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CHAPTER 1

Introduction: cosmological models in field theory and topological defects.

1.1 Cosmology

During the 20th century our knowledge of the history of the universe experienced a spectacular development, which was triggered by important breakthroughs both in theoretical physics and in observations. The discovery of the recession of distant galaxies at the beginning of the 20th century is often referred to as one of the most significant contributions to the birth of modern cosmology. For a review on cosmology see [1].

Between 1925 and 1929 E. Hubble was able to determine the velocities of 18 galaxies by measuring the Doppler shift (*redshift*) of the light they emit. He confirmed that most of the galaxies appeared to move away from us, and moreover, he found that their recession velocity v satisfied a simple relation with the distance x from us:

$$v = H\,x.$$

This is the so called *Hubble law* and H is the *Hubble parameter*. At first sight, the fact the galaxies are moving away from us, seems to imply that we live on a very special place in the universe. However an observer at any arbitrary point of an expanding universe which is homogeneous and isotropic would see exactly the same thing. Indeed, the galaxy surveys seem to indicate that the distribution of galaxies is homogeneous and isotropic on scales larger that about 100 Mpc, and, therefore, *there are no privileged places in our universe.* This statement, which is one of the conceptual cornerstones of modern cosmology, is known as the *cosmological principle.*

From the fact that the universe is currently expanding we could anticipate that at very early times the universe should have been very dense and hot and, extrapolating even further, that at some instant in the past everything was together at the same point. This scenario is known as the *hot Big-Bang cosmology.* The first works on the physics at such early times appeared during the 1940's [2, 3]. These early attempts suggested that the universe had undergone a phase where all the matter was ionized, with both matter and radiation in thermal equilibrium. Eventually, the temperature would be low enough for the first atoms to form, and photons would be able to travel freely, leaving a universe filled with relic radiation.

This relic radiation, known as the *Cosmic Microwave Background*, was detected in 1965 by A. Penzias and R. W. Wilson [4], and became one of the crucial observations that supported the Big-Bang cosmology. The most recent measurements of the Cosmic Microwave Background with the *Wilkinson Microwave Anisotropy Probe* (WMAP) indicate that it is extremely isotropic, with temperature fluctuations of one part in 10^5, and it is known to have a very accurate black-body spectrum with a temperature of about 2.7 K [5]. The fluctuations of the Cosmic Microwave Background reflect the energy density perturbations which, under the action of gravity, grew to form the large scale structures of the universe.

The first rigorous attempts to describe a dynamical universe appeared at the beginning of the 20th century, shortly after Einstein formulated the equations of General Relativity [6, 7, 8, 9]. The fundamental object in General Relativity is the space-time metric, which in the case of a homogeneous and isotropic universe takes the form (we work in the units $c = \hbar = 1$)

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right). \qquad (1.1.1)$$

This is known as the Friedmann-Lemaître-Robertson-Walker metric, and the time dependent function $a(t)$, the *scale factor*, characterizes the evolution of the universe. This metric is written in terms of spherical *comoving coordinates*, so that the galaxies have the same fixed coordinates at all times, provided that we neglect small random velocities. The constant k can take the values $+1, -1, 0$ corresponding to an closed, open and flat universe respectively.

It is very easy to check that the scale factor and the Hubble parameter are related to each other. The distance between our galaxy and the observed one can be obtained from the metric, for instance in a nearly flat universe like ours, $k \approx 0$, it is simply $x = a(t)r$. In this language an expanding universe corresponds to a scale factor that grows with time $\dot{a}(t) > 0$. The Hubble law follows immediatly from this since:

$$v = \frac{dx}{dt} = \frac{\dot{a}}{a}\, x \equiv H\, x. \tag{1.1.2}$$

The evolution of the scale factor $a(t)$ is governed by the Friedmann equation, which follows from the Einstein equations of General Relativity:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1.1.3}$$

where G is Newton's constant and ρ is the energy density in the universe. In order to determine the cosmological evolution, the Friedmann equation has to be supplemented by the equation for the conservation of energy

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = 0, \tag{1.1.4}$$

and an equation of state relating the pressure p with the energy density, $p = p(\rho)$.

The energy density of the different constituents of the universe, such as matter and radiation, evolve in different ways as the universe expands. For example, in the case of non-relativistic matter (dust) the equation of state is simply $p = 0$, and therefore, from (1.1.4), we conclude that the energy density of non-relativistic matter evolves as $\rho_m \sim a^{-3}$. On the other hand the equation of state for radiation is given by $p = \frac{1}{3}\rho$, which leads to the following relation between its energy density and the scale factor $\rho_r \sim a^{-4}$. Since the energy content of the universe determines its evolution, (1.1.3), this observation implies that it is possible to infer the composition of the universe from the time evolution of the scale factor.

