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CHAPTER 4

Stability of consistently decoupled sectors.

4.1 Introduction

Supersymmetric vacua are unsuitable to describe the present day cosmology. First, we know supersymmetry to be broken at low energies, and therefore any cosmologically viable model should include a mechanism of supersymmetry breaking. And second, supersymmetric vacua can never a have a positive cosmological constant (2.3.3), and thus they cannot describe the present day acceleration of the universe. The different methods to break supersymmetry and lift the energy of these vacua to de Sitter are known as *uplifting mechanisms*. Many uplifting mechanisms have been proposed, for example in the original KKLT construction this was achieved by including anti-D3 branes in the compactification. Subsequently the work was concentrated on F-term [120, 121, 93], and D-term uplifting [122, 123, 124], where supersymmetry is only broken spontaneously in the light sector, which gives better calculational control. In F-term and D-term uplifting supersymmetry breaks due to the expectation value of the auxiliary fields of the chiral F^I and vector multiplets D^a respectively. Other approaches are discussed in [125, 126], where supersymmetry is broken due to Kähler corrections.

The problem in flux compactifications involving D-term and F-term uplifting mechanisms is that, in general, it cannot be taken for granted that the presence of the light sector is compatible with the supersymmetric integration of the heavy moduli [22, 76, 127, 77, 78]. As we discussed in the previous chapter, in order to be able to stabilize the heavy moduli while preserving supersymmetry the couplings between the light and heavy sectors have to satisfy certain constraints. Moreover, even when these conditions are satisfied, if the light sector breaks supersymmetry nothing guarantees the stability of the heavy field configuration. If the breaking of supersymmetry by the light sector renders this configuration unstable, any perturbation could make the heavy fields evolve far away from if, and thus integrating out this sector would be inconsistent. In this chapter we study the stability of the would-be heavy sector for two classes of models which satisfy the conditions for the supersymmetric integration derived in the last chapter.

4.2 Uplifting with a separable Kähler function.

We now study the perturbative stability of vacua in theories where the heavy sector is integrated out in a supersymmetric way, i.e. the action satisfies the conditions derived in chapter 3, and the light sector is in supersymmetry breaking configuration. In this chapter we will assume that the Kähler invariant function is separable in the heavy and light sectors, eq. (3.5.2). To simplify notion we use the definitions $G = \widehat{G}(H, \bar{H}, L, \bar{L})$, $A = \widehat{G}_1(H, \bar{H})$, and $B = \widehat{G}_2(L, \bar{L})$:

$$G(H, \bar{H}, L, \bar{L}) = A(H, \bar{H}) + B(L, \bar{L}). \tag{4.2.1}$$

The F-term potential derived from this ansatz can be seen to be, using (2.2.32):

$$V_F = \mathrm{e}^{A+B}\left(A^{\alpha\bar{\beta}} A_\alpha A_{\bar{\beta}} + B^{i\bar{\jmath}} B_i B_{\bar{\jmath}} - 3\right), \tag{4.2.2}$$

and the D-term potential reads:

$$\begin{aligned} V_D \;=\; & \tfrac{1}{2}(\mathrm{Re}\, f)^{-1\tilde{a}\tilde{b}}\, k^\alpha_{\tilde{a}} k^{\bar{\beta}}_{\tilde{b}}\, A_\alpha A_{\bar{\beta}} + \tfrac{1}{2}(\mathrm{Re}\, f)^{-1ab}\, k^\alpha_a k^{\bar{\jmath}}_b\, B_i B_{\bar{\jmath}} + \\ & (\mathrm{Re}\, f)^{-1\tilde{a}b}\, k^\alpha_{\tilde{a}} k^{\bar{\jmath}}_b\, A_\alpha B_{\bar{\jmath}}. \end{aligned} \tag{4.2.3}$$

