CHAPTER 5

Fayet-Iliopoulos terms and supersymmetric decoupling.

5.1 Introduction

Fayet-Iliopoulos terms, which appear as a constant contribution to the moment map (2.2.18), have many important applications. The presence of FI-terms can provide a mechanism of spontaneous supersymmetry breaking, and also, as the D-term potential is positive definite, they can be used to construct field theories with de Sitter solutions, i.e. with a positive vacuum energy (see [113]). These field theories are suitable to describe an inflationary period, or the present accelerated expansion of the universe.

In the last section of chapter 2, we discussed how Fayet-Iliopoulos terms also play an essential role in the construction of supersymmetric cosmic string solutions. The scalar potential of the supersymmetric Abelian-Higgs model is a pure D-term potential, it has no contribution from the F-terms. The existence of supersymmetric cosmic string solutions in this model is closely related to the presence of a FI term. Actually the tension of the string is proportional to the magnitude of the FI term (2.5.36).

So far it is not known how to derive supergravity theories with a constant FI-term from superstrings/M-theory. One of the first attempts was made in [128], where Dine *et al.* proposed a mechanism to generate such FI-terms in string theory models with pseudo-anomalous U(1)'s. In these models the shift symmetry associated to an axio-dilaton, $S = \mathrm{e}^{\rho} + \mathrm{i}a$, is gauged under the U(1) as a result of the Green-Schwarz mechanism of anomaly cancellation [129]. This gauging leads to a field dependent contribution in the moment-map of the 4-dimensional theory:

$$\mathcal{P} \sim \mathrm{i}(\mathrm{e}^{-\rho} + \ldots). \tag{5.1.1}$$

The authors of [128] suggested that, provided the dilaton gets an expectation value, such a contribution could act as an effective Fayet-Iliopoulos term in the low energy effective theory. However, as was later shown in [75], the field dependence of the dilaton cannot be integrated out in a supersymmetric way at a high energy scale without integrating out the full vector multiplet too. Thus, if the dilaton is truncated consistently then the $D-$term potential receives no contribution from the pseudo-anomalous $U(1)$, and in particular no effective FI-term is generated.

More recently the debate was reactivated due to the results presented by Komargodski *et al.* [81, 130], where it was shown that rigid $\mathcal{N} = 1$ supersymmetric theories with a FI-term cannot be the effective low energy theory of any consistent theory of quantum gravity. This conclusion would explain part of the difficulties found when trying to construct string theory models where the FI-term is dynamically generated at low energies. However later works [131, 82] have shown that it is still possible to construct intrinsically supergravity models which do not suffer from the same problems as the ones discussed by Komargodski *et al.* In any case the value of the FI-terms is very constrained, since they have to satisfy quantization conditions [131, 132], and they are also subject to anomaly cancellation conditions in order to ensure the consistency at the quantum level (see [82]).

In view of these discussions, we would like to address the question of how do the FI-terms of a low energy effective theory arise after the integration of the heavy fields of a given model. In particular in this chapter we will discuss the connection between the FI terms of the full $\mathcal{N} = 1$ theory and those appearing in the model obtained after truncating some of the fields while preserving supersymmetry.

In section 5.3 we will show that the size of the FI-terms is unaffected by the supersymmetric truncation, or in other words, that FI-terms cannot be generated during the stabilization of heavy fields without breaking supersymmetry. This study extends the conclusions presented in [75] about supersymmetric truncations and effective FI-tems to more general gauge couplings. For the discussion we have used the recent work by Catino *et al.* [82] which clarifies the issue of identifying the FI-term in non-linear sigma models with general gauge couplings.

In section 5.4 we analyze the gauged axio-dilaton system proposed by Dine *et al.* [128] to construct effective FI-terms. We recover the results by Binetruy et al. [75] which show that with the gauge couplings considered by Dine *et al.* it is not possible to obtain an FI-term after the supersymmetric truncation of the axio-dilaton. We also present an alternative gauging of the sigma-model which allows the supersymmetric integration of the axio-dilaton leaving a reduced theory with a non-vanishing FI-term. As we will see in the next chapter the axio-dilaton system gauged in this way can be embedded in $\mathcal{N}=2$ supergravity, giving models with a effective FI-terms which can be used to construct de Sitter vacua, to break gauge symmetries and to obtain cosmic string solutions.