The evolution of the scale factor was characterized accurately for the first time during the 1990s. Two teams, the *Supernova Cosmology Project* and the *High-z Supernova Search Team*, measured the redshift of large samples of distant galaxies, for which the distance could be determined from type Ia supernova observations. The results from these experiments were a surprise for many cosmologists, because they showed that our universe was expanding at an increasing rate instead of decelerating, which was the most extended belief. This discovery earned S. Perlmutter (from Supernova Cosmology Project), A. G. Riess and B.

P. Smith (both from the High-z Supernova Search Team) the 2011 Nobel prize in physics [10]. Their data implied that the most abundant constituent of our universe (73 %) was an exotic form of energy with negative pressure, which has equation of state of the form $\rho_{de} = \omega\, p$ and $\omega < -1/3$. Nowadays the microscopic origin of this energy, known as *Dark Energy*, is still uncertain. These are some of the ideas which have been proposed to describe cosmic acceleration (see [11]):

- The simplest one is the introduction of the *cosmological constant* in Einsteins equations. When this extra term is added the Friedmann equation reads:
$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{1.1.5}$$
Although this term is permitted by General Relativity it does not provide an explanation of the physics underlying cosmic acceleration.

- The cosmological constant can be interpreted as the *vacuum energy of empty space*. In such a case $\omega = -1$, which is the value preferred by the observations. However particle physics theories predict a cosmological constant which is orders of magnitude too large to be consistent with the data.

- Another possible explanation is that the Dark Energy is related to the vacuum energy of a scalar field. Scalar fields are ubiquitous in theories of high energy physics such as the Standard Model (the Higgs), Grand Unification Theories (GUTs) and superstring theories.

- It has also been suggested that cosmic acceleration could be a result of gravitational physics, such as non-linear effects due to energy density inhomogeneities, or modifications of General Relativity which only become relevant at cosmological scales.

Recently the composition of the universe has been determined very accurately with the observations of the CMB provided by the WMAP satellite [5]. Usually the abundances are given relative to the critical energy density, $\rho_c \equiv 3H^2/8\pi G$, and thus they are represented by the quantities of the form $\Omega = \rho/\rho_c$, and the curvature of the universe is represented by $\Omega_k = -k/a^2H^2$. The CMB data alone can bound the curvature of the universe between the limits $-0.273 < \Omega_k < 0.013$ [5], indicating that we live in a closed, almost flat universe. Assuming a completely flat universe, $\Omega_k = 0$, the WMAP data alone give the following results for the abundances [5]:

$$\Omega_b = 0.0449 \pm 0.0028 \qquad \Omega_{dm} = 0.222 \pm 0.026 \qquad \Omega_{de} = 0.734 \pm 0.029 \tag{1.1.6}$$

From these data we can see that the universe is dominated by Dark Energy, Ω_{de}. Another striking feature is that most of the matter content of the universe (22 %) is non-relativistic non-baryonic matter, known as *dark matter* Ω_{dm}, whose origin is still undetermined. On the other hand ordinary matter, characterized by Ω_b, only constitutes 4.5 % of the total energy content.

This is often called the Concordance Model or ΛCDM, where the acronym refers to the most abundant constituents of the universe, the dark energy or cosmological constant Λ, and Cold Dark Matter (CDM).

1.1.1 Cosmic Inflation.

Despite the many successes of the standard Big-Bang cosmology, which describes very well the high redshift supernova observations, the Cosmic Microwave Background and the formation of large scale structure, there are several observations that cannot be explained in this framework. These are the *flatness problem*, the *horizon problem* and the *monopole problem*.

The flatness problem is related to the fact that the universe today is close to having flat (Euclidean) geometry $\Omega_k \approx -0.08$. From the Friedmann equation it is possible to see that a Euclidean universe, $\Omega_k = 0$, is an unstable point, meaning that during the expansion Ω_k tends to move away from zero. Actually, according to the Big-Bang cosmology, if we extrapolate backwards to the time when the CMB was formed, we find that Ω_k should have had an extremely small value $|\Omega_k| \sim 10^{-5}$, and as we extrapolate to earlier times it gets even closer to zero, indicating a severe finetuning of this parameter.

