as actual *sounding qualities* (which is a definition of ‘timbre’), continuously variable and in a state of flux. Proportionality, on the other hand, is independent of the timbre with which it is instantiated, lying ‘behind’ and exceeding sensory qualities. Proportionality is a pattern inferred from the actual sounds, being more like platonic *eidos*, in the sense of pure Forms detached from their sensory presentation, not directly

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53 This is probably the earliest definition of musical harmony we find in ancient Greeks harmonics, and it is noteworthy to point out its resonances with contemporary thought in order to construct of a concept of harmony fit for our times. Thus, Philolalus could be paraphrased in Deleuzian/Badiouian terms as: incompossible elements coexisting in a ‘disjunctive synthesis’. See Badiou, A. (2000). *Deleuze: the clamor of Being*. Minneapolis: Minnesota University Press, 44 and 58.
accessible to hearing. They are not, however, eternal Ideas preexisting and separate from their presentation, but rather, to use an Aristotelian concept, substantial forms, that is to say, intelligible rather than sensible. These eide are the essential patterns indwelling at the core of sense objects, after their accidental or inessential qualities have been subtracted by reason, but they are not separate from their individual instantiation. A ratio (logos) is the definition pattern or form of an interval, the word logos being synonymous with all three of these notions, as well as with ‘speech’, ‘discourse’ and ‘reason’, depending on the context.

We never encounter the harmonic aspect in isolation, devoid of timbre, not even if the interval is made out of sine waves, as they still have a timbral quality. These timbral qualities are the medium through which harmonic patterns are encountered. A harmonic pattern or form always comes sensually attached with timbre. Timbre is the sensual medium for these patterns which are alluded from it.

One fundamental difference between the two schools concerns measurement. The empirical approach seeks to find the minimum interval which can serve as a unit of measurement for others (called a diesis). The search for this ‘just noticeable difference’ of pitch was pursued by Aristoxenus through comparing two closely tuned strings until the difference between them was no longer distinguishable. From this small interval springs the analysis of tunings and scales (composed out of genos, and forming systema). Aristoxenus achieved his analysis and divisions of the tetrachord (which has the size of a fourth, a diatessaron) inside a grid of 30 steps, corresponding to a sixth of a semitone.

Ratios, on the other hand, not being directly accessible to hearing, represent either relations between lengths of strings or pipes, or correspond to aspects of the physical events that cause the perceptions. Being indirect to perception, they are arrived at through reflection and observation by means of measuring devices like the monochord. Aristoxenus’ criticism of mathematical harmonics was aimed not at denying the existence of ratios and the phenomena they explain but at their relevance to the study of music and to harmonic science. Contrary to this view, our research finds it important to maintain a flow of information between the accounts given by both domains in a way similar to Aristotle, for whom empirical harmonics provides the facts while mathematical harmonics – which works in a different domain – provides the principles from which these facts are explained and demonstrated. This resonates with our experimental approach to harmony in relation to sciences of perception and mathematics, which in my opinion is compositionally the most productive.

Concerning units of measurement, it is paradoxical that the basic ratio from which other proportions derive is the octave, 2/1, which is not a small interval. Aristotle tried to find these units of measurement to no avail, precisely because it is the smaller ratios which are derived from the octave, but not the other way around; also because most ratios – the ones that matter – cannot be divided into equal parts, so there cannot be a fundamental measure that adds up to them. We can also think of units of harmonic ratios – units of harmony or harmonemes – as corresponding to the prime numbers that factorize their terms, or the prime numbers that constitute the axes of harmonic lattices, but that is an extrapolation only present in latent form in Greek harmonics.

Empirical accounts describe facts about music and musical perception. They provide information which cannot be proved from a mathematical perspective. The fact that the octave, fourth and fifth are consonant is primary and cannot be demonstrated from arithmetic alone, not following from any mathematical theorem. Once this consonance is acknowledged as an irreducible fact, it is relatively easy to correlate it with properties of numbers and ratios – Euclid actually tried, unsuccessfully, to prove consonance from mathematics alone. The mathematical approach, on the other hand, once the connection between arithmetics and music has been made, can provide explanations and reveal underlying patterns for some more complex and derivative facts, some
theorems of numbers having relevant consequences for music. This shows, however, that the mathematical approach, when its aim is not completely metaphysical, actually requires an empirical departure point, a ‘sensory axiom’. The empirical route, by the way, is not a purely qualitative discipline, but one that seeks out quantities from which the notation of melodic sequences and thus the classification of scales and tunings is made possible. These quantities, consisting of half, quarter, third, sixth or full tones will provide the idea on which temperaments will be based much later on, although Greek music had no need for them. Both approaches, even though irreconcilable in spirit, provide complementary information that feeds into each other.

1.3.2 Pythagorean tuning systems

Mathematical harmonics provides a connection between qualitative attributes of sensation and properties of numbers. It explains the ‘how’ of consonance and dissonance (as we saw, it cannot explain the ‘why’ of it) by relating properties of numbers and ratios with sounding qualities in formal terms, which is how Pythagorean harmonists classified ratios. They had six categories: equal, multiple, epimore, epimere, multiple epimore and multiple epimere. Equal ratios are those whose terms are the same (unison in musical parlance), multiple are those whose terms are multiple of each other (2/1, 3/1, 4/1, etc, forming overtone series); epimores are those whose difference between terms is 1 (3/2, 4/3, 5/4, 6/5, etc, also known as ‘superparticular’ ratios) and epimeres those whose difference is not a portion of the smaller term – in epimores 1 is a part of both numbers and the mark of their difference – but some more complex part, or rather ‘parts’. Relative to today’s overtone series, multiple ratios occur between a fundamental and its overtones; epimores happen between successive adjacent overtones and epimeres between non adjacent ones. This classification, together with the precedence given to ratios with small numbers provides the hierarchies with which to organize the qualities of intervals. This hierarchy has fallen into disuse by now, but I find no intrinsic objection to reject it as a possible harmonic classification among others.

Pythagorean music theory originates before Pythagoras with Near-Eastern civilizations such as Babylonians and Egyptians. A distinction should be made between the well known Pythagorean tuning (based solely on multiples of perfect fifths, thus limited to numbers with prime factors not larger than 3, or ‘3-limit’) and Pythagorean music theory, as the latter is not restricted to the former, which is more a theoretical construct than a derivation from musical practice. The tunings and theory used in Greek music come from the Sumerians, who based their number system on the number 60 and its divisors, corresponding to scale intervals and correlated to their main deities. In this mythological framework, numbers had, in connection to the attributes and powers of the gods, important qualitative properties. Pythagorean arithmetics also classified numbers according to their qualities (square, triangular, cubic, oblong, etc) and not just their size. The prime factors of 60, {2, 3, 5}, form 5-limit intervals, 3-limit Pythagorean tuning not corresponding to ancient musical practice. Whence, the divisor set (the set of all numbers that divide another) of 60 can be considered the seed of most heptatonic scales.