Finally in section 5.4.2 we couple the gauged axio-dilaton system to the supersymmetric Abelian-Higgs model discussed in section 2.5. The coupling allows for the supersymmetric truncation of the axio-dilaton leaving the Abelian-Higgs model as the reduced theory. This construction illustrates an important property of consistent truncations, BPS solutions of the reduced theory, i.e. those preserving part of the supersymmetries, are also BPS solutions of the parent theory.

5.2 FI-terms in non-linear sigma models.

In chapter 2 we introduced the compensators $r_a(\xi)$, which characterize the gauge transformations of the Kähler potential $K(\xi,\bar{\xi})$ and the superpotential $W(\xi)$ under the killing vectors $k_a^I(\xi)$

$$\delta_a K(\xi,\bar{\xi}) = 3r_a(\xi) + 3\bar{r}_a(\bar{\xi}), \qquad \delta_a W(\xi) = -3r_a(\xi)W(\xi). \tag{5.2.1}$$

Given the Kähler potential and the killing vectors the compensators are only determined up to a constant shift, the Fayet-Iliopoulos term, which also appears as a constant contribution to the moment-map $\mathcal{P}_a$ (2.2.16).

From (5.2.1) we can see that gauge transformations induce a Kähler transformation (2.2.28) with the holomorphic function given by $h(\xi) = 3r_a(\xi)$. Actually, the invariance of the action under (abelian) gauge transformations also requires that the fermions transform as:

$$\begin{aligned}
\delta_a\psi_\mu &= -\tfrac{\mathrm{i}3}{2}\gamma_5\,\mathrm{Im}(r_a)\,\psi_\mu,\\
\delta_a\chi^I &= \partial_J k_a^I\,\chi^J + \tfrac{\mathrm{i}3}{2}\gamma_5\,\mathrm{Im}(r_a)\,\chi^I,\\
\delta_a\lambda^b &= -\tfrac{\mathrm{i}3}{2}\gamma_5\,\mathrm{Im}(r_a)\,\lambda^b.
\end{aligned} \tag{5.2.2}$$

If the compensator $r_a(\xi)$ has a constant contribution, i.e. an FI-term

$$r_a(\xi) = \ldots + \frac{\mathrm{i}\eta_a}{3}, \tag{5.2.3}$$

then the gravitino and the gaugini become charged under the abelian gauge symmetry generated by the killing vector k_a^I, and the gauge couplings of the chiralini are modified. These new couplings introduced by the FI-terms are responsible for the quantization and the anomaly cancellation conditions which constrain their allowed values.

However in non-linear sigma models it is not obvious how to identify such constant contribution of the compensators, which are field dependent in general. Moreover, the gauge couplings of the fermions induced by the compensator (5.2.2) are not well defined since they are both Kähler dependent and U(1) gauge dependent. Indeed, the compensator transforms in the following way under Kähler and U(1) gauge transformations:

$$\delta_K r_a = \tfrac{1}{3} k_a^I \partial_I h, \qquad \delta_a r_b = k_a^I \partial_I r_a, \tag{5.2.4}$$

where $h(\xi^I)$ is the holomorphic function characterizing the Kähler transformation. Catino *et al.* proved in [82] that FI-terms are only present in those theories where the compensator cannot be 'gauged away' everywhere in field space. As a matter of fact, Kähler transformations relate equivalent theories, and therefore if we can find a Kähler gauge where the compensator is zero everywhere, in particular the U(1) gauge couplings of the gravitino and the gaugino become also vanishing, and therefore the FI-term must be vanishing too.

In order to find the appropriate Kähler transformation to gauge away the FI-term we have to solve the following differential equation for the holomorphic function $h(\xi)$

$$k_a^I(\xi)\partial_I h(\xi) = -3 r_a(\xi). \tag{5.2.5}$$

Assuming $h(\xi)$ to be regular, this can only be solved provided the compensator vanishes at the fixed point of the killing vector[1] ξ_0, i.e. where $k_a^I(\xi_0) = 0$. Then, if the killing vector has no fixed points then the compensator can always be gauged away and no FI-terms can be present [82]. This equation has an obvious solution whenever the theory has a superpotential $W(\xi)$ which is non-vanishing everywhere in field space. Indeed, from the gauge transformation of the superpotential (2.2.19) we see that the appropriate Kähler transformation is given by

$$h(\xi^I) = \log W(\xi^I). \tag{5.2.6}$$

[1] We can always choose a local coordinate system where $k_a(\xi) = \mathrm{i}q_a\xi^1\partial_1$ (see [87]). In these coordinates the integration of equation (5.2.5) is trivial provided the compensator vanishes at the fixed point of the killing vector.