The second problem of the standard Big-Bang cosmology is related to the finite age of the universe. Since light only had a finite time to travel, there is a maximum distance beyond which we couldn't have received any information. Light emitted beyond that distance did not have time to reach us yet. The boundary of the observable universe is known as the *horizon*. Therefore two regions separated by a distance larger than the size of the horizon could never have been in causal contact. As we mentioned above the Earth is bathed in a relic radiation, the CMB, which is extremely isotropic with a temperature of about 2.7 K. Since the temperature is the same in every direction of the sky this seems to indicate that all the universe we observe was once in thermal equilibrium, and thus in causal contact. However, according to the standard Big-Bang cosmology, two opposite points of the sky are separated by a distance over a hundred times the horizon size, and could never have been in causal contact.

The last puzzle, the monopole problem, appears in the context of Grand Unified Theories, which we will discuss briefly in a later section. These theories would describe the physics of the universe at energy scales of order 10^{15} GeV, and thus are relevant at very early stages of the universe. A generic prediction of these theories is the formation of extremely massive particle-like objects, the *magnetic monopoles*. These particles would rapidly dominate the energy density of the universe, leading to an evolution incompatible with the present observations.

The most widely accepted explanation to all these problems was proposed in 1981 by A. Guth, *cosmic inflation* [12]. The basic idea of inflation is that the universe underwent an extremely fast, almost exponential accelerated expansion, at a very early stage of its evolution. During inflation, which typically could last 10^{-34} seconds, the universe was stretched by a factor of at least 10^{28}, enough to solve the flatness, horizon, and monopole problems. In addition, "slow roll" inflation can give an explanation for the adiabatic, near gaussian, and almost scale invariant spectrum of primordial density perturbations that is observed.

The idea of inflation can be implemented in the context of field theory introducing a scalar field, *the inflaton* ϕ. In the basic picture the scalar field is in a homogeneous configuration ϕ_0, with a very large potential energy, slowly rolling down an extremely flat potential. The slow roll condition is

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1. \tag{1.1.7}$$

In this setting the scalar field contributes to the energy density of the universe with its potential energy, which acts as an effective cosmological constant:

$$\rho_{infl} = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}(\nabla\phi)^2 + V(\phi) \approx V(\phi_0). \tag{1.1.8}$$

If the potential energy is sufficiently large to dominate the energy density of the universe, from the Friedmann equation it is possible to see that the scale factor evolves exponentially:

$$a(t) = \exp\left(\sqrt{\frac{V(\phi_0)}{3M_p^2}}\, t\right). \tag{1.1.9}$$

where M_p is the reduced Plank mass[1]. In the present example it can be written in terms of the scalar potential $V(\phi)$, leading to so called slow roll conditions:

$$\epsilon_V \equiv \frac{M_p^2}{2}\frac{V'}{V} \ll 1, \qquad |\eta_V| \equiv M_p^2\frac{|V''|}{V} \ll 1. \tag{1.1.10}$$

Since the scalar field is varying in time, the expansion is not exactly exponential. Eventually the scalar field reaches a point where the conditions for inflation are no longer satisfied and the exponential expansion ends. During this process the inflaton decays leading to the initial state of the standard Big-Bang cosmology.

[1] The Plank mass is defined as $m_p^2 = \hbar c/G$, but in our current units is $m_p^{-2} = G$. However, for convenience in this chapter we use the reduced Plank mass, which in our units reads $M_p^{-2} \equiv 8\pi G$. In the remaining chapters we will work in units of the reduced Planck mass $M_p = 1$.

In contrast with standard Big-Bang evolution, during the exponential expansion the universe approaches a Euclidean geometry with $\Omega_k = 0$:

$$\Omega_k = \frac{k}{a^2 H^2} \propto \exp\left(-\sqrt{\frac{4}{3M_p^2}V(\phi_0)}\, t\right), \tag{1.1.11}$$

which would explain the finetuning of the parameter Ω_k at the initial stage of Big-Bang Cosmology. Inflation also solves the horizon problem. During inflation a patch of the universe small enough to be in thermal equilibrium, and thus in causal contact, could have been stretched to be much larger that the present observable universe. Therefore the CMB radiation has the same temperature in every direction because all these points of the sky were once in thermal equilibrium. Finally the monopoles, as non relativistic matter, have an energy density that evolves as $\rho_{monop} \sim a^{-3}$, which implies that the cosmic inflation would have diluted them to the point that they cannot be detected.

Inflation also provides an explanation for the origin of structure formation. According to the standard picture the irregularities observed in the CMB originated from quantum fluctuations, which during inflation were stretched to a cosmic size. This mechanism is currently the most accepted one, due to the excellent agreement between its predictions and the WMAP observations of temperature fluctuations in the CMB.