Early Pythagoreans found their scalar unit in the tone 9/8, which appears among the main consonances as the measure of their difference: that between two fifths and an octave, an octave and two fourths, and as the space separating the fourth and fifth within the octave. The Pythagorean tetrachord can be seen as two 9/8 tones and their residue with respect to the fourth which is the limma (‘remnant’), 256/243 (a step of around 90¢). This parsimonious method of dividing the octave, as five tones plus a limma, is one that was adopted in the West up to the Middle Ages:

“It uses the principles inherent in the beginning of the integer series more economically than any other. For if we reflect on the matter, we see that in some sense the fourth itself is a limma, left over
from the projection of the fifth back into the octave. The fifth, in its own way, is a *limma*, left over from the projection of the octave forward into the twelfth (3/1). Only the octave seems to remain aloof from this process, being generated in some more mysterious way directly from the womb of unity itself."\(^{54}\)

The Pythagorean *harmoneme* was, for practical purposes, 9/8, though it is clear that the only irreducibly given interval not deducible mathematically is the octave, which in my contention has more reasons to be considered an ‘atomic’ constituent of proportional harmony, though not the only one.

Of the Pythagoreans, it was Archytas who took on a different and novel route by dividing intervals according to mathematical means, namely the geometric, arithmetic and harmonic, which he introduces for the first time into theory\(^ {55}\). The geometric mean is the simplest, used to test the equality of ratios. However, it deals mostly with continuous proportions not expressible as whole number ratios (as they are *alogos*, irrational), and cannot be applied to consonances (*epimoric* ratios). It is correctly named as it belongs to the continuous constructions of geometry. The arithmetic mean may have arisen from purely numerical considerations, but it is in harmonic science where it shows its usefulness, as it permits the division of consonances. Its drawback, however, was dividing intervals ‘the wrong way around’, with the larger one at the bottom, inducing Archytas’ search for a ‘subcontrary’ mean, also called *harmonic* because of the solution it provides to a musical problem. Taken together there is a symmetry between the three: the arithmetic and harmonic means gradually approximate, as their terms grow, the geometric mean, one from above and the other from below. They divide the first multiple proportion, 1:2, the octave, by considering it as the multiple-multiple 6:12. The arithmetic mean yields 9 because 12 - 9 = 9 - 6; the series 6:9:12, equal to 2:3:4, gives a fifth and a fourth. The harmonic mean, a relationship between ratios instead of a difference between their terms, yields 8 because \((12 - 8)/(8 - 6) = 12/6\); this 6:8:12 series is the same as 3:4:6, a fourth and a fifth. Taking both means together as 6:8:9:12 we can see the 9/8 tone in between the divisions. This is the most important construction in Pythagorean mathematical harmonics, the one that provides Plato with his cosmogony and which Richard Crocker calls the *harmony*.

Archytas then applied this process once more to both the fifth and the fourth, yielding a sequence of *epimores* which sorted become: 1:2, 2:3, 3:4, 4:5, 5:6, 6:7, 7:8 and 8:9. This is not merely a mathematical exercise, but the starting point for a correlation with the way musicians tuned their instruments by, for instance, tuning slightly off from a ditone 81/64 to a ‘sweeter’ 80/64 = 5/4 by what is called the ‘method of concordance’, in which the ear is the judge. He adapted mathematics to musical practice as it was impossible to measure these ratios by ear, so they were arrived at through a combination of mathematics and musical skills, adapting these ratios to the given genera of his time, which meant producing, by addition or subtraction, other intervals needed to adjust these deduced ones to the different tetrachords. Some of these derivative intervals are the 28/27 third tone, the 36/35 quarter tone, the semitone 16/15, the 32/27 Pythagorean minor third and a very strange 243/224, close to a neutral second of 141\(^ {\circ}\). They don’t mean much by themselves until we consider the constructions within the pentachord formed with a note a whole tone below the tetrachord, as John Chalmers suggests, revealing important intervals that appear between these degrees, such as the 6/5 and 5/4 thirds, the notable 7/6 (a subminor third, 266\(^ {\circ}\) in size which was ubiquitous enough to deserve a name of its own, *ekbole*), the 9/7 (a supermajor third, the difference between the fourth and a third tone) as well as the large whole tone 8/7.

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54 Crocker, R., *op. cit.*, 197.
55 In today’s terms: Geometric mean: \((a+b)/(b-c) = a/b = h/c; ac = b^2\). Arithmetic mean: \((a+b)/(b-c) = a/b = b/c\). Harmonic or subcontrary mean: \((a+b)/(b-c) = a/c; 1/a+1/c = 2/b\). From Chalmers, J., *Divisions of the Tetrachord*, 29. Crocker gives a detailed and fascinating account of these means in the second of his aforementioned articles.
Figure 4. Archytean intervals within an enharmonic pentachord. (From Chalmers, 1992.)

His constructions are subtle, not merely theoretical but derived in conjunction with a keen sense for musical practice and the perception of intervals.

"Archytas' work opened the way for richer and more detailed explorations of ... abstract patterns of order, most notably by turning the spot-light on the special status of epimoric ratios, by demonstrating techniques for manipulating them in the construction of harmonic divisions, by proving these ratios' resistance to equal division, by his classification and definition of the three 'musical means' and by his deployment of these means in his analysis of attunements. ... [H]is studies in physical acoustics point also to a scientific interest in sound and pitch themselves, and reinforce the impression given by his tetrachordal divisions that he was concerned, much more directly than earlier Pythagoreans, with the domain of the audible for its own sake."

"Archytas is interested in the numbers by which phenomenal things are known and of which they give signs." This again resonates strongly with the aims and procedures of the present project. He is a paradigmatic harmonist, combining mathematical and empirical approaches, as well as an interest in physics and the correlations between the domains. Archytean intervals diverge sharply from 3-limit tunings, going as far as 7-limit and including some non-epimoric ones as well. These ratios are more complex and rare than most intervals used nowadays, some of them still presenting problems as to their use. Through Boethius' reading of Pythagorean music theory, medieval theories and the practice of plainchant became anchored to 3-limit tunings. Ptolemaic 5-limit tuning, which accounts for the heptatonic modes of Greek music, remained in use in secular and folk musics, to be recovered in the practice of vocal polyphony since the 14th century and given theoretical legitimation, within a Neoplatonic framework, until the 16th century, in Gioseffo Zarlino's senario. He proceeded by dividing the fifth with harmonic and arithmetic means, this time interpreted as chords with 5/4 major and 6/5 minor thirds, hence authorizing major and minor chords as the harmonic units of tonal harmonic practices.