Conversely, suppose that the compensator has a finite value at ξ_0

$$r_a(\xi_0) = \tfrac{\mathrm{i}}{3}\eta_a. \tag{5.2.7}$$

At the point $\xi = \xi_0$ the Kähler potential should not transform, and therefore from (5.2.1) we see that η_a must be a real constant. In such a case it is still possible to find a regular Kähler transformation which sets the compensator to a constant everywhere solving the equation

$$k_a^I(\xi)\partial_I h(\xi) = -3r_a(\xi) + \mathrm{i}\eta_a. \tag{5.2.8}$$

After this transformation the compensator is simply $r_a(\xi) = \mathrm{i}\eta_a/3$, and thus the Kähler potential becomes invariant under gauge transformations. Moreover, recalling the expression for the moment map (2.2.16) we find

$$\mathcal{P}_a|_{\xi_0} = -\mathrm{i}3r_a(\xi_0) = \eta_a, \tag{5.2.9}$$

and therefore we can identify the FI-term with the value of the moment map at the fixed point of the killing vector k_a^I. The previous expression is both Kähler invariant and gauge invariant. Notice that, if the scalar manifold is simply connected and the gauged isometry is compact, then the corresponding killing vector has a fixed point and thus it is possible to have a non-zero FI-term.

In situations where the killing vector has more than one fixed point $\xi_0, \xi_1, \ldots$, we can repeat the procedure locally in patches $U_i \subseteq \mathcal{M}$ of the Kähler manifold small enough to contain a single fixed point ξ_i. In each patch we will have to change the Kähler gauge in order to transform the compensator into a constant. In general the resulting Kähler potential and the superpotential will be different in different patches, but on the overlap regions $U_i \cap U_j$ they will be related to each other by Kähler transformations [90]:

$$K_i - K_j = h_{ij} + \bar{h}_{ij}, \tag{5.2.10}$$

where h_{ij} are holomorphic functions. These transition functions will determine the relation between the values of the FI-term in different patches, and are constrained by the topology of the Kähler-Hodge manifold [90].

Example: FI-terms on the sphere

Consider the supersymmetric $\mathbb{C}P^1$ model coupled to a U(1) gauge field A_μ. This model leads to an anomalous quantum theory which can be cured adding extra matter fields [133, 134], but in order to keep this discussion simple we will not consider this issue. The target space $\mathcal{M}$ is a Kähler manifold, the two-sphere $S^2 \cong \frac{\mathrm{SU}(2)}{\mathrm{U}(1)}$. In order to parametrize the sphere completely we divide it into two patches, the northern and the southern hemispheres.

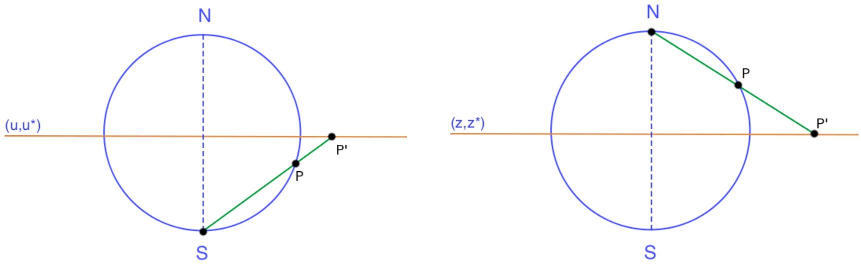

Figure 5.1 *– LEFT: Stereographic projection from the South pole on the complex plane parametrized by $\{u, \bar{u}\}$. RIGHT: Stereographic projection from the North pole on the complex plane parametrized by $\{z, \bar{z}\}$.*

