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**Title:** Molecular charge transport : relating orbital structures to the conductance properties  
**Issue Date:** 2012-11-06
**THE SIMMONS MODEL**

**Full formulation of the Simmons formula.** According to ref [1], a full expression for the current density, $J$, through a barrier between two similar metal electrodes over the entire voltage range is given by:

$$J = c(\tilde{A} + \tilde{B} + \tilde{C})$$

$$c = \frac{4\pi me}{\hbar^3}$$

$$\tilde{A} = eV \int_{\eta - eV}^{\eta} \exp(-A \sqrt{\eta + \bar{\phi} - E_x}) dE_x$$

$$\tilde{B} = -\bar{\phi} \int_{\eta - eV}^{\eta} \exp(-A \sqrt{\eta + \bar{\phi} - E_x}) dE_x$$

$$\tilde{C} = \int_{\eta - eV}^{\eta} (\eta + \bar{\phi} - E_x) \exp(-A \sqrt{\eta + \bar{\phi} - E_x}) dE_x.$$
By substituting $y^2 = \eta + \phi - E_x$ and $d(-E_x) = d(\eta + \phi - E_x) = dy^2 = 2yd_y$, this becomes:

$$- \int_{y_1}^{y_2} \exp(-Ay) \cdot 2yd_y$$

Here, $y_{1,2} = \sqrt{\eta + \phi - e_{1,2}}$. These integrals can be solved by partial integration [1]. Boundaries for $\tilde{A}$ are $e_1 = 0, e_2 = \eta - eV, y_1 = \sqrt{\eta + \phi}, y_2 = \sqrt{\phi + eV}$, yielding:

$$\tilde{A} = \frac{2eV}{A^2}((A\sqrt{\phi + eV + 1})\exp(-A\sqrt{\phi + eV}) - (A\sqrt{\eta + \phi + 1})\exp(-A\sqrt{\eta + \phi})).$$

Boundaries for $\tilde{B}$ are $e_1 = \eta - eV, e_2 = \eta, y_1 = \sqrt{\phi + eV}, y_2 = \sqrt{\phi}$, yielding:

$$\tilde{B} = \phi\frac{2}{A^2}((A\sqrt{\phi + 1})\exp(-A\sqrt{\phi}) - (A\sqrt{\phi + 1})\exp(-A\sqrt{\phi + eV})).$$

Like $\tilde{A}$ and $\tilde{B}$, $\tilde{C}$ can again be solved by substituting $y^2 = \eta + \phi - E_x$ and $d(-E_x) = d(\eta + \phi - E_x)$ and partial integration.

$$\tilde{C} = -2\int_{y_1}^{y_2} y^3 \exp(-Ay)dy$$

Boundaries for $\tilde{C}$ are $e_1 = \eta - eV, e_2 = \eta, y_1 = \sqrt{\phi + eV}, y_2 = \sqrt{\phi}$, so that:

$$\tilde{C} = \frac{2}{A}((\phi^{3/2} + \frac{3}{A} \phi + \frac{6}{A^2} \sqrt{\phi} + \frac{6}{A^3})\exp(-A\sqrt{\phi})$$

$$-((\phi + eV)^{3/2} + \frac{3}{A} (\phi + eV) + \frac{6}{A^2} \sqrt{\phi + eV} + \frac{6}{A^3})\exp(-A\sqrt{\phi + eV})).$$

Taking all integrals together, we can calculate $J$. Note that for relatively high and/or thick barriers, i.e. if $A\sqrt{\phi \pm eV} \gg 1$, the full expression for $J$ reduces to eq. (26) of reference [1]:

$$J = J_0((\phi - eV/2)\exp(-A\sqrt{\phi - eV/2}) - (\phi + eV/2)\exp(-A\sqrt{\phi + eV/2})).$$

where, $J_0 = e/(2\pi \hbar s^2)$.

Figure 1 shows $V_m$ versus $1/d$ for each of the three equations mentioned above; eq. 26 of ref [1], (black), eq. 1 (Stratton) in the main text (blue) and the full Simmons expression (red). For thick barriers all three collapse on a single line. The maximum deviation between the three is in the order of a few percent for thin barriers (around $d = 5\text{Å}$). These differences are negligible compared to the spread in the experimental data as discussed in the Letter.
REFERENCES