1.3.3 Consonance and Dissonance

This is an initial discussion into one of the most intricate topics within harmonic theory. It will now

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56 Barker, op. cit., 306.
58 Furthermore, he was, amongst other things, prince, warrior, mathematician (with long lasting contributions to the field of physical mechanics) and a teacher of Plato (for whom he served as a model of the 'philosopher king'). *Ibid.*
revolve around harmonic duality and Greek music theory, in section 2.1.4 it will involve the psychoacoustics and psychology of the late nineteenth and early twentieth century, and in section 3.1.5 it will revolve around harmonic metrics.

We should recognize, as James Tenney makes us aware, that there are at least five ways of conceiving consonance and dissonance in the history of Western music. Each conception can span different styles and genres as it refers more to the underlying assumptions behind the consonance/dissonance distinction than to the aesthetic attitudes toward them, as these can vary considerably within each conception. They can also be coextensive, operating simultaneously in different musics of a single historical period. The one relevant for the Greeks, the horizontal conception, is still in use today in many sorts of heterophonic and tonal melodic music.

For the Greeks, symphona and diaphona (concordance/discordance) were understood melodically, employed to discern 'degrees of affinity, agreement, similarity or relatedness between pitches sounding successively.' Harmonia is the attunement of strings to a scale and concordance/discordance refers to relatedness between tones. Both empirical and mathematical harmonists set a sharp cutoff point in demarcating the line between sym- and dia-phon, allowing only octaves, fourths and fifths as consonances. For mathematical harmonists its was required that a ratio be eppi-phon and that its terms lie within the tetraktys of integers 1 to 4. This restriction entailed that only 2/1, 3/2, and 4/3 could qualify as consonances, when it was clear that some consonant intervals, particularly 8/3, an octave plus a fourth, would have to be considered, but not perceived as dissonant. Greek theorists swept this aporia under the carpet until Ptolomy solved it with a law stating that octave compounds of consonances are themselves consonant. This further evidences the octavicity of harmonic relations, the octave being a harmoneme under which other relationships are confined.

The problem with 8/3 also points to the fact that consonance and dissonance can be conceived either as a continuum of gradations or as being antinomic, in a binary opposition, where an interval belongs either to one category or the other. The former grasp suggests a coloristic approach to intervalllic use and choice, abounding in gradations of hues, suitable for transitional purposes, the actual sounding qualities of intervals being the main focus of attention, however broad the possible logics constructed upon these materials might be. This is timbral conception, pertaining (but not limited) to late nineteenth and early twentieth century composition. It lies behind Schoenberg's concept of the emancipation of dissonance, which describes the harmonic situation at his time and place: there is no absolute dissonance but rather a spectrum of 'sonances' to which the ear must accustom itself as they progress further towards unexplored areas. By the combination and compounding of even a few intervals together one can plainly see that a high number of potential sonance levels arise from this conception. These levels are closely connected to matters of voicings, registers, dynamics and sound source. This type of harmony encompasses aspects of Tenney's CDC-2 – vertical, polyphonic – and CDC-5 – timbral, psychoacoustic – conceptions, including various degrees of fusion between proportionality and sound for its own sake.

The second conception, on the other hand, is governed by contrast. Consonance/dissonance can be the cause of both melodic and harmonic movement by means of tensions and resolutions, as in CDC-4, functional tonality, or can be an effect resulting from movement, as in counterpoint, CDC-3. A decision has to be made as to where to draw the line between the two poles, and this alters the resulting logics that set the materials into motion. It is less attached to sensory considerations than to functional and contextual ones, as notes can be treated as dissonant without actually sounding dissonant. This is the case when a note in functional tonality does not belong to either the prime,

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60 Ibid., 4.
third or fifth of a triad but nevertheless forms a consonance with the fundamental root (sixth chords for example, or a six-four major chord which is considered dissonant without being aurally so). This kind of proportional harmony is less dependent on the sounds which populate its intervals, leading to the consideration of widely different chords as belonging to the same harmonic category, as is the case with triadic inversions, or when a fundamental is implied but not sounded.

In terms of harmonic duality, the terms ‘consonance’ and ‘dissonance’ refer to the timbral dimension. This aspect depends on spectral constitution, perceptual salience of the components (their degree of fusion, called tonalness), and the interaction between partials which produces psychoacoustic roughness (or sensory dissonance). On the other hand, the terms ‘harmonicity’ and ‘inharmonicity’ refer to the proportional feature which is independent of spectrum, only pertaining, as it were, to the fundamentals that carry the interval and thus, to the commensurability or simplicity of the numbers defining their ratio, as is the case with time based pitch perception theories. Historically there has not been much of a distinction between these two senses and some aspects of both have been conflated together in different ways. In the case of the Greeks, they were mostly referring to harmonicity, but they also discussed consonance in relation to two simultaneous sounds and the occurrence of fusion between them, involving the timbral aspect.

The following is list of factors involved in the harmonicity/consonance of an interval. Some of these factors involve both attributes, some only one, but for now they mainly concern harmonicity more than consonance. They are:

- The form of an interval’s ratio. The dividing line is drawn between epimore and non-epimores;
- The magnitude of the pitch distance (a timbral consideration always present within proportionality). For pure sounds (sine waves), intervals larger than a critical band are more consonant, but with complex timbres it depends on the interaction between partials;
- The independence of an interval’s harmonicity with respect to register, which is the same as saying that octave compounds of intervals retain their harmonicity (Ptolemy’s law);
- The ‘simplicity’ of the numbers involved in its ratio:
  - Considerations based on the ordinal index of a number, i.e. its size, such as their inclusion in the tetraktys for the early Pythagoreans, or within {1..5} for Ptolemy;
  - Consideration of the prime factors of the terms in a ratio, a concept which we do not find in the Greeks although it is clearly a corollary of Greek harmonics and math;
- The melodic/harmonic function of an interval in context: the way intervals follow and precede each other, along with their function within a scale (their dynamis);
- The metric weight or rhythmic accentuation of an interval;
- The ‘aesthetic attitude’ or compositional/cultural conventions towards an interval’s sonority. Perceptual qualities can be ignored or pursued in different and arbitrary directions. This consideration stems from the point of view of twentieth century composition, but has been present in all kinds of musics and epochs.

