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# Consistent Supersymmetric Decoupling in Cosmology.

Kepa Sousa Sánchez

**On the front cover:** artistic representation of a supersymmetric truncation and a spontaneous supersymmetry breaking. Aquarel by *Ana Sánchez Trujillo*.

# Consistent Supersymmetric Decoupling in Cosmology.

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*A mis padres,  
por estar siempre a mi lado.*



*Para el adulto que padece  
una impotencia creativa (...) la única posibilidad que le queda es remontarse a su propia niñez y empezar de nuevo, a partir del momento en que le arrebataron sus sueños. Que no eran sueños, en absoluto, sino el fundamento de su propia vida, las raíces de su existencia, sin la cuales nunca será persona verdaderamente.*

*The only cure for the adult incapable of any act of creativity (...) is to return to his own childhood, to the instant where his dreams were taken from him. Which were not just dreams, (...) but the very roots of his existence, without which he will never really be a person.*

F. Hundertwasser





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# Preface

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This thesis contains the results of a PhD project carried out at the University of Leiden, in collaborations with my advisor Ana Achúcarro, with Mboyo Esole and Sjoerd Hardeman. The present work discusses several problems related to the stability of ground states with broken supersymmetry in supergravity, and to the existence and stability of cosmic strings in various supersymmetric models.

In the last decades there has been a lot of interest in building cosmological models within the frameworks of the main candidate theories to explain the physics at very high energy scales, such as Grand Unification Theories, supersymmetric theories and Superstrings.

The main reason is that the phenomena described by these theories take place at energy scales way above those probed by any particle accelerator built on earth, now or in the near future. On the other hand, due to the extremely high energies involved in the very early stages of the evolution of the universe, cosmology provides an excellent framework to test all these theories.

The standard model in cosmology is the so called  $\Lambda$ CDM, where the acronym refers to the main components of the universe, the cosmological constant  $\Lambda$ , which drives the observed accelerated expansion of the universe, and the Cold Dark Matter, an exotic new type of matter which has only been detected due to its gravitational effects. The  $\Lambda$ CDM model has been very successful in explaining, among other things, the element abundances, galaxy formation and the large scale distribution of galaxies. However, there are still many questions

that need to be answered, for instance the microscopic origin of the cosmological constant, or the nature of the Dark Matter particles.

Another issue of the  $\Lambda$ CDM model is related to the initial conditions of the universe which led to the observed galaxy distribution. According to the currently accepted ideas about galaxy formation, the large scale structure of the universe was seeded by primordial density perturbations which grew mainly due to the gravitational effects of Dark Matter, and can be seen directly as temperature fluctuations on the Cosmic Microwave Background (CMB). These irregularities were originated from quantum fluctuations stretched to cosmic size during an early period of extremely fast accelerated expansion called inflation.

The CMB temperature spectrum is consistent with the predictions of inflation, however the physical processes which are responsible for it remain unclear. In the simplest models inflation is driven by the vacuum energy of a scalar field which is slowly rolling down a very flat potential. The development of inflationary models can provide very useful information about the high energy regime. On the one hand the peculiarities of the theory where inflation is implemented would leave their imprint on the CMB, and on the other hand we have at our disposal extremely precise measurements of the CMB spectrum provided by the WMAP mission, which will be improved even further by observations of the Atacama Cosmology Telescope and the PLANCK satellite.

The topics discussed in this thesis are relevant for cosmological models based on supersymmetric theories. Supersymmetry, a symmetry which transforms bosons into fermions and viceversa, is an attractive framework which provides a solution to the hierarchy problem. Its local version, supergravity, was initially proposed as a method to cure the divergences which typically appear in theories of quantum gravity, but this idea did not succeed. Nowadays, supersymmetry and supergravity are mainly used to describe the low energy regime of the most prominent theory of quantum gravity, String Theory.

Cosmological models derived from superstrings typically involve a large number of scalar fields, such as the moduli. Partly with the aim of gaining control on the analysis, and also because inflationary models based on a single field predict very accurately the spectrum of perturbations of the CMB, cosmological models assume most of these scalar fields stabilized at some high energy scale, leaving behind a low energy supersymmetric theory involving the minimum necessary to implement inflation. Moreover, since supersymmetry is known to be broken at low energies both during inflation and at present (the Standard Model particles cannot be fitted in supermultiplets without introducing new particles), the sector surviving at low energies must also provide the mechanism to break it.