Where we are using the same notation as in chapter 3. In the reduced theory, with the heavy fields stabilized at the supersymmetric configuration $H^\alpha = H^\alpha_0$, the scalar potential reads:

$$V = \widehat{V}|_{H_0} = \left[\mathrm{e}^{A(H)} V_F^{light}(L) + \tfrac{1}{2}(\mathrm{Re}\, f)^{-1ab}\, k^\alpha_a k^{\bar{\jmath}}_b\, B_i B_{\bar{\jmath}}\right]_{H_0} \tag{4.2.4}$$

where $V_F^{light} = \mathrm{e}^B(B^{i\bar{\jmath}} B_i B_{\bar{\jmath}} - 3)$ is the F-term potential of the uplifting sector when considered alone. Since heavy fields are integrated out in a supersymmetric way, if H_0^α is a supersymmetric critical point of the heavy sector, and L_0^i is a critical point the reduced potential V, then the field configuration (H_0^α, L_0^i) is a critical point of the full potential. Moreover, the scalar potential is block diagonal in the two sectors (3.2.38), meaning that there are no quadratic interactions between the fluctuations of the heavy and light fields. Therefore the stability of the two sectors can be studied separately. In the next two sections we will study the perturbative stability of the critical point (H_0^α, L_0^i) along the heavy directions. First in section 4.2.1 we will study the situation where all the fields are uncharged, and in section 4.2.2 we will consider the most general case.

In the absence of gauge couplings it is possible to derive a few general results without a detailed stability analysis. In this case, since $V_D = 0$, the scalar potential of the reduced theory is simply

$$\widehat{V}|_{H_0} = \mathrm{e}^{A(H_0)} V_F^{light}(L). \tag{4.2.5}$$

This expression implies that the critical points of the reduced potential L_0^i coincide with the critical points of the F-term potential of light sector, $V_F^{light}(L)$. Moreover, the value of the potential of the light sector at the critical point L_0^i determines whether the supersymmetric vacuum is lifted to dS, Minkowski or remains AdS:

$$\begin{aligned} V_F^{light}(L) > 0 \qquad &\Longrightarrow \qquad (H_0^\alpha, L_0^i) \text{ is a dS vacuum} \\ V_F^{light}(L) = 0 \qquad &\Longrightarrow \qquad (H_0^\alpha, L_0^i) \text{ is a Minkowski vacuum} \\ V_F^{light}(L) < 0 \qquad &\Longrightarrow \qquad (H_0^\alpha, L_0^i) \text{ is an AdS vacuum.} \end{aligned}$$

In particular, when there is more than one supersymmetric configuration of the heavy sector, all of them become degenerate when uplifted to Minkowski (note that this makes the possibility of topological inflation quite natural). As the stability along the heavy and light direction can be studied independently, the relation (4.2.5) leads to a remarkably simple result. Namely, the vacuum (H_0^α, L_0^i) is perturbatively stable with respect to fluctuations of the light fields as long as L_0^i is a minimum of the potential of the light sector $V_F^{light}(L)$.

Although the stability analysis along the heavy directions is more involved, it is possible to derive some general results. Due to the block diagonal structure of the Hessian of the potential at the point (H_0^α, L_0^i), in order to analyze the stability of this configuration along the heavy directions it is enough to study the potential evaluated at L_0^i

$$V|_{L_0^i} = \mathrm{e}^{B(L_0)} \mathrm{e}^{A} \left(A^{\alpha\bar{\beta}} A_\alpha A_{\bar{\beta}} + (b-3) \right) \tag{4.2.6}$$

Note that the stability analysis for fluctuations of the heavy fields depends on the light sector only through a single parameter $b = B^{i\bar{\jmath}} B_i B_{\bar{\jmath}}|_{L_0}$ that controls the amount of uplifting,

$$b - 3 = e^{-G} V|_{H_0,L_0} = \left(\frac{3H}{m_{3/2}}\right)^2. \quad (4.2.7)$$

A remarkable property of this model is that all configurations (H_0^α, L_0^i) are stable or marginally stable after uplifting to Minkowski vacuum ($b = 3$). To see this, we set $b = 3$ in the expression (4.2.6), then the full potential evaluated at L_0^i reads

$$V_{L_0} = \mathrm{e}^{B(L_0)} \mathrm{e}^A \; A^{\alpha\bar{\beta}} A_\alpha A_{\bar{\beta}} \geq 0 \quad \text{for all } L^i. \quad (4.2.8)$$