1.2 High energy physics.

1.2.1 Symmetry and spontaneous symmetry breaking.

Two of the most important developments in theoretical physics during the 20th century were Quantum Field Theory and Einstein's theory of General Relativity (see [13, 14]), which have been essential in the understanding of the four fundamental interactions of nature. While General Relativity treats gravity, Quantum Field Theory led to the formulation of the Standard Model of particle physics, which describes the other three fundaments forces: electromagnetic, strong and weak interactions.

The idea of symmetry plays a fundamental role in these two theories. A physical system presents a symmetry when the laws that govern its evolution are invariant under a particular *group of transformations*. In field theory the symmetries can be classified in various types, such as *space-time* and *internal*. The first ones are related to the transformations that involve space and time, while the internal symmetries relate the different fields that characterize the system to each other. Thus the fields must form a representation of the group of transformations associated with the internal symmetry. A symmetry is said to be *global* when the system is invariant under transformations which act in the same way at every point of space time. If instead the transformations are allowed to vary from point to point then it is called a *local symmetry*. The last type of symmetries are the basis for *gauge theories*. In order to gauge a local internal symmetry it is necessary to introduce a spin-1 field, a gauge boson, which has to be massless for the theory to be renormalizable.

In some situations the ground state of a system is only invariant under a subgroup H of the full symmetry group, G of the equations of motion. In such a case the group G is said to be *spontaneously broken* to its subgroup H. The classical example is the Heisenberg ferromagnet, an array of spin-1/2 magnetic dipoles interacting only with their nearest neighbors. The equations of motion describing this system are invariant under the group of spatial rotations SO(3), the evolution does not depend on the original orientation of the sample. When the system is in its ground state all the spins are aligned in an arbitrary direction, and therefore the different orientations of the sample are physically distinguishable. However there is a set of rotations that can still act in the ferromagnet without producing any physical change: those which have the axis of rotation aligned with the dipoles. Thus the rotation group SO(3) is spontaneously broken by the ground state to its subgroup SO(2).

1.2.2 Spontaneous breaking of local symmetries, the Abelian-Higs model.

The case of the Heisenberg ferromagnet is an example of the spontaneous breaking of a global symmetry. Let us consider now what happens when the broken symmetry is gauged. A simple gauge theory that exhibits spontaneous symmetry breaking is the *Abelian-Higgs model.* This model is defined in $3+1$ dimensions, it contains a complex scalar field ϕ, the *Higgs*, coupled to a U(1) gauge field A_μ, i.e. electro-magnetism. The action is given by:

$$\mathcal{S} = \int d_4x \left[-D_\mu\phi D^\mu\bar{\phi} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right], \qquad V(\phi) = \lambda(|\phi|^2-\eta^2)^2, \quad (1.2.1)$$

where the covariant derivative is defined as $D_\mu\phi = \partial_\mu\phi - \mathrm{i}gA_\mu\phi$, and the field strength of the gauge boson by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The Euler-Lagrange equations corresponding to the action (1.2.1) are:

$$D_\mu D^\mu\phi - 2\lambda(|\phi|^2-\eta^2)\phi = 0 \quad (1.2.2)$$

$$\partial_\nu F^{\mu\nu} + \mathrm{i}g(\bar{\phi}D^\mu\phi - \phi D^\mu\bar{\phi}) = 0. \quad (1.2.3)$$

Note that the action and the equations of motions are invariant under the following local U(1) transformations:

$$\phi \to \mathrm{e}^{\mathrm{i}g\Lambda}\phi \qquad \bar{\phi} \to \mathrm{e}^{-\mathrm{i}g\Lambda}\bar{\phi} \qquad A_\mu \to A_\mu + \mathrm{i}\partial_\mu\Lambda, \quad (1.2.4)$$

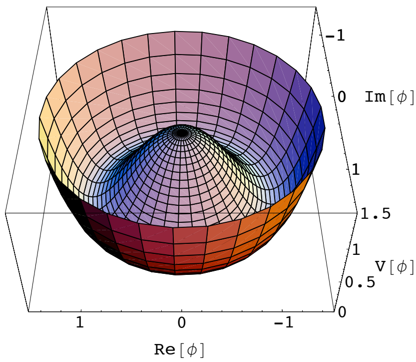

Figure 1.1 *– The scalar potential $V(\phi)$ in (1.2.1) with $\lambda = \eta = 1$.*

where $\Lambda(x^\mu)$ is a function of the point on space-time. The covariant derivative is defined so that it has the same transformation rule as the Higgs $D_\mu\phi \to \mathrm{e}^{ig\Lambda} D_\mu\phi$.