In supergravity the couplings between fields are highly constrained, and

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since gravity couples to everything, it is non-trivial to integrate out a sector of the theory consistently while preserving supersymmetry. Moreover, since supersymmetry has to be broken in the surviving sector, there is no easy way to guarantee that the truncated fields remain massive. If the truncation is not made in a consistent way the inflaton will possibly couple to the truncated degrees of freedom, leaving distinctive features on the CMB spectrum. The study of such effect could provide us important information about the high energy regime.

In chapters 3 and 4 we study the necessary conditions to truncate consistently a sector of the theory while preserving supersymmetry, and on those models where these conditions are met we will discuss how the breaking of supersymmetry in the light sector affects the stability of the truncated fields. The basis of this work are the journal papers

- ‘*F-term uplifting and moduli stabilization consistent with Kahler invariance.*’ Ana Achúcarro and Keba Sousa. *JHEP* **0803** (2008) 002; [arXiv:0712.3460](#).
- ‘*Consistent decoupling of heavy scalars and moduli in  $\mathcal{N} = 1$  supergravity.*’ Ana Achúcarro Sjoerd Hardeman and Keba Sousa. *Phys.Rev.* **D78** (2008) 101901; [arXiv:0806.4364](#).
- ‘*F-term uplifting and the supersymmetric integration of heavy moduli.*’ Ana Achúcarro, Sjoerd Hardeman and Keba Sousa. *JHEP* **0811** (2008) 003; [arXiv:0809.1441](#).

Although inflation is widely accepted as the main mechanism to produce the primordial energy density perturbations, the observations are still compatible with a minor contribution from other sources, such as cosmic strings. Cosmic strings are extremely thin line-like concentrations of energy of cosmic length which move at relativistic speeds. The formation of cosmic strings is a generic prediction of many promising cosmological models based on supersymmetric Grand Unified Theories and superstrings.

Among this type of solutions supersymmetric cosmic strings, those preserving a fraction of the supersymmetries of the original model, are especially interesting because the unbroken supersymmetries protect them from quantum corrections. In particular, their tension is preserved in the quantum theory along the renormalization flow, and therefore they constitute a probe of the high energy regime. In this thesis we will discuss a new type of supersymmetric cosmic string solutions in  $\mathcal{N} = 2$  supergravity. Extended supergravity theories  $\mathcal{N} > 1$  are not a common framework for cosmology since  $\mathcal{N} = 1$  supergravity, which involves chiral fermions, is more suitable to describe phenomenology. However many compactifications of string theory have a description in terms of  $\mathcal{N} = 2$  supergravity, and thus these models can be used as a bridge between the more



phenomenological  $\mathcal{N} = 1$  models, and String Theory. In particular  $\mathcal{N} = 1$  models are much less constrained than those built in  $\mathcal{N} = 2$  supergravity and therefore they have less predictability.

In chapter 7 we present a  $\mathcal{N} = 2$  model that which admits a family of cosmic string solutions with the same energy but different core radius. The family is parametrized by the expectation value of a field appearing in the model which has similar couplings to the dilaton and the Kähler moduli in string theory compactifications. The research in this chapter can be found in the following article

- ‘*Half-BPS cosmic string in  $\mathcal{N} = 2$  supergravity in the presence of a dilaton.*’ Mboyo Esole and Keba Sousa. *JHEP* **0703** (2007) 079; [hep-th/0610124](#).

We will also discuss the stability of these strings against perturbations. In particular, in a cosmological context the zero mode associated to the expectation value of the dilaton-type field could be excited, leading to the spread of the magnetic flux and the disappearance of the string. Here we will explore this possibility. The arguments used in this discussion are based on a previous publication where we analyze a class of cosmic string solutions appearing on a  $\mathcal{N} = 1$  supersymmetric model with a similar zero mode

- ‘*A Note on the stability of axionic D-term strings.*’ Ana Achúcarro and Keba Sousa. *Phys.Rev.* **D74** (2006) 081701; [hep-th/0601151](#).

From the summary above it is clear that in this thesis we consider two different topics, on the one hand the consistent supersymmetric decoupling of a sector in  $\mathcal{N} = 1$  supergravity models and the stability of the decoupled sector, and on the other hand supersymmetric cosmic strings solutions in  $\mathcal{N} = 2$  supergravity, and the stability of these configurations in cosmological context. Nevertheless for convenience we have not made an explicit splitting, instead both topics appear together through the thesis, and we discuss them as we introduce the necessary concepts for doing it.