Since, by assumption, $V(H_0, L_0) = 0$, the condition (4.2.8) implies that no fluctuation of the fields on the heavy sector can decrease the energy, and therefore the point (H_0^α, L_0^i) is either a local minimum or a plateau along the heavy directions. For large final values of the cosmological constant the stability analysis simplifies considerably. In this limit the full potential (4.2.6) becomes approximately:

$$V(H, L) \approx b \, \mathrm{e}^{A+B}, \quad (4.2.9)$$

and therefore the stable configurations of the heavy sector are those minimizing the Kähler function $A(H, \bar{H})$.

4.2.1 Stability of uplifted vacua with zero D-term potential

In order to study the stability of the configuration (H_0^α, L_0^i) along the heavy directions we will apply the same technique we used in the previous chapter for supersymmetric critical points. First we need to calculate the derivatives of the potential $V_{\alpha\bar{\beta}}(H_0, L_0)$ and $V_{\alpha\beta}(H_0, L_0)$ from (4.2.2):

$$\begin{aligned} V_{\alpha\bar{\beta}}(H_0, L_0) &= \mathrm{e}^{A+B}|_{H_0,L_0} \left[A^{\gamma\bar{\delta}} A_{\alpha\gamma} A_{\bar{\beta}\bar{\delta}} + (b-2) A_{\alpha\bar{\beta}}\right]_{H_0} \\ V_{\alpha\beta}(H_0, L_0) &= \mathrm{e}^{A+B}|_{H_0,L_0} \, (b-1) A_{\alpha\beta}(H_0), \end{aligned} \quad (4.2.10)$$

In the following we will use the notation $X = A_{\alpha\beta}(H_0)$. As in the previous chapter we will assume the fields H^α to be canonically normalized at H_0^α. Moreover, we will use the residual freedom to define the fields to make the matrix $X^\dagger X$ real and diagonal

$$X^\dagger X = Diag(|x_1|^2 \mathbb{1}_{n_1}, \ldots, |x_p|^2 \mathbb{1}_{n_p}), \qquad |x_\lambda| \geq 0. \quad (4.2.11)$$

Here the index $\lambda = 1, \ldots, p$, labels the subspace corresponding to the eigenvalue $|x_\lambda|^2$, which has dimension n_λ. With this choice we obtain the following expression for the Hessian of the potential at (H_0^α, L_0^i):

$$\begin{pmatrix} V_{\alpha\bar{\beta}} & V_{\alpha\beta} \\ V_{\bar{\alpha}\bar{\beta}} & V_{\alpha\bar{\beta}} \end{pmatrix}_{H_0,L_0} = \mathrm{e}^{A+B}|_{H_0,L_0} \begin{pmatrix} XX^\dagger + (b-2)\,\mathbb{1} & (b-1)\,X \\ (b-1)\,X^\dagger & X^\dagger X + (b-2)\,\mathbb{1} \end{pmatrix}. \quad (4.2.12)$$

The mass spectrum can be calculated along the same lines of section 2.4, what leads to our final result:

$$\begin{aligned} m^2_{\pm\lambda} &= \mathrm{e}^{A+B}|_{H_0,L_0}\Big[|x_\lambda|^2 + (b-2) \pm |(b-1)x_\lambda|\Big] = \\ &\quad \mathrm{e}^{A+B}|_{H_0,L_0}\left[\Big(|x_\lambda| \pm \tfrac{1}{2}(b-1)\Big)^2 - \tfrac{1}{4}(b-3)^2\right]. \end{aligned} \tag{4.2.13}$$