The energy functional of the Abelian-Higgs model is given by

$$E = \int dx_3 \left[|D_t\phi|^2 + |D_i\phi|^2 + \tfrac{1}{2}B^2 + \tfrac{1}{2}E^2 + \lambda(|\phi|^2 - \eta^2)^2\right] \geq 0, \qquad (1.2.5)$$

where the magnetic and electric field are defined by $B = \partial_1 A_2 - \partial_2 A_1$ and $E_i = \partial_t A_i - \partial_i A_t$ respectively. The ground state of the system is defined by a homogenous configuration with $A_\mu = 0$ and the Higgs fixed at one of the minima of the potential, $|\phi_0|^2 = \eta^2$, such that $V(\phi_0) = 0$. Note that this configuration is stable since it has zero energy and the energy functional is positive definite. The set of minima of the potential can be parametrized as $\phi_0 = \eta\,\mathrm{e}^{\mathrm{i}\alpha}$, thus they transform non-trivially under the U(1) symmetry:

$$\phi_0 = \eta\,\mathrm{e}^{\mathrm{i}\alpha} \to \eta\,\mathrm{e}^{\mathrm{i}(\alpha+g\Lambda)}. \qquad (1.2.6)$$

In other words, when the Higgs field is stabilized at any of these points, the gauge symmetry is spontaneously broken. The fluctuations around the background can be parametrized as:

$$\phi = (\eta + \rho)\,\mathrm{e}^{\mathrm{i}\alpha}, \qquad A_\mu = \tfrac{1}{g}\partial_\mu\alpha + a_\mu, \qquad (1.2.7)$$

where $\alpha(x^\mu)$ is now a space-time dependent. We can use the gauge freedom to make ϕ real everywhere[2] (unitary gauge), so that $\phi = \eta + \rho$. Using this parametrization, and after expanding the lagrangian to second order in the fluctuations around the vacuum, it reads:

$$\mathcal{L} = -\partial_\mu\rho\partial^\mu\rho - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu} - g^2\eta^2 a_\mu a^\mu + 4\lambda\eta^2\rho^2 + \mathcal{O}(3), \qquad (1.2.8)$$

[2] Actually, in the presence of topological defects this is not possible. We discuss topological defects in a later section.

where $f_{\mu\nu}$ is the field strength corresponding to the fluctuation of the gauge field a_μ. Note that the condensation of the Higgs field has led to the appearance of a mass term in the lagrangian for the gauge boson. Thus, in the spontaneously broken theory, the spectrum of fluctuations consists of a real scalar field and a gauge boson with masses given by $m_s = 2\sqrt{\lambda}\eta$, and $m_v = \sqrt{2}g\eta$ respectively. The process where the spontaneous breaking of a symmetry leads to the appearance of mass terms for the gauge boson is known as the *Higgs mechanism*.

The Abelian-Higgs model is the relativistic version of the Landau-Ginzburg model describing a superconductor. Here the complex scalar field is what would play the role of the condensate of Cooper pairs, and A_μ would be the usual vector potential from electro-magmetism. The fact that the vector boson becomes massive reflects in the inability of the magnetic field to penetrate the condensate, this is the so called *Meissner effect*.

The Higgs mechanism acts in a similar way in the case of Yang-Mills theories with a non-abelian gauge group, such as SU(N), $N \geq 2$. In that case only the gauge fields associated to the broken generators of the symmetry group acquire a mass, while the ones associated to the unbroken subgroup remain massless.

1.2.3 The Standard Model and Grand Unification Theories.

The three fundamental forces of the Standard Model can be described by a Yang-Mills theory based on the gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. In particular the strong interaction is described by the SU(3) factor, and the electro-magnetic and weak interactions are given in terms of the unified theory $\mathrm{SU}(2) \times \mathrm{U}(1)$, the Weinberg-Salam model.

In spite of being formulated as a unified theory, the electromagnetic and weak interactions have very different properties at low energies. For instance, while the electromagnetic interaction is mediated by a massless gauge boson, the photon, the mediators of the weak force (the W and Z bosons) are massive, and thus the interaction is short range. Therefore, at low energies, the symmetry group $\mathrm{SU}(2) \times \mathrm{U}(1)$ has to be broken to $\mathrm{U}_{em}(1)$, the gauge group of the electromagnetic interactions. The Standard Model contains a spin-0 field, the Higgs, which transforms non-trivally under the electroweak symmetry group. At low energies the Higgs is stabilized at a configuration that minimizes its energy functional, and which breaks the electroweak symmetry group to its subgroup $\mathrm{U}_{em}(1)$.