The northern hemisphere can be parametrized using the stereographic projection from the South pole onto the complex plane with coordinates $\{u, \bar{u}\}$, see Fig. 5.1. This parametrization excludes the south pole, which is projected to the point at infinity on the complex plane. For the southern hemisphere we use the stereographic projection from the north pole on the complex plane with coordinates $\{z, \bar{z}\}$. Both parametrizations are related by the holomorphic change of coordinates

$$z = \frac{1}{u}, \qquad \bar{z} = \frac{1}{\bar{u}}, \tag{5.2.11}$$

An appropriate Kähler potential in the northern hemisphere is $K^{(N)}(u, \bar{u}) = n \log(1 + u\bar{u})$, with n being an even integer [90], leading to the following kinetic terms for the scalars

$$\mathcal{T} = -n \, \frac{\partial_\mu u \partial^\mu \bar{u}}{(1 + u\bar{u})^2}. \tag{5.2.12}$$

On the southern hemisphere we choose the Kähler potential $K^{(S)}(z, \bar{z}) = n \log(1 + z\bar{z})$, which is related to $K^{(N)}$ on the overlap of both patches, i.e. the equator, by the Kähler transformation

$$K^{(S)} - K^{(N)} = h_{NS} + \bar{h}_{NS}, \qquad \text{with} \qquad h_{NS}(u) = n \log u. \tag{5.2.13}$$

Suppose we gauge the isometry associated to rotations of the sphere around the North-South axis. The corresponding gauge transformations of the coordinates on the northern and southern hemispheres are respectively

$$\delta u = \mathrm{i} g u \, \alpha, \qquad \delta z = -\mathrm{i} g z \, \alpha, \tag{5.2.14}$$

Since the Kähler potentials $K^{(N)}$ and $K^{(S)}$ are invariant under the gauge symmetry the compensators must be a constant proportional to the FI-terms, and therefore we do not have to perform any additional Kähler transformation in order to gauge away the field dependence of the compensator. Thus the moment maps are given by

$$\mathcal{P}^{(N)} = -g\left(\frac{n\,u\bar{u}}{(1+u\bar{u})} - \eta^{(N)}\right), \qquad \mathcal{P}^{(S)} = g\left(\frac{n\,z\bar{z}}{(1+z\bar{z})} - \eta^{(S)}\right). \tag{5.2.15}$$

Where $\eta^{(N)}$ and $\eta^{(S)}$ are the FI-terms on the northern and the southern hemisphere respectively. Note that, since the gauged isometry has fixed points at the South pole, $u = 0$, and the North pole, $z = 0$, the FI-terms cannot be gauged away by a Kähler transformation and thus, according to the results in [131, 132], the FI-terms have to be quantized

$$\eta^{(N)} = p \quad \text{and} \quad \eta^{(S)} = q, \qquad \text{with } p \text{ and } q \text{ being even integers.} \tag{5.2.16}$$

Moreover, according to (5.2.4) the compensators on the two hemispheres are related by the Kähler transformation (5.2.13)

$$r^{(S)} - r^{(N)} = \mathrm{i}gu\,\partial_u h_{NS} = \mathrm{i}g\,n \qquad \Longleftrightarrow \qquad q - p = n. \tag{5.2.17}$$

Therefore we can see that, since in this model the values of the compensator at the two fixed points of the gauged isometry are related to each other, and we are only free to choose one of them.

5.3 FI-terms and supersymmetric truncations

As we discussed in the introduction it is interesting to understand whether if it is possible to generate effective FI-terms by the stabilization of part of the fields of a theory. Here we will show that the size of the FI-term cannot change if the truncation leaves supersymmetry unbroken. In other words, the size of the FI-term in the reduced theory is the same as the size of the FI-term in the full parent theory.

Suppose we perform a supersymmetric truncation of a theory with target space in the Kähler-Hodge manifold $\widehat{\mathcal{M}}$ where we have gauged a U(1) isometry. The manifold $\widehat{\mathcal{M}}$ is parametrized by the fields $\{H^\alpha, L^i\}$, and the truncation is done fixing a sector of the fields H^α at the point $H^\alpha = H^\alpha_0$. Let us denote by $\hat{k}$ the killing vector associated to the gauged U(1) symmetry in the full theory, we call $\widehat{\mathcal{P}}$ the corresponding moment map, and $\hat{\eta}$ the FI-term.