To obtain the last expression we assumed that $b > 1$, but in the case $b < 1$ then $m^2_{+\lambda}$ and $m^2_{-\lambda}$ have to be exchanged. For each energy level characterized by $m^2_{\pm\lambda}$ there are n_λ different excitations with the same mass. The stability condition after uplifting the minimum of the potential to Minkowski or de Sitter, $b \geq 3$, reduces to $m^2_{\pm\lambda} > 0$ for all $\lambda = 1, \ldots, p$, but if the minimum remains AdS after the uplifting, $b < 3$, the masses have to satisfy the Breitenlohner-Freedman bound (2.3.4):

$$\begin{aligned} &\text{for } \ b < 3 \qquad \Longrightarrow \qquad \left[\Big(|x_\lambda| \pm \tfrac{1}{2}(b-1)\Big)^2 - \tfrac{1}{4}(b-3)^2\right] \geq \frac{3}{4}(b-3), \\ &\text{for } \ b \geq 3 \qquad \Longrightarrow \qquad \left[\Big(|x_\lambda| \pm \tfrac{1}{2}(b-1)\Big)^2 - \tfrac{1}{4}(b-3)^2\right] \geq 0. \end{aligned} \tag{4.2.14}$$

Recalling that $b \geq 0$, and after a little bit of algebra, it is possible to see that the first of the two inequalities is always satisfied. The second shows that there are no instabilities when the minimum is uplifted to Minkowski $b = 3$, although zero modes are possible if any $|x_\lambda| = 1$. Thus, the instabilities can only arise for upliftings to dS. These results are summarized in figure 4.1.

Before the uplifting, i.e. if the light sector does not break supersymmetry, the results we obtained in section 1.3 also apply here, since (H_0^α, L_0^i) is a supersymmetric configuration. In particular, the maxima of the scalar potential before the uplifting coincide with the minima of the total Kähler function $G(H, \bar{H}, L, \bar{L})$, which are the minima of $A(H, \bar{H})$ due to the ansatz (3.5.2). According to the result (2.4.11) the configuration H_0^α is a minimum of $A(H, \bar{H})$ provided that all the eigenvalues of $X^\dagger X$ satisfy $|x_\lambda| < 1$. Therefore the results (4.2.14) imply that local AdS minima and saddle points before the uplifting are only stable for small values of the cosmological constant, while local AdS maxima of the potential, which coincide with the local minima of the Kähler function, are always stable.

4.2.2 Stability of uplifted vacua with non-zero D-term potential

Now we study the stability of uplifted vacua when the gauge couplings are turned on. Including gauge interactions is specially relevant in the case of the light sector, since it includes the visible sector. As we discussed in section 3.1.3, the conditions necessary for the supersymmetric decoupling of the heavy sector ensure that the Hessian of the D-term potential is block diagonal in the heavy and

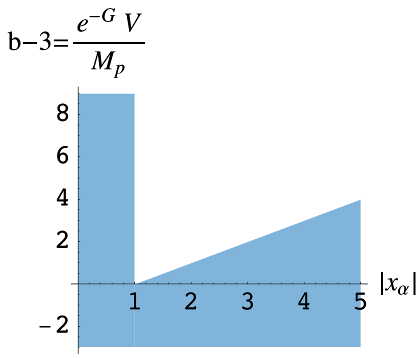

Figure 4.1 – *Stability of supersymmetric critical points after the uplifting. The quantity on the vertical axes* $b-3$ *is proportional to the cosmological constant (or the Hubble parameter during inflation). The horizontal axes represents the curvature of the Kähler function at the critical point along one of the heavy field directions* H^α*:* $|x_\lambda| < 1$ *corresponds to local minima and* $|x_\lambda| >$ *to saddle points. The shaded region represents stable configurations under perturbations of the heavy fields. For* $b < 3$ *and* $b = 3$ *the uplifted vacua, which are AdS and Minkowski respectively, are always stable. Local AdS minima of the scalar potential at zero uplifting,* $|x_\lambda| > 2$*, are always destabilized for large uplifting. Local AdS maxima,* $|x_\lambda| < 1$*, remain stable for arbitrary large uplifting.*

light sectors, $V_{D|\alpha i}(H_0, L_0) = V_{D|\alpha\bar{\imath}}(H_0, L_0) = 0$. Therefore in order to calculate the contribution of the D-term potential to the Hessian of $\widehat{V}$ at (H_0^α, L_0^i) we just have to calculate the second derivatives of V_D along the heavy directions:

$$V_{D|\alpha\beta}(H_0, L_0) = \tfrac{1}{2}\big(\mathrm{Re}\, f(H_0, L_0)\big)^{-1\,ab}\, k_a^\gamma(H_0, L_0)\, k_b^{\bar{\delta}}(\bar{H}_0, \bar{L}_0)\, A_{\gamma\alpha} A_{\bar{\delta}\beta},$$
$$V_{D|\alpha\bar{\beta}}(H_0, L_0) = \tfrac{1}{2}\big(\mathrm{Re}\, f(H_0, L_0)\big)^{-1\,ab}\, k_a^\gamma(H_0, L_0)\, k_b^{\bar{\delta}}(\bar{H}_0, \bar{L}_0)\, A_{\gamma\alpha} A_{\bar{\delta}\bar{\beta}} \quad (4.2.15)$$

The mass matrix of the heavy fields at (H_0^α, L_0^i), is given by the sum of the Hessian of the D-term potential with respect to the heavy fields and the one we found for the F-term potential (4.2.12). Following the discussion in section 2.4.1, we choose our heavy scalar fields so that they have trivial kinetic terms $G_{\alpha\bar{\beta}} = 1$ and that the matrices $X^\dagger X$ ($X = A_{\alpha\beta}$) and $k_a^\alpha k_a^{\bar{\beta}}$ are both real and diagonal. We also define the gauge fields $A_\mu^{(h)\,a}$ such that the real parts of the gauge kinetic functions of the heavy sector are proportional to the identity matrix:

$$\mathrm{Re}\, f_{ab}^{(h)} = \mathrm{e}^G|_{H_0, L_0}\, \delta_{ab}. \quad (4.2.16)$$

The calculation of the D-term contribution to the Hessian can be done along the same lines as in section 2.4.3. Is not difficult to check that the properties (2.4.28) and (2.4.29) still hold, thus here again the matrices $kk^\dagger$ and kk^T, with

$k = k_a^\alpha$, have non-vanishing components only in the eigenspace corresponding to the eigenvalue $|x_1|^2 = 1$. In particular, the block $k_1^\alpha k_1^{\bar\beta}$ is of the form:

$$k_1 k_1^\dagger = Diag(|k_1|^2, \ldots, |k_{n_1}|^2). \tag{4.2.17}$$

Thus, after some simplifications, the Hessian of the total scalar potential reads:

$$\begin{pmatrix} V_{\alpha\bar\beta} & V_{\alpha\beta} \\ V_{\bar\alpha\bar\beta} & V_{\alpha\bar\beta} \end{pmatrix}_{H_0,L_0} = \mathrm{e}^{A+B}|_{H_0,L_0} \begin{pmatrix} XX^\dagger + kk^\dagger + (b-2)\,\mathbb{1} & (b-1+kk^\dagger)\,X \\ (b-1+kk^\dagger)\,X^\dagger & X^\dagger X + kk^\dagger + (b-2)\,\mathbb{1} \end{pmatrix}. \tag{4.2.18}$$

From this expression it is straightforward to find the mass spectrum of fluctuations of the heavy sector along the heavy directions:

$$\begin{aligned} m^2_{\pm\lambda} &= e^{G(\xi_0)}\left(\left(|x_\lambda| \pm \tfrac{1}{2}(b-1)\right)^2 - \tfrac{1}{4}(b-3)^2\right) && \text{if } |x_\lambda|^2 \neq 1, \\ m^2_{+1i} &= 2\, e^{G(\xi_0)}\left(|k_i|^2 + b - 1\right) && \text{if } |x_\lambda|^2 = 1, \\ m^2_{-1i} &= 0 && \text{if } |x_\lambda|^2 = 1. \end{aligned} \tag{4.2.19}$$

We can see that figure 4.1 is still valid when we include the gauge interactions. The only difference with the result in the previous section is that if some of the gauge symmetries are spontaneously broken the mass degeneracy in the eigenspace with $|x_\lambda| = 1$ is destroyed. From (4.2.19) we can see that the presence of gauge interactions only increases the stability of the critical point.