There is no doubt to the fact that epimore ratios possess a special quality. They are easier to tune by ear, as it happens with acoustic instruments, where non-epimores lack a special resonance that epimores do possess. Archytas’ theorem proves that epimores have no integer geometric mean, that is, that they cannot be divided into halves. This is in direct conflict with the Aristoxenian practice that divides

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61 What ‘simple’ and ‘commensurable’ mean is not so obvious and will be developed in section 3.1.5.
any pitch distance by half (or any other part) without difficulty by considering them as linear distances. We can now make sense of this apparent contradiction by distinguishing each procedure as referring to a different and independent dimension of harmony. Proportionally it makes no sense to talk about the middle point of an interval (half an octave, say) as they are unitary characters. It is also absurd to talk of an interval not expressible as a ratio, as would be the strict case with any interval of equal temperament, excluding octaves. On the other hand, notes can be placed wherever we want in the pitch-distance continuum, but they only acquire harmonic meaning by being understood as tolerably close to a proportion. The size of this tolerance depends on harmonicity as well as the function of the intervals in context. Thus, there are intervals whose function is timbral, when they do not refer to a harmonic context (as in the case of ornamentation, transposition, register changing, doublings, and so on) and intervals that function proportionally, independently of register and timbre and which do not have to be exactly tuned to function as such.

There is also the issue of complementarity or inversion. An interval can be complemented modulo another interval, the most common case being inversion modulo octaves. A ratio is reflected in its opposite direction and then octave transposed back to its original register. For instance, the 5/4 major third reflects into a 4/5, which when moved up an octave gives 8/5, a minor sixth. The proportional harmonicity of complements is maintained but its timbral aspect of height changes, so the resulting consonance perception will diminish or increase depending on the simplicity of the new interval with respect to the first as well as its pitch distance. Intervals other than octaves can also serve as moduli, but as with the case of non-octave related equivalences, their use has to be clearly constructed with an appropriate context.

The symmetry stemming from interval complementarity is a fundamental harmonic feature. These symmetric properties can be applied to whole intervalllic systems, as an arbitrary pitch set is only ‘complete’ when all its intervals have complements, making the set a mathematical group under the operation of intervalllic addition, where every interval has an inverse and there is a neutral element, 1/1. Much like overtone and undertone series, o-tonalities and u-tonalities in Harry Partch, the quadrants in harmonic space and the relation between arithmetic and harmonic means (which tend towards the geometric mean), inversion is a harmonic feature that forms symmetric clusters around intervalllic classes. It is likewise expressed inside an interval by the property of standing either ‘up’ or ‘down’, that is to say, of the two notes that compose it, which of them has the most weight and feels like the root. If it is the lower note, then the interval is standing up, otherwise it is ‘upside down’. There is an intriguing interrelation between the topics of complementarity, symmetry and the tendency from above and below towards logarithmicity, the linear pitch distance embodied by the geometric mean and approximated asymptotically by the integer means. Logarithmicity is a way to fit or compress more information into a limited space. It is the final limit tendency of periodicity, a tendency achieved from above and below. Together, periodicity and logarithmicity form a dialectic which lies at the heart of the thinking about harmonic duality.

1.3.4 Dynamis

Dynamis is a technical term found in Aristoxenus’ Elementa Harmonica referring to the ability of intervals to follow each other and the ways in which this happens. It is a relational potential inherent in notes, independent of their intervalllic size. Melodic function (dynamis) is a higher perceptual concept in harmonic science that is encountered through hearing (akoe) and thought (dianoia) over the course of a melody. This incorporation of thought and perception is referred to as aisthesis: perception sensitive to musical meanings of notes and intervals in various contexts.

The Greek word dynamis has several meanings, all of them useful for elucidating harmonic
properties: it is most literally potential, or capacities *in potentia*, powers obtained through relations; it is also movement, dynamics as the capacity to produce (melodic) movement or change; finally, it is translated from Aristoxenus, who refers to it as one of the non-quantitative discriminations found in harmonic science, as function. This function is independent of interval sizes since melodic characters can remain stable while the magnitude of the intervals that instantiate them can vary (within limits). The concept is closely connected to the Greek *genera*, and it is valuable that we pay them a short visit.

We have seen that the octave is the fundamental interval from which harmonic and arithmetic means are used to divide it and produce other concords, namely the fourth and the fifth. Proceeding a step further, each of these intervals can in turn be divided. The fifth produces harmonic intervals (4:5 and 5:6, as mentioned above) while the fourth produces melodic ones (7:8 and 6:7, which are not used harmonically but melodically). This suggests that the fourth is a primary unit from which to generate melodic divisions. Dividing it with an ‘infix’, a semitone or a tone, it produces 3 tones, giving rise to pentatonic scales when two of these structures are conjoined within an octave (for instance, dividing the fourth A-d to form A-c-d and then adding the complementary fourth e-g-a to complete the scale A-c-d-e-g-a). Adding another infix to fill-in the wide interval produces a 4 note tetrachord which yields heptatonic scales. As with the Pythagorean harmonic derivation of intervals, we can make sense of melodic intervals as spanning from divisions of larger, stable ones.

Tetrachords are the building blocks of scales, consisting of two tetrachords, either conjunct (a-d-g) or disjunct (a-d and e-a, separated by a tone and forming an octave). The notes bounding the tetrachords are fixed, forming the scalar structural framework. Changes in the positions of the two movable notes inside tetrachords determine their genera, which are of three types: diatonic, chromatic and enharmonic. Each genus has different variations or ‘shades’ in tuning (*chroai*, such as ‘soft’, ‘tense’, ‘tonic’ and ‘even’). Diatonic has no interval smaller than a semitone or larger than a tone, but chromatic and enharmonic have crowded intervals at the bottom and larger ones at the top. Intervals in all genera are unequal in magnitude, and the fact that Aristoxenus divided the fourth equally into 30 steps does not imply the common reading according to which this idea anticipates equal temperament: there is no place for equal intervalllic spacing in Greek music, equal temperament arising out of different musical problems, namely modulation in just intonation for keyboard and fretted instruments. Aristoxenus’ divisions are theoretical means of measuring and classifying the unequally spaced intervals of his time, and never did he refer to the actual stacking of more than two or three equal intervals together.