According to our discussion in the previous section, in order to have a non vanishing FI-term the killing vector $\hat{k}$ must have at least one fixed point. For simplicity we restrict ourselves to a patch $U \subseteq \widehat{\mathcal{M}}$ containing is a single fixed point of $\hat{k}$. Then, if $\xi^I_0 = \{H^\alpha_0, L^i_0\}$ is the fixed point of the U(1) isometry, $\hat{k}(\xi^I_0) = 0$, in the full theory we have the following relation

$$\widehat{\mathcal{P}}(\xi^I_0, \xi^{\bar{I}}_0) = \hat{\eta}. \tag{5.3.1}$$

The most interesting situation is when the corresponding gauge field survives in the low energy theory, so that the FI-term in the reduced theory can be used for the spontaneous breaking of gauge symmetries. In such a case the moment map in the reduced theory is given by:

$$\mathcal{P}(L^i, L^{\bar{\imath}}) = \widehat{\mathcal{P}}(H_0^\alpha, H_0^{\bar{\alpha}}, L^i, L^{\bar{\imath}}). \tag{5.3.2}$$

In the discussion of section 3.2.2 we saw that if we want the gauge symmetry to survive in the reduced theory, the components of the killing vector $\hat{k}$ along the H^α directions must vanish in the reduced theory, $\hat{k}^\alpha|_{H_0^\alpha} = 0$. Actually the killing vector in the reduced theory is given by

$$k(L^i) = \hat{k}(H_0^\alpha, L^i) = \hat{k}^i(H_0^\alpha, L^j)\,\partial_i, \tag{5.3.3}$$

which implies that the fixed point of the killing vector k of the truncated theory is L_0^i, since the killing vector $\hat{k}$ of the parent theory vanishes at the configuration $\xi_0^I = \{H_0^\alpha, L_0^i\}$. From this result, together with equation (5.3.2), it follows that the FI-term of the reduced theory η must be equal to the FI-term before the truncation, that is, $\hat{\eta}$:

$$\eta = \mathcal{P}(L_0^i, L_0^{\bar{\imath}}) = \hat{\mathcal{P}}(H_0^\alpha, H_0^{\bar{\alpha}}, L_0^i, L_0^{\bar{\imath}}) = \hat{\eta}. \tag{5.3.4}$$

Therefore the size of the FI-term cannot change by the supersymmetric truncation of part of the field content of the theory. In other words, in order to generate an FI term by the stabilization of some fields supersymmetry must be broken necessarily.

5.4 The axio-dilaton system

In this section we study the *axio-dilaton* system, a sigma model with target space on the hyperboloid $H^2 \cong \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$. As we mentioned in the introduction, this model was considered by Dine *et al.* in [128], where they proposed a mechanism to obtain low energy effective supergravity lagrangians with FI-terms from string theory. In their construction the shift symmetry of the axio-dilaton, $S = \mathrm{e}^\rho + \mathrm{i}a$, was coupled to a U(1) gauge field as a result of the Green-Schwarz mechanism of anomaly cancellation.

Using arguments from our discussion in the previous two sections we recover the results by Binetruy et al. [75]. We show that with the gauge coupling given by the Green-Schwarz mechanism it is not possible to obtain an FI-term after the supersymmetric truncation of the axio-dilaton. We also present an alternative gauging of the sigma-model which allows the supersymmetric integration of the axio-dilaton leaving a reduced theory with a non-vanishing FI-term.

The idea of generating effective FI-terms from field dependent moment maps is also especially relevant in the context of $\mathcal{N} = 2$ supergravity. As we will see in the next chapter, $\mathcal{N} = 2$ supergravity theories are highly constrained. In particular, in $\mathcal{N} = 1$ supergravity models which admit an embedding in $\mathcal{N} = 2$ it is no longer allowed to introduce FI terms as shifts of the moment map (2.2.18). In this type of models the compensator $r_a(\xi)$ is completely determined by the gauging of isometries. Then, in order to find constant FI-terms in $\mathcal{N} = 2$ supergravity the only freedom we have is the gauging of isometries, i.e. the choice of the killing vectors.