4.3 Uplifting with more general couplings

An interesting question to consider is whether it is possible apply our results to other systems where heavy moduli are supersymmetrically decoupled, but its interactions with the heavy are given by an ansatz more general than (4.2.1). As we discussed in section 3.2, the condition that the potential has to be block diagonal on the light and heavy fields is necessary in order to integrate out the heavy fields consistently. Therefore, in any scenario where part of the moduli are going to be integrated out the stability of these fields can be studied independently considering only the "heavy" directions in field space.

Let us assume only the mild condition that the Kähler potentials are separable in the light and heavy sectors:

$$K = K^h(H, \bar H) + K^l(L, \bar L).$$

This condition ensures that mixed derivatives of the Kähler function of the form $G_{i\bar\alpha}(H_0, L_0)$, $G_{i\bar\alpha\beta}(H_0, L_0)$ et cetera... involving both holomorphic and antiholomorphic indices from the two sectors must vanish. The condition (3.2.9) also implies that the derivatives of the form $G_{\alpha i}|_{H_0} = 0$. Since the Hessian of the

Kähler function is block diagonal in the light and heavy sectors, it makes sense to study the curvature of $G(H, L)$ at the critical point H_0^α only along the heavy directions. Thus, we can repeat the analysis of section 2.4.1 arriving at similar conclusions:

- The Kähler function $G(L, H)$ has a local minimum at H_0^α along the heavy directions if the eigenvalues of the matrix $X^\dagger X$ satisfy the conditions $|x_\lambda| < 1$ for all $\lambda = 1, \ldots, p$, with $X = G_{\alpha\beta}(H_0, L_0)$.
- If any of the eigenvalues of $X^\dagger X$ satisfies $|x_\lambda| > 1$ the function $G(H, L)$ has a saddle point at H_0.
- For each eigenvalue of $X^\dagger X$ satisfying $|x_\lambda| = 1$ the Kähler function has a neutrally stable direction and a minimum along some complex direction H^α.

Using all these results, we can now study the stability of the scalar potential along the heavy directions as in sections 2.4.2 and 2.4.3. The second derivatives of the scalar potential are given by

$$V_{\alpha\beta}(H_0, L_0) = \mathrm{e}^G|_{H_0,L_0}\Big[(b-1)G_{\alpha\beta} + G^{i\bar{\jmath}}G_{i\alpha\beta}G_{\bar{\jmath}}\Big]_{H_0,L_0}, \quad (4.3.1)$$

$$V_{\alpha\bar{\beta}}(H_0, L_0) = \mathrm{e}^G|_{H_0,L_0}\Big[G^{\gamma\bar{\delta}}G_{\alpha\gamma}G_{\bar{\beta}\bar{\delta}} + (b-2)G_{\alpha\bar{\beta}}\Big]_{H_0,L_0}, \quad (4.3.2)$$

where we have used the notation $b = G^{i\bar{\jmath}}G_iG_{\bar{\jmath}}|_{H_0,L_0}$.

Note that, apart from the second term in the equation (4.3.1), the result we have obtained is of the same form as (4.2.10). If the quantity $G_{i\alpha\beta}$ stays of order $\mathcal{O}(1)$, the extra term that we have obtained is roughly of order $\mathcal{O}(b^{1/2})$, which means that for large values of the uplifting, $b \gg 3$, it will become subdominant. Therefore, in this limit, the mass matrix becomes proportional to the Hessian of the Kähler function at H_0^α:

$$\begin{pmatrix} V_{\alpha\bar{\beta}} & V_{\alpha\beta} \\ V_{\bar{\alpha}\bar{\beta}} & V_{\alpha\bar{\beta}} \end{pmatrix}_{H_0,L_0} = b\begin{pmatrix} \mathbb{1} & X \\ X^\dagger & \mathbb{1} \end{pmatrix}\mathrm{e}^{A+B}|_{H_0,L_0} = b\begin{pmatrix} G_{\alpha\bar{\beta}} & G_{\alpha\beta} \\ G_{\bar{\alpha}\bar{\beta}} & G_{\bar{\alpha}\beta} \end{pmatrix}_{H_0,L_0}\mathrm{e}^{A+B}|_{H_0,L_0}, \quad (4.3.3)$$

indicating that the minima of the Kähler function along the heavy directions will always survive uplifting to an arbitrary large value of the cosmological constant. Note also that before uplifing, $G_i(H_0, L_0) = 0$, the mass matrix given by (4.3.2) coincides with (2.4.14), so we can again identify the AdS maxima of the scalar potential with the local minima of the Käher function along the heavy directions.