After t'Hooft and Veltman [15] proved the renormalizability of the Weinberg-Salam model in the early 1970's, the idea of unifying all fundamental forces under a single symmetry principle became one of the main quests in theoretical physics. The basic idea behind Grand Unification Theories is that the Standard Model can be formulated as the broken phase of a theory with a larger symmetry group. Before the phase transition that led to the breaking of the unified group there would be no difference between the fundamental interactions. Only after the transition some of the gauge bosons become massive, and the corresponding interactions short range, destroying the symmetry between the various forces.

The Standard Model involves three coupling constants, g_3, g_2 and g, one for each of the factors of its gauge symmetry group, SU(3), SU(2) and U(1) respectively. These couplings depend logarithmically on the energy. For instance, while g_3 and g_2 decrease for growing energies, g becomes larger. Grand Unified Theories based on simple groups, such as SU(5), involve a single coupling constant. The energy scale where the GUT phase transition takes place can be estimated requiring the three couplings of the Standard Model to be roughly equal. Due to the logarithmic dependence of gauge couplings on energy the obtained symmetry breaking scale (*GUT scale*) is extremely high: $10^{15} - 10^{16}$ GeV.

1.2.4 Supersymmetry and supergravity.

When trying to embed the Standard Model into a Grand Unified Theory we have to confront an important conceptual puzzle, the so called *hierarchy problem.* The hierarchy problem consists of explaining the smallness of the Higgs mass, as compared with the GUT scale.

In the Standard Model the Higgs mass has an upper bound implied by the unitarity of the theory[3], $M_H < 10^3$ GeV [17]. However, the mass terms of scalar fields generically receive renormalization corrections which are quadratically divergent $\delta M_H^2 \sim g^2 \Lambda^2$, with g a gauge coupling, and Λ a cutoff scale. When the standard model is embedded in a Grand Unified Theory, the appropriate value for Λ should be of order of the GUT scale or larger $\Lambda \geq 10^{15}$ GeV. Therefore, for the Higgs to have a mass of order of the electroweak scale, the bare mass squared should be of order of $-\Lambda^2$, and cancel the radiative correction with extreme accuracy.

This argument implies that it is not possible to have a large hierarchy of spontaneous symmetry breaking scales without introducing a severe fine tuning of the parameters of the theory. An alternative, more natural, solution to fine tuning would be to invoke a symmetry that protects the Higgs mass from receiving corrections at all orders in perturbation theory. A prominent proposal

[3]There are studies which suggest stronger bounds on the Higgs mass, $M_H < 225$ GeV [13, 16]. At the time of writing, the Higgs has not been found yet by the LHC but the experiments have bounded its mass to be in the range 116-130 GeV, and the are hints suggesting that the most likely value is around 126 GeV.

for such a symmetry is *supersymmetry* (see [18]).

Supersymmetry is a symmetry that transforms particles of different spin into each other, in particular bosons into fermions and vice versa. In theories invariant under supersymmetry every boson must have a fermionic partner with equal mass. Moreover the mass terms of scalar fields are no longer quadratically divergent, since the contribution from fermions and bosons to the radiative corrections have the same magnitude but opposite sign, leading to an exact cancellation.

The particles in supersymmetric theories are arranged in *supermultiplets*, which are the irreducible representations of the supersymmetry algebra. Each supermultiplet always contains the same number of bosonic and fermionic degrees of freedom, and moreover, all the particles within a supermultiplet have the same mass. The field content of each supermultiplet depends on the number of supersymmetry generators, which is denoted by $\mathcal{N}$. The theories invariant under the action of more than one supersymmetry generator $\mathcal{N} \geq 2$ are called *extended supersymmetry theories*.

The known elementary particles cannot be fitted in supermultiplets, thus supersymmetry cannot be an exact symmetry of nature. An appealing idea is to consider that supersymmetry is only spontaneously broken at low energies. As long as the supersymmetry breaking scale is around 1 TeV, supersymmetry still provides a solution for the hierarchy problem. Interestingly, an analysis of the running of the gauge couplings seems to favor this possibility. In the simplest cases, when the GUT symmetry group breaks down to the Standard Model via a single phase transition, the couplings do not quite meet. However, in the supersymmetric extension of these models, the matching of the gauge couplings at the GUT scale is substantially improved.

Although no evidence of supersymmetry has been found so far in the Large Hadron Collider (LHC), nowadays some of the most prominent candidates to explain the physics above the electroweak scale are supersymmetric theories, such as the Minimal Supersymmetric Standard Model (MSSM), supersymmetric GUTs, and superstrings. These theories provide a framework to investigate important cosmological problems such as the origin of inflation, dark matter or the present day accelerated expansion.