In the next chapter we will present a technique to obtain $\mathcal{N} = 1$ models with an effective FI-term from the truncation of $\mathcal{N} = 2$ parent theory [73]. This technique is also based on the idea of truncating a sector of the theory in a supersymmetric way, and therefore is rather similar to our discussion here. The axio-dilaton system is an important building block in this construction. Indeed, the gauged axio-dilaton model we present here can be embedded in $\mathcal{N} = 2$ supergravity, and the truncation of the axio-dilaton in the $\mathcal{N} = 2$ theory leaves behind a reduced $\mathcal{N} = 1$ theory with an FI-term.

5.4.1 Gauging of isometries and FI-term

We consider a sigma model with target space $\mathcal{M} = \frac{\mathrm{SU(1,1)}}{\mathrm{U(1)}}$, i.e. an axio-dilaton system, coupled to a U(1) gauge field A_μ. An appropriate Kähler potential for the axio-dilaton system would be:

$$K(S, \bar{S}) = -\log(S + \bar{S}), \qquad \text{with} \qquad S = \mathrm{e}^\rho + \mathrm{i}a, \tag{5.4.1}$$

For simplicity we choose a vanishing superpotential and the gauge kinetic function to be a constant $f(S) = 1$. The scalar manifold $\frac{\mathrm{SU(1,1)}}{\mathrm{U(1)}}$ has three different isometries characterized by the killing vectors:

$$k_1 = 2S\,\partial_S \qquad k_2 = \mathrm{i}\,\partial_S \qquad k_3 = -\mathrm{i}S^2\,\partial_S. \tag{5.4.2}$$

The isometry defined by k_2 corresponds to the invariance under shifts of the axion a which is manifest in the Kähler potential. We can see that none of the killing vectors has fixed points within the Kähler manifold, since the point $S = 0$ does not belong to H^2.

The compensators corresponding to the killing vectors can be determined, up to an arbitrary imaginary constant, by the equation (2.2.17). In order to illustrate the situation in $\mathcal{N} = 2$ supergravity, where the only freedom we have to obtain non vanishing FI-term is the choice of gauging, we will fix completely the value of the compensators from the beginning, thus removing our freedom to shift them. One possible choice is

$$r_1 = -\tfrac{1}{3} \qquad r_2 = 0 \qquad r_3 = \tfrac{1}{3}\mathrm{i}S, \tag{5.4.3}$$

and therefore the moment maps are given by[2]

$$\mathcal{P}_1 = \frac{S-\bar{S}}{\mathrm{i}(S+\bar{S})}, \qquad \mathcal{P}_2 = \frac{1}{S+\bar{S}}, \qquad \mathcal{P}_3 = \frac{S\bar{S}}{S+\bar{S}}. \tag{5.4.4}$$

Since none of the three killing vectors in (5.4.2) has a fixed point, gauging any one of them would result into a theory with vanishing FI-term.

In the construction by Dine *et al.* [128] the gauge coupling appeared as a result of the Green-Schwarz mechanism of anomaly cancellation. This gauge coupling is obtained promoting to a local symmetry the shift generated by k_2

$$\delta S = \mathrm{i}g\alpha, \qquad D_\mu S = \partial_\mu S - \mathrm{i}gA_\mu, \tag{5.4.5}$$

where α is the gauge parameter. The gauge symmetry is broken everywhere in field space and thus this coupling leads to a theory with a vanishing FI-term. Moreover, since there is no configuration where the gauge boson and the axion decouple, from the discussion in chapter 3 it follows that it is not possible to truncate S consistently and preserve supersymmetry.

An alternative gauging

Instead we choose to gauge the linear combination $k_{FI} = k_2 + k_3$, which has a fixed point at $S = 1$. The killing vector k_{FI} has a very simple geometrical interpretation, actually it generates the isometry associated to rotations of the hyperboloid around its symmetry axis, and the fixed point $S = 1$ corresponds to the vertex of the hyperboloid. The moment map associated to k_{FI} is given by:

$$\mathcal{P}_{FI} = \mathcal{P}_1 + \mathcal{P}_2 = \frac{(1+S\bar{S})}{S+\bar{S}} = \tfrac{1}{2}\left[e^{2h} + (a^2+1)e^{-2h}\right] \geq 1, \tag{5.4.6}$$

where we have assumed that the moment maps combine in the same way as the killing vectors, linearly, as occurs in $\mathcal{N} = 2$ supergravity models. In particular, in $\mathcal{N} = 1$ supergravity models which admit an embedding in $\mathcal{N} = 2$ the moment maps combine in this way. The scalar potential can be obtained from (2.2.14)

$$V = V_D = \frac{1}{2}\mathcal{P}^2 = \frac{1}{2}\frac{(1+|S|^2)^2}{(S+\bar{S})^2}. \tag{5.4.7}$$