Enharmonic was considered the principal genera by Aristoxenus (its meaning was to be in tune, a *harmonia*, although already by his time it was falling in disuse and was alluded to as ‘old style’). Chromatic is a deviation from enharmonic (*chroma* meaning coloring) and diatonic was initially only used in certain regions, but in time this changed and ended up dominating the music in Roman times, later inherited to Europe. Using the letters *q*, *s*, *t*, *x*, and *d* to stand for quartertone, semitone, tone, three semitones (trihemitone) and ditone, their basic structures are the following:

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  enharmonic:  q, q, d
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62 An example of this misunderstanding can be seen in Xenakis, I. (1985). Music composition treks. In C. Roads (Ed.), *Composers and the Computer*. Los Altos, CA: W. Kaufmann, and Xenakis, I. (1992). *Formalized Music: Thought and Mathematics in Composition*. Stuyvesant, NY: Pendragon Music Press, 182. He claims that Aristoxenus ‘invents, in theory, a complete, equally tempered chromatic scale with the twelfth of a tone as the modulus (step),’ Moreover, his terminology designates Pythagorean theory as geometric and Aristoxenian as additive. This because adding proportions is a multiplicative operation, while distances become additive through their logarithmicization. His terminology is founded upon an operational, algebraic, viewpoint, not on the correlation between music theory and the two main fields of mathematics, as in our case (the former related to arithmetic, the latter to geometry). More than an inconsistency it is a difference in terminology.
There are more variations on each genos depending on how some of the intervals are defined for each shade, a change in one of them evidently modifying the others. The intervals, once defined in one of the possible shades cannot be further divided: they are incomposite, no matter how large. A tetrachord is characterized by its largest interval: enharmonic is distinguished by the incomposite ditone (construed by Archytas as a 5/4 but without tertial implications, used as a melodic step; other rationalizations including 81/64 – the Pythagorean ditone – and the unusual 19/15); chromatic is characterized by some variety of trihemitone (regarded as 32/27, 6/5 or 7/6); and the diatonic by a tone, which could be different from the disjunction tone (interpreted as 10/8, 9/8, 8/7, and even 7/6).

Melodic functions appear when each note in a tetrachord is identified as carrying a specific behavior. Because several interval sizes are shared by different genera, this recognition depends on function, not size. Notes are not just pitches, in the sense of only occupying a position within a system, but functions acquired through their relational roles within that structure. A ‘route’ (hodos, the root word for ‘method’) is a progression, a specific melodic formula reminiscent of melodic patterns that form part of the structure of hindu raags and arabic maquam. It refers to the possible movements melodic successions can take when traveling between stable pitches, defining the features of intervallic progressions, where each note has the potential to influence future alternatives that depend on a wider pattern of relationships beyond the previous, current and following note. It is not just a fixed point in the structure, but something with its own ‘power’ or dynamic properties which impel the melody to move forward.

### 1.3.5 Corollary. Dynamis: the horizontal core of harmony

Harmonic and melodic tensions (the propensities to move) are not limited to the intrinsic properties of chords and notes, but result from context. Differences between genera were comprehended through aisthesis: through hearing and thought, which also means through training, and this is where cultural schemata play a role in the organization of these contexts. Harmonists were interested in explaining the nature (physis) of melos, not the conventions built on these structures. Greek systema were not given phenomena but constructions created by theoreticians, so the question centered on why these systems and not others reproduced the melodic roles taken by the notes. The rules of melodic progression are not just arbitrary but stem from the features of pitch that produce melodies and determine the scales. This is the reason why dynamis is a non-quantitative aspect of melos: an interval is not to be defined by a magnitude but by a character present to the ear. This is one of Aristoxenus’ greatest contributions to harmonic science and was to be taken on by harmonists of every persuasion: it belongs both to the proportional and to the timbral facets of harmony. Proportional (although harmonic seems to be the better term here) in the sense that it defines characters and roles, potentials spanning beyond immediate sonic qualities, defining structures at larger time frames, one level of organization above intervals and notes in isolation. Some roles can belong to the timbral sphere (ephemeral, ornamental and coloristic roles, tensions aiming towards goals – the ‘leading note effect’, etc), while some are harmonic (stabilities, goals, the use of intervals larger than fourths, pivot notes that can change function in modulation and so on).

Potential for movement is a significant feature of harmony and melos is its operating principle, whether it is movement that produces tension (as in contrapuntal conceptions of consonance and dissonance, Tenney’s CDC-3) or tension that produces movement (as in functional tonality, CDC-4). In this way, context, progression and potential are properly harmonic roles, expressed through
melody whether it is explicit or merely implied, which is another important lesson to be taken from dynamis. A scale is not a neutral structure, it involves not just a collection of notes or intervals but also specific and conditional comportments. If scales are associated with colors or moods, this functional conception plays a salient role in defining those higher level features by involving characters, routes and functions. These roles determine the tension and movement from which a melody is said to imply a harmony and from which a verticality induces a melody.

With ratios we are dealing not so much with magnitude as with qualitative attributes, so magnitude belongs to the timbral aspect and character to proportion. There is no such irresolvable incompatibility between the mathematical and the empirical approach: they describe different aspects of music, each having advantages and disadvantages over the other in different circumstances, but without overlap or contradiction. It is similar to the way in which harmonic duality plays itself out, as an intertwining or entanglement, always in tension, no aspect being independent of the other. Their paradoxical relationship can be turned into a concept.

From this perspective, we must also distinguish in intervals the difference in function between commas, alterations, steps, and leaps. A comma changes the tuning of an interval without changing its scale degree, as it happens when an interval is tuned in accordance with one set of fundamental intervals rather than another (as when a third is tuned according to fifths as opposed to pure thirds, for example). An alteration does not alter the degree but changes its mode or quality into that of another genera, as in the case of a change from minor to neutral to major within a degree. A step is an adjacent change of scale degree, independent of its size (in enharmonic it could range from a quarter tone to a ditone). A leap is a non-adjacent change of degree. Moreover, inversion can be considered an operation upon these roles, altering their properties in interesting ways: intervals larger than a fourth can be inverted to fall within the fourth, so that their roles are related to the smaller versions with additional characteristics brought by size and direction.

Functions within the tetrachord could also be related to posterior functional harmony such as tonic-dominant, supertonic-submediant, mediant-leading-tone and subdominant-tonic, but the difference is that these are harmonic properties of chords constructed on these scale degrees while the functions we’re discussing are eminently horizontal. Modulatory harmony takes dynamis a step further to higher levels of organization by applying it to chords progressions and, even further, to tonalities.

\[1.3.6\] **Numbers and perception: Pythagoreanism**

‘There is no difference between composing music and thinking about the stars’ (Karlheinz Stockhausen)

The mathematics issuing from arithmetic and harmonic means are by no means trivial, even if their initial musical application might seem a bit innocent. The harmonic mean, discovered as a result of musical problems, shows that musical phenomena testify to patterns lying beyond the auditory realm, providing departure points for mathematics, and not the other way around, as the relation between music and mathematics is usually understood. The discoveries of Pythagoreans have since

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64 Taken from the documentary film *Tuning In*, by Robin Maconie, 1983. Last retrieved August 13, 2012, from [http://www.youtube.com/watch?v=gGnkZmn9MPw](http://www.youtube.com/watch?v=gGnkZmn9MPw).