We would like to emphasize that in order to obtain this result we have made very mild assumptions. We have required that the Kähler potential is separable in the two sectors, we have also imposed the condition that the effective action left after integrating out the heavy moduli is invariant under supersymmetry, and finally we asked the quantity $G_{i\alpha\beta}$ to stay of order $\mathcal{O}(1)$ for large values of the uplifting. In this scenario we have proved that the AdS maxima of the potential along the heavy directions at zero uplifting ($G_i(H_0, L_0) = 0$), which are perturbatively stable configurations, remain stable after the uplifting for arbitrary large values of the cosmological constant.

4.4 Summary

We have studied the perturbative stability of the heavy sector in two different classes of models that allow for the supersymmetric integration of the heavy moduli. In the first case the couplings between the heavy and light sectors are characterized by a separable Kähler function

$$\widehat{G}(H, \bar{H}, L, \bar{L}) = G_h(H, \bar{H}) + G_l(L, \bar{L}), \tag{4.4.1}$$

which can be expressed in terms of the Kähler potential and the superpotential as follows

$$\widehat{K}(H, \bar{H}, L, \bar{L}) = K_h(H, \bar{H}) + K_l(L, \bar{L}), \qquad \widehat{W}(H, L) = W_h(H) W_l(L).$$

An interesting property of this type of couplings is that there is always choice of fields such that the mass matrix and the Kähler metric can be diagonalized simultaneously. This allows expressing the stability requirement of having a positive definite mass matrix as a constraint on the curvature of the Kähler function at the vacua of the full theory (H_0^α, L_0^i). Our results, which are displayed in fig. 4.1, show that if the heavy fields are fixed at a minimum of the Kähler function, an AdS maximum of the scalar potential, the configuration remains stable for any final value of the cosmological constant. However, if the heavy fields are fixed at a saddle point of the Kähler function, (the Kähler function cannot have maxima), the configuration always becomes unstable for large enough values of the cosmological constant.

This analysis complements that of Covi *et al.* [79], who formulated a necessary (and in most practical situations sufficient) condition for the existence of (meta)stable de Sitter vacua, following earlier work by Gomez Reino and Scrucca [120, 121, 93]. The constraint restricts the Kähler geometry of the non-linear sigma model associated to the chiral multiplets. Expressed in terms of the metric $G_{I\bar{J}}$ and the Riemann tensor $R_{I\bar{J}M\bar{N}}$ of the Kähler manifold it reads:

$$\sigma \equiv \left[\frac{1}{3}\left(G_{I\bar{J}} G_{M\bar{N}} + G_{I\bar{N}} G_{M\bar{J}}\right) - R_{I\bar{J}M\bar{N}}\right] G^I G^{\bar{J}} G^M G^{\bar{N}} > 0. \tag{4.4.2}$$

This condition, they point out, is e.g. not satisfied by moduli with no-scale Kähler functions of the form $K = -3\log(\xi + \bar{\xi})$, or more generally

$K = -\sum_I n_I \log(\xi^I + \bar{\xi}^{\bar{I}})$, $\sum_I n_I = 3$. Clearly, the constraint (4.4.2) is only sensitive to the geometry of the Kähler manifold along the direction of the goldstino vector G_I, and therefore it can say nothing about the perturbative stability of moduli with zero F-terms, $G_I = 0$. In particular, it cannot be used to restrict the interactions of those fields that are supersymmetrically decoupled from the sector that breaks supersymmetry. Our work provides necessary and sufficient conditions for the perturbative stability of these $G^I = 0$ fields in the class of models where the Kähler function is of the form (4.4.1).

Finally, we have been able to prove that even in more general scenarios where the integrated heavy moduli do not satisfy (4.4.1), the supersymmetric AdS maxima are always stable for large values of the cosmological constant.