It is now easy to check that with this choice of gauging our model has a non vanishing FI-term. Its size is given by the value of the moment map at the fixed point of the killing vector, $S = 1$, where the moment map saturates its lower bound

$$\eta_{FI} = \mathcal{P}_{FI}|_{S=1} = 1. \tag{5.4.8}$$

[2] In order to fix the compensators completely we have chosen the moment maps to be consistent with the gauging of the full $\mathrm{SU}(1,1)$ group [18].

Using equations (2.2.5) and (2.2.12) to calculate the kinetic terms of the axio-dilaton we arrive at the following expression for the bosonic sector of the lagrangian

$$e^{-1/2}\mathcal{L} = -\frac{1}{2}R - \frac{D_\mu S\, D^\mu \bar{S}}{(S+\bar{S})^2} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\frac{(1+|S|^2)^2}{(S+\bar{S})^2}, \tag{5.4.9}$$

where we have defined the covariant derivative as

$$D_\mu S = \partial_\mu S - \mathrm{i}\eta(1-S^2)A_\mu. \tag{5.4.10}$$

The axio-dilaton system gauged in this way can be embedded in $\mathcal{N}=2$ supergravity, giving a theory with an effective FI-term which can be used to obtain de Sitter vacua, to break gauge symmetries, and to construct cosmic string solutions.

5.4.2 Supersymmetric cosmic strings and supersymmetric truncations

An important property of supersymmetric truncations is that they respect BPS configurations, i.e. those which leave unbroken some of the supersymmetries. In other words, if a field configuration is BPS in the reduced theory, it must be BPS in the parent theory. This can be understood noting that the supersymmetric truncation can be considered as an ansatz which ensures that the supersymmetry transformations of the truncated fields are zero, and therefore supersymmetries which survive on the BPS configuration of the reduced theory must also be unbroken in the full theory.

In this section we present a simple model obtained coupling the axio-dilaton system described above with the supersymmetric Abelian-Higgs model discussed in section 2.5. The coupling is done in such a way that the axio-dilaton can be truncated supersymmetrically leaving the Abelian-Higgs model as the reduced theory. Thus, the half-BPS cosmic string solutions found in section 2.5, will also be solutions of the full model which preserve half of the supersymmetries.

Let us consider a sigma model defined on the target space $\widehat{\mathcal{M}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \mathbb{C}$, containing an axio-dilaton S and a complex scalar field ϕ, both coupled to a U(1) gauge boson A_μ. We choose the Kähler potential to be

$$K(\phi,\bar{\phi},S,\bar{S}) = -\log(S+\bar{S}) + \phi\bar{\phi}. \tag{5.4.11}$$

As in the previous section we will assume that the superpotential is vanishing $W=0$, and that the kinetic function is a constant $f(\phi,S)=1$. In order to couple the gauge boson A_μ to the fields ϕ and S we promote to local the isometry associated to the killing vector

$$k = gk_4 + g\eta\, k_{FI}. \tag{5.4.12}$$

Here k_4 is killing vector associated to the standard U(1) gauge coupling, and k_{FI} characterizes the isometry of the axio-dilaton system we gauged in the previous section

$$k_4 = \mathrm{i}\phi\,\partial_\phi, \qquad k_{FI} = \mathrm{i}(1-S^2)\partial_S. \tag{5.4.13}$$

The compensator and the moment map associated to the killing vector k can be chosen to be

$$r = \mathrm{i}g\eta\, S, \qquad \mathcal{P} = -g|\phi|^2 + g\eta\,\frac{1+|S|^2}{S+\bar{S}}. \tag{5.4.14}$$