65 ‘[Around 500 B.C.,] music gives a marvelous thrust to number theory and geometry [...] Music theory highlights the discovery of the isomorphism between the logarithms (musical intervals) and exponentials (string lengths) more than 15 centuries before their discovery in mathematics; also a premonition of group theory is suggested by Aristoxenos.’, Xenakis, I., Music Composition Treks, 171-192.
taken a life of their own, integer harmonic means providing further inroads into puzzling and deep number-theoretic insights. Numbers with integer harmonic means are quite rare and from them derive harmonic divisor numbers, positive integers whose divisors have an integer harmonic mean – they are non trivial, the first few being 1, 6, 28, 140, 270, 496, 672, 1438, 2970, 6200, 8128, 8190, possessing interesting number-theoretic properties. Euler proved that there are infinitely many primes – a result having been proved through reductio ad absurdum by Euclid – with the aid of reciprocals of harmonic series. This resulted in Riemann’s zeta function which gives clues to the distribution of the primes over the integers, providing insights into the relation between the continuous and the discrete as well as their relation to periodicity, which, as we saw, lies at the heart of pitch.

The related prime number theorem states that \( x/\ln(x) \) (a real number divided by its natural logarithm) approximates \( \pi(x) \) (the number of primes smaller than \( x \)) as \( x \) approaches infinity, connecting primes (integers) and logarithms (reals) in a single expression.

Simple questions related to integers and arithmetic (counting, addition and multiplication) lead quite quickly to the frontiers of knowledge. Perception arrives very early to the shores, but knowledge also gets there quite fast, either to extremely complex mathematics or simply to questions that nobody, not even the best mathematicians know how to answer.

It would be quite far fetched to try to read these results back into music, but it nevertheless manifests that the notion of harmonic duality, giving evidence of the polarity between the continuous and the discrete intrinsic to musical phenomena, an hypothesis not resulting from cultural conventions, is closely involved with a long tradition of thinking about the opposition between integers and the reals, the mathematical dialectic between arithmetic and geometry. These mathematical results attest to a deep link between the two aspects on which auditory harmonic perception rests, even if these connections lie far beyond the direct field of apprehension.

Music (perception, theory and practice) offers awareness of these mathematical structures. Better still, it can convey some mathematical ideas as phenomena. The connection happens through aisthesis: not only as immediate sensation but requiring thought and reflection for its recognition. Aisthesis captures only the lowest confines of these numerical traits, but it is exposed to their whole ‘frequency range’, not isolated from their full ramifications, which could also re-fold back into the aisthetical spectrum. Discussion of Pythagorean topics at least refreshes (or resuscitates) the problem of the relation between mathematics and music, indicating that the issues at stake are intricate, complex and quite relevant today. Musical phenomena are caught between continuity and discreteness, sharing forms and properties with numbers and ratios, communicating to and fro between abstract ideas of order, pattern, relationality and the empirical regions of ‘tone color’, intervallic character, degree of dissonance, potential, movement, rhythm and other musical qualias known through sensory experience. Musical questions open up to issues bearing on the nature of numbers and their relation to the empirical universe (or, which is the same, of mathematics to physics), as well as the relations of the latter back into music. This is all closely related to Pythagorean science, which incorporates mathematics, philosophy, natural science and music. Music understood not simply as an art form among others, but also as a gateway, a sensory entry into and between these disciplines.

Though arithmetic has very little relevance in today’s mathematics, it still has contemporary

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66 They are a recent discovery. See the online lecture notes by Goto, T. On Ore’s harmonic numbers [PDF document]. Last retrieved May 18, 2011, from http://www.ma.noda.tus.ac.jp/~tg/files/uts.pdf

67 The zeta function can be expressed as series of periodic functions: “A physicist will think of a sum of periodic functions as a superposition of waves, a vibration or sound. This is what the physicist Sir Michael Berry meant by ‘we can give a one-line nontechnical statement of the Riemann hypothesis: The primes have music in them.’” Webpage of Jeffrey Stopple: Stopple, J. (n.d.), Riemann's Explicit Formula. Last retrieved May 12, 2011, from http://www.math.ucsb.edu/~stopple/explicit.html

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relevance is in musical harmony. The relation between qualitative and quantitative aspects of numbers in and through music is a good reason to glance back at Pythagoreanism. Pythagoreans noticed that operations on numbers applied to natural phenomena, conceptualizing this as ‘reality is structured by number’, a kind of mathematical empiricism. This came to an end with Hypassus, who discovered quantities exceeding numbers as they were understood then: the diagonal of a square of size 1, the area of a circle, the golden ratio, all being *alogos*. These irrationals found their intervallic equivalence in Aristoxenian *dieses* (the diagonal of the unit square, $\sqrt{2}$, corresponds to the tempered tritone). Contemporary mathematicians had the tools (extraction of square and cube roots) to compute the string lengths for tempered scales. Aristoxenian dieses are equivalent to taking 3 geometric means (square roots) and 2 cubic roots to the octave\(^{68}\). The Greek musical system did not require temperament nor logarithms. Unless we acknowledge music as emerging solely within the mind, rational intervals have more musical primacy than the otherwise more practical logarithmic intervallic divisions, which are closer to sensation, approximating but not replacing their musical meanings.

This Pythagorean catastrophe leads to Aristotelian instrumentalism, where sublunary phenomena are subject to degradation, as opposed to the celestial realm, where exact mathematical relationships still hold: beauty was not terrestrial but cosmic. Kepler reunited both realms under a single mathematical physics, renewing the Pythagorean dream. Music, particularly harmonics, was crucial to this dream, figuring prominently as a model for the universe in the thought of many of the involved thinkers up to the Renaissance. The quadrivium of sciences inaugurated *avant la lettre* by Archytas set music side by side with mathematics – divided into the discrete/continuous twofold of arithmetics and geometry – and astronomy – with which it shared its cosmological perfection. The music of the spheres was a realm for contemplation and amazement: a correspondence between the structure of the heavens, that of music (particularly the proportions determining consonance and dissonance) and the human soul. Microcosm and macrocosm were in accord, mediated through music through sympathetic resonance.