The physical phenomena that occurred during the first stages of the universe involve enormous energies, in particular in the case of inflation they might be as high as 10^{15} GeV. This energy scale is rather close to the Planck scale $M_p \sim 10^{18}$ GeV, where the strength of gravitational interactions becomes comparable to the other three fundamental forces. This suggests that at these energy scales gravity should be treated on an equal footing with the rest of the interactions,

$\mathcal{N}=1$ **Supersymmetry**	$\mathcal{N}=2$ **Supersymmetry**
Chiral multiplet $(0,\frac{1}{2})$	Hypermultiplet $(0,\frac{1}{2},\frac{1}{2},0)$
Vector multiplet $(\frac{1}{2},1)$	Vector multiplet $(0,\frac{1}{2},\frac{1}{2},1)$
Gravity multiplet $(\frac{3}{2},2)$	Gravity multiplet $(1,\frac{3}{2},\frac{3}{2},2)$

Table 1.1 *– Massless supermultiplets of globally and locally supersymmetric models with $\mathcal{N}=1$ and $\mathcal{N}=2$ in 4 space-time dimensions. We denote the supermultiplets by the helicity s of the particles they involve: $(s_0, s_0+\frac{1}{2})$ for $\mathcal{N}=1$, and $(s_0, s_0+\frac{1}{2}, s_0+\frac{1}{2}, s_0+1)$ for $\mathcal{N}=2$ supersymmetry.*

and in particular should be treated as a quantum theory. Similarly to the other three interactions, gravity can be described as a gauge theory, where the symmetries that are gauged are space-time symmetries. The corresponding gauge group is the Poincaré group, which involves space-time translations, rotations and boosts. In contrast with gauged internal symmetries, the gauge boson is a spin-2 field, the *graviton*, which represents the gravitational field itself.

If all four fundamental interactions are to be treated in the same way, then in supersymmetric theories we should require the gravitational interaction itself to be invariant under supersymmetry. Such theories are called *supergravity theories* (see [18]). Supergravity is the *local version of supersymmetry*, and is obtained by allowing the supersymmetry transformations to vary from point to point in space-time. Actually, supersymmetry is intermediate between internal and space-time symmetries, and any theory invariant under local supersymmetry also contains gravity. Indeed, the supersymmetry algebra contains the Poincaré algebra, and therefore by promoting global supersymmetry to local supersymmetry we are also gauging the Poincaré group. The gauge particle associated to local supersymmetry is the *gravitino*, the spin 3/2 partner of the graviton. In supergravity theories the number of gravitinos is equal to the number of the supersymmetry generators, $\mathcal{N}$. The different types of massless supermultiplets of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry are summarized in table 1.1.

Supergravity plays an important role in the construction of cosmological models, since it describes the low energy regime of the most prominent theory of quantum gravity to date, *String Theory*.

1.2.5 String theory and the integration of heavy moduli.

In contrast with conventional quantum field theory, where the elementary particles are mathematical points, the fundamental entities in string theory are one-dimensional extended objects, namely *the strings* (see [19]). The basic idea is that the elementary particles of the Standard Model would arise as oscillation modes of these fundamental strings. An appealing feature of string theory is that it involves a single fundamental parameter, the characteristic length of the strings l_s, or the corresponding energy scale $E_s = 1/l_s$. Any other quantity, such as the strength of the interactions between strings, is field dependent.

Another remarkable property of superstring theories is that they predict the number of space-time dimensions. Indeed, for these theories to be consistent, the fundamental strings must live in a $10-$ or $11-$dimensional space-time. So far there is no experimental evidence of extra dimensions, thus in order for string theory to describe the $4-$dimensional world we experience, we must invoke a mechanism to hide the 6 or 7 extra dimensions. The most accepted solution is that the extra dimensions are *compactified* on some internal manifold, so small that it remains invisible even at the largest energy scales, (the shortest length scales), tested with current accelerators.

Simple compactifications can be characterized by a single length scale[4], l_c, or its associated energy scale $E_c = 1/l_c$. Many results in string theory have been derived in the limit $E_c \ll E_s$, because, in this regime, the physics at the energy scales $E \ll E_s$ admit an effective description in terms of 10 or 11 dimensional supergravity. However, given the absence of evidence of extra dimensions, most of the proposed cosmological models assume the extra condition $E \ll E_c$, so that they can be described in the framework of 4-dimensional supergravity.