With a vanishing superpotential the full scalar potential coincides with the D-term potential, and therefore it reads

$$V = V_D = \tfrac{1}{2}\mathcal{P}^2 = \frac{g^2}{2}\left(|\phi|^2 - \eta\,\frac{1+|S|^2}{S+\bar{S}}\right)^2, \tag{5.4.15}$$

Gathering everything together we find that the bosonic sector of the lagrangian is given by

$$\begin{aligned} e^{-1/2}\mathcal{L} \;\; = \;\; & -\frac{1}{2}R - D_\mu\phi D^\mu\bar{\phi} - \frac{D_\mu S D^\mu \bar{S}}{(S+\bar{S})^2} + \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ & -\frac{g^2}{2}\left(|\phi|^2 - \eta\,\frac{1+|S|^2}{S+\bar{S}}\right)^2, \end{aligned} \tag{5.4.16}$$

where the covariant derivatives are defined as

$$D_\mu\phi = \partial_\mu\phi - \mathrm{i}g\phi A_\mu \qquad \text{and} \qquad D_\mu S = \partial_\mu S - \mathrm{i}g\eta\,(1-S^2)A_\mu. \tag{5.4.17}$$

The field configuration $S = 1$ is consistent with the supersymmetric integration of the axio-dilaton. With the axio-dilaton fixed at this point the scalar manifold reduces to

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \mathbb{C} \longrightarrow \mathbb{C} \tag{5.4.18}$$

which, due to the product structure of $\widehat{\mathcal{M}}$, is a totally geodesic Kähler submanifold of the full target space. Moreover at $S = 1$ the gauge boson A_μ and the axio-dilaton decouple, since the component of the gauged killing vector along S becomes zero. From our discussion in chapter 3 it follows that these conditions are enough to ensure the supersymmetric decoupling of the axio-dilaton. In particular it is easy to check that $S = 1$ is an extremum of the scalar potential along the direction S.

In the reduced theory, i.e. with $S = 1$, the compensator reduces to an imaginary constant $r|_{S=1} = \mathrm{i}g\eta$, an FI-term

$$\mathcal{P}|_{S=1} = -g|\phi|^2 + g\eta \quad \Longrightarrow \quad V_{S=1} = \frac{g^2}{2}\left(|\phi|^2 - \eta\right)^2. \tag{5.4.19}$$

Note that the reduced theory coincides with the abelian-Higgs model studied in section 2.5. This reduced model admits half-BPS cosmic string solutions, which preserve a fraction of the supersymmetries of the system. Since the reduced theory is obtained after a supersymmetric truncation, the conditions that ensure the BPS condition in the reduced theory are sufficient to guarantee that the cosmic strings are also BPS in the full theory.

5.5 Summary

In this chapter we have considered the possibility of dynamically generating FI-terms by the supersymmetric truncation of a sector of a $\mathcal{N}=1$ supergravity theory. We have shown that the size of the FI-terms is unaffected by the supersymmetric truncation, or in other words, that FI-terms cannot be generated during the stabilization of heavy fields without breaking supersymmetry.

We have discussed the proposal by Dine *et al.* [128] to construct effective FI-terms from an axio-dilaton system with a U(1) gauge coupling induced the Green-Schwarz mechanism of anomaly cancellation. This study recovers the results by Binetruy *et al.* [75] which show that in this model it is not possible to truncate the axio-dilaton in a supersymmetric way giving an effective theory with an FI-term. We have shown that it is still possible to find a coupling between the axio-dilaton and the U(1) gauge field which is consistent with truncating the axio-dilaton in a supersymmetric way, so that the reduced theory contains a non-vanishing FI-term. Although the isometry associated to this gauging has an easy geometrical interpretation, it is not so clear what would be its physical meaning within the framework of string theory.

Finally we have discussed a model where we couple the version of the gauged axio-dilaton system presented here to the supersymmetric Abelian-Higgs model discussed in section 2.5. The coupling allows for the supersymmetric truncation of the axio-dilaton leaving the Abelian-Higgs model as the reduced theory. With this model we have illustrated an important property of consistent truncations, that BPS solutions of the reduced theory are also BPS solutions of the full theory. That is, if a particular solution of the reduced theory leaves unbroken part of the supersymmetries, these supersymmetries remain unbroken in the parent theory, i.e. if we consider again the effects of all the truncated fields.