Newton’s decomposition of planetary orbits into terrestrial linear movements ruined the separation between cosmos and earth, displacing it into an intra-terrestrial one between nature and culture. Sound is now understood within the domain of the laws of motion and not as a separate phenomenal field, and the theory of sound propagation is founded on Newton’s model of the harmonic oscillator (the pendulum) which was inspired by harmonic science, later loosing its musical derivation and taking on an independent history as a mathematical abstraction which would serve physics a great deal. Since the Enlightenment, harmonics lost its importance as an intellectual model in a transition that began with Galileo and Mersenne and went all the way up to Helmholtz, whose aim was explaining the consonance of simple ratios, the Pythagorean problem par excellence, from a sensory instead of a formal standpoint – effecting a change from a proportional to a timbral explanation. Music theory’s foundations underwent an ‘empirical turn’, speculation being replaced by physical and physio-psychological explanations. Furthermore, after Kant’s suspension of metaphysics as dogmatic and naïve, speculation lost whatever impetus it had left. When all that is permitted or possible to talk about is the conditions of access for subjective experiences, and not things in themselves, which are confused as grounded objectively instead of subjectively, reasoning in music turns toward the perceptual. As the human subject becomes the new center of the universe, the aim of theorizing about music is transformed from that of gaining knowledge about music in general to that analyzing individual compositions.

The qualitative aspects of numbers have gradually been lost since those days, regarded as they are in purely quantitative terms and leading to a ‘quantocentrism’ which cedes the monopoly over

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\(^{68}\) Dividing an octave into 72 equal steps: 72 is $2^3 \cdot 3^2$, equivalent to taking 3 square and 2 cubic roots to 2.
numeric qualities to new age numerology.\textsuperscript{69} Mathew Watkins\textsuperscript{70} proposes that to recover these qualitative aspects without losing some kind of verifiable support implies seriously studying number theory, ‘because as far as I’m concerned, that is numerology – you’re looking at the properties of integers and if you study it to a certain depth it takes you into the realms of what you could only call the mystical or uncanny, where cracks seem to open in your normal understanding of reality.’\textsuperscript{71} Music, though rarely mentioned in relation to this topic, provides one of the most direct instances of the qualitativeness of numbers, and in this sense harmony can be considered an audible interface between these realms.

‘Although number is widely considered as a mental construct, at the same time it manifests directly in the world of matter: when you consider a quartz crystal or a five-petalled wildflower, it’s hard to deny there’s an essential “sixness” or “fiveness” there. So, number itself is a bridge of sorts between psyche and matter.’\textsuperscript{72}

Philosophy was in closer contact with music in its Greek origins. These numeric qualities are audible proportionally, both as intervals and durational distributions; ‘twoness’ is one of its basic attributes, relating to octave equivalence and interval classes, as well as basic rhythmic binary divisions; ‘threeness’ has to do with fifths and fourths, hemiolas and sesquialtera; ‘fiveness’ with thirds, sixths; and so on, each being a distinctive and easily recognizable audible \textit{qua莉}. Moreover, from purely harmonic considerations, we should discern in ‘sixness’ a combination of ‘threeness’ and ‘twoness’: the fact that composite numbers adopt and combine the qualities of their prime factors, which become of \textit{prime}-ary importance not only to arithmetic, as in the Greek Fundamental Theorem of Arithmetic\textsuperscript{73}, but also to harmony. They are the atomic constituents of proportional intervals, their formal causes or ‘definition patterns’. If according to Aristotle the formal cause of the octave is 2/1, then 2 is the main content or the interval’s quality. In something more complex like 15/8 (a major seventh), its formal causes would be 2, 3, and 5 (the 2 compounded 3 times)\textsuperscript{74}. Another issue is how high in the prime series can human auditory \textit{aisthesis} encompass. Some say up to 7, some up to 13 and higher: this contemporary debate will have to be waged with musical rather than purely theoretical hypotheses.

‘Pythagoreanism’ can refer to many schools and eras beginning with early Pythagoreans, split between the \textit{acusmatici} – a mystical, non-scientific tradition based on the aurally revealed word of Pythagoras, emphasizing ethical precepts for living ascetically – and the \textit{mathêmatici} – referring to rational Pythagorean science: mathematicians and natural philosophers, culminating with Archytas.

\textsuperscript{69} I am correcting this paragraph on 11/11/11 and cannot believe all the vacuous frenzy around such a fortuitous date. While all this complacency takes place, yesterday the Western Black Rhinoceros was declared extinct. Many of today’s superstitions pass unexamined (at times with awful consequences, as the news shows) while the depth and subtlety of reasoning behind some of the most influential models of the universe in history tends to be lost and considered unsophisticated belief compared to our current ‘advanced’ knowledge.


\textsuperscript{71} Ibid. 166-7.

\textsuperscript{72} Ibid, 183.

\textsuperscript{73} Proven by Euclid, the Fundamental Theorem of Arithmetic ‘states that any integer greater than 1 can be written as a unique product (up to ordering of the factors) of prime numbers.’ That these factorizations exist is evident, that they are unique is not so trivial. Extended to harmony it implies that every interval is determined by a unique combination of fundamental intervals. Fundamental Theorem of Arithmetic (2010). In \textit{Wikipedia}. Last retrieved November 16, 2010, from \url{http://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic}.

\textsuperscript{74} ‘“Cause” means (1) that from which, as inmanent material, a thing comes into being, e.g. the bronze is the cause of the statue and the silver of the saucer, and so are the classes which include these. (2) the form or pattern, i.e. the definition of the essence, and the classes which include these (e.g. the ratio 2:1 and number in general are the causes of the octave), and the parts included in the definition. (3) That from which the change or the resting from change first begins; e.g. the adviser is a cause of the action, and the father a cause of the child, and in general the maker a cause of the thing made and the change-producing of the changing. (4) The end, i.e. that for the sake of which a thing is...’’, Aristotle. (1996). \textit{Metaphysics}. In J. Barnes (Ed.), \textit{The Complete Works of Aristotle}. Princeton, NJ: Princeton University Press, Book V. §2 (1015a), Emphasis added.
Pythagoreanism is also attributed to other thinkers and schools who assimilate and transform some of their chief ideas such as the dyad of the limited and unlimited and the correlation between the ideal and empirical realms. Plato and Aristotle are examples, as well as Aristoxenus, who provides the earliest direct account of their theories, having lived under Archytas’ rule. The unification and creation of a canonic tradition was pursued by the Neopythagoreans of later centuries (Porphyry, Iamblichus) and carried further by medieval neoplatonists (Boethuis, Nichomacus) all the way up to the Renaissance (Copernicus, Zarlino, Kepler, Galileo) and the Baroque (Euler, Leibniz). Even today, many modern scientists accept the basic tenet that knowledge of the natural world is to be expressed in mathematical formulae, which is rightly regarded as a central Pythagorean thesis, since it was first rigorously formulated by the Pythagoreans Philolaus and Archytas and may, in a rudimentary form, go back to Pythagoras himself.\(^{75}\)

Recovering this scientific, philosophical and mathematical strand is central for us, not dismissing it as numerological mysticism (or superstition) nor as a highly innovative but easily surpassable idea. This would miss the subtlety, importance and permanence of some of these discoveries, which stretch beyond the possible interpretations and uses given to them. It refers more to the structure of these ideas rather than their content. In fact, we can state in the same vein that harmony is both discovered and created (and composition an act of observation more than just authorship). The model of the string is still in use and provides one of the most tangible links between the discrete, the continuous, the mathematical and the physical, as well as supplying the link between place and time theories in pitch perception models. Moreover, probably the principal characteristic of this Pythagoreanism is the gesture ‘number is a bridge between psyche and matter’ more than the regularly acknowledged ‘being is number’. It is with Archytas then, that the inspiration for a renewed Pythagoreanism must be found, where perception and empirical observation are coupled with deduction and logical rigor. The ‘pitching’ of melodic space, as Aristoxenus named the puncturing of the pitch continuum with stable pitches, is discovered by induction from observation. Through this abstraction, an ‘astonishing orderliness’\(^{76}\) is uncovered, attesting to a physis, a nature or essence of melos. An orderliness revealing patterns of rational numbers.

1.3.7 Concluding remarks

The Romantic nature-culture pair that ensued from the Enlightenment is replaced by a modern symbol of history as a non-human, non-natural force driving an impersonal process.\(^{77}\) Modernity shifts the focus from the human strife with nature towards indifferent forces, and in music this involves turning our ears from self-expression towards receptivity, abstraction, aloof forms, as well as all kinds of things ‘out there’.\(^{78}\) This very contemporary concept of attuning to all sorts of entities is implied by Pythagoreanism and consists in paying attention to the structure of what surrounds us. It

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76 Aristoxenus as quoted in a discussion of this topic in Barker, op. cit. 150.


78 In this vein, the avant-garde serialists of the 1950’s (Eimert, Stockhausen, Pousseur, etc) are also weird heirs to Pythagoreanism, mostly due to their absolute and anti-language approach to music. Nevertheless, if there is any paradigmatic Pythagorean composer inspiring this research it is Iannis Xenakis (even if his approach to pitch is highly Aristoxenian). If we make sense of Pythagoreanism as in the Renaissance, as a conjunction of music, philosophy, mathematics, poetry as well as the common origin of arts, then is not he a most Renaissance figure who conjoins heart, mind, science and philosophy, adapting it to the needs of his own time? He is perhaps the greatest emancipator of the continuum in music which is a point of departure for our advocating a reassessment of discreteness and proportionality.
should not be understood either in a mimetic nor positivistic sense of mechanical knowledge or as an optimistic-progressive metaphor, but instead as something to probe or dig into, to be reckoned in its full uncanniness. According to Graham Harman’s metaphysics of objects, essences, although having been considered either elsewhere (outside this world) or non-existent, are in fact in objects, yet withdrawn: they are not simply present. They are somehow like substantial forms in relation to proportions: definition patterns that establish objects as autonomous realities that emerge over and above their constituent parts as well as being independent of their outward relations. An object’s relation to other objects siphons some of their withdrawn qualities creating a sensus, phenomenal realm in the object that makes contact with another, though the object itself is never exhausted by these relations, always holding some novelty in reserve (its withdrawn, noumenal realm). These qualities are highly singular, not ‘bare particulars’, each imbued in the unique style of its object, not being pale specters of ideal ones (a 3/2 is the 3/2 of the timbre that instantiated it, not a chimerical interval whose timbre is a deficient approximation of an ideal one). For that matter, these essences are not indestructible nor preexist the object. There is more inside the object than outside it. As we will see with more detail further on in defining proportional and timbral aspects of harmony in relation to rhythm and form, in this static model space and time are emergent features of objects and we are fundamentally embedded in an infinite regress of objects, not in a meaningful, blissful way, but in an uncanny, expressionist, Lovecraftian, disturbed manner. It is not matter which is at the service of subjectivity, but a matter of subjectivity caught inside the strangeness of other entities.

This suggests an aesthetic stance of attuning to musical and sonic phenomena, instead of trying heroically to dominate them (which would fall into the Romantic attitude). This position leads to sincerely (instead of ironically) approach all kinds of objects for translation into music and sonic forms, seeking some of their hidden harmonies. The task calls for the sonification of the formal as well as the empirical worlds, rendering entities into sound, as well as proceeding from sounds towards abstract entities.

One aim of this research has been to give back to harmony its speculative and arithmetic dimension without disregarding but even emphasizing the developments made after the study of music and harmony changed from having a metaphysical towards a physical and subjective grounding. Many concepts of Greek harmonics can be incorporated and used in a contemporary harmony, chiefly by admitting melos as one of its pivotal aspects, a structuring and motion producing device established through the relationality of tones. Other notions such as division by means, atomic constituents, tetrachordal (and other moduli) divisions, functional conceptions, etc., can be further extended, inviting us to recuperate long forgotten intervals, with ideas about possible generative and relational strategies to deal with them. The demarcation between the proportional and timbral aspects of Greek harmonics as well as the detailed delineation of melodic consonance/dissonance also provides a refreshing perspective on these issues, which have been clouded by layers of sedimented theories covering them up throughout the centuries. Finally, its metaphysical perspective can be transposed today by rethinking the question of numbers in relation to perception, of mathematics in relation to music and of music in relation to structural aspects of reality.

Attuning, contemplating, ceding control, receptivity: to reinterpret ideas from the past as part of the construction of the future and in order to have the largest possible present.

If the first section on pitch perception ended by delineating a perceptual ontology of sorts, this section has tried to identify a link between perception and arithmetic, but this time perception is linked with a phenomenological first person perspective and the arithmetical with the aisthetical deduction of the formal causes behind these perceptions. The arithmetic standpoint is a kind of ‘formal ontology’ for harmony, while the phenomenological attitude belongs to its precondition, its

79 The main books where I draw these ideas from are Guerilla Metaphysics and The Quadruple Object.
‘axiomatic’ starting point. What must now be brought forward is the connection of these in the direction of a properly musical ontology, not dependent on neither arithmetic nor perception, lying at a certain distance from them, but in constant tension and correspondence.