A common problem that has to be treated in all cosmological models derived from string compactifications is the presence of a large number of massless scalar fields, for which so far there is no observational evidence. In particular, this set of fields involves the *moduli fields*, which characterize the size and shape

[4] Such a characteristic length scale might be related, for example, to the volume of the internal manifold $\mathcal{V}_n \sim l_c^n$.

of the extra dimensions. The mechanisms to generate a potential that can stabilize the moduli have only been developed recently [20, 21]. This requires compactifying string theory in a new type of backgrounds, *flux compactifications*, which require the presence of D-branes[5] wrapping the internal dimensions, and certain generalizations of the magnetic flux sourced be the branes.

In flux compactifications some of the moduli are stabilized at a high energy scale and decouple from the low energy effective theory. On the other hand, as we have mentioned before, it is phenomenologically appealing that supersymmetry remains unbroken at scales as low as 1 TeV, therefore the stabilization of these heavy fields has to leave supersymmetry unbroken. However, integrating out heavy scalars in such a way that the effective theory is still invariant under supersymmetry is not a trivial problem, the couplings between the light and heavy sectors have to satisfy certain conditions. Thus, it is important to characterize the type of couplings between the two sectors that allow for the supersymmetric integration of the heavy fields. This problem will be discussed in detail in chapter 3.

There are two situations where details of the decoupling of heavy moduli become important. The first one is in inflationary models derived from supergravity or superstrings with moduli stabilization. In these models the inflaton belongs to the light sector of the theory, and the slow roll conditions can be easily spoilt by its couplings to the heavy fields [22]. Thus the predictions of these models can only be trusted after having a precise characterization of the interactions between the inflationary and heavy sectors [23] (see also [24]). The second situation is supersymmetry breaking. Since supersymmetry is not an exact symmetry of nature, any realistic cosmological model should involve the mechanism for supersymmetry breaking. The breaking of supersymmetry in the light sector will affect the stability properties of the heavy fields that were integrated out. In particular, if the heavy fields become unstable due to the supersymmetry breaking effects, the integration of heavy moduli no longer makes sense. This is especially relevant in string theory models describing late time accelerated expansion of the universe. We will come back to this problem in chapter 4.

1.3 Cosmic strings and other topological defects.

We continue this introductory chapter with a discussion about the formation of topological defects in the early universe and their cosmological implications, paying particular attention to cosmic strings.

[5] A D-brane is a hypersurface where open strings can end, which can be extended along several of the ten or eleven space-time dimensions.

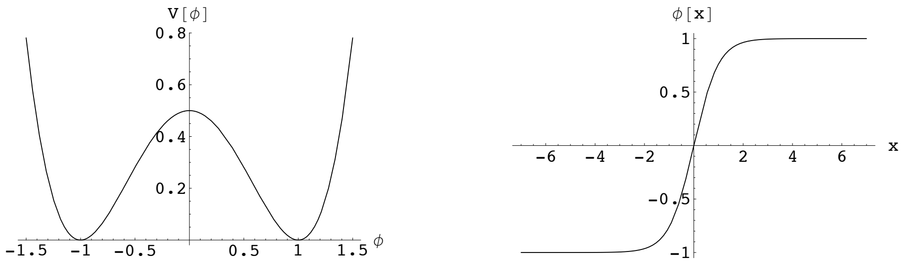

Figure 1.2 *– The scalar potential $V(\phi)$ in (1.3.2) (left) with $\lambda = 2$, $\eta = 1$ leads to the kink solution (1.3.4) $\phi(x)$ on the right.*

As we have mentioned in the previous sections, phase transitions are a fundamental ingredient of many interesting cosmological scenarios, such as those based on Grand Unified Theories (GUT). An important example are *hybrid inflationary* models, where the exponential expansion ends in a natural way as the inflaton reaches a point of the potential with a tachyonic instability leading to a phase transition. Like in the more familiar condensed matter systems, phase transitions in the early universe also lead to the formation of localized, non-dissipative objects called *topological defects* (see [25, 26]). In particular, it has been proven that cosmological models based on Supersymmetric Grand Unified Theories generically predict the formation of cosmic strings at the end of inflation [27].

In this section we will introduce topological defects without considering the gravitational effects. In a theory without gravity, where the energy density is given by the T_0^0 component of the energy-momentum tensor (see [25]), the energy is bounded below $T_0^0 \geq 0$ and the ground states satisfy $T_0^0 = 0$. We understand by *dissipative solutions* of the classical equations of motion those satisfying:

$$\lim_{t\to\infty} T_0^0(x^i, t) = 0, \tag{1.3.1}$$

at every point of space x^i.

The simplest example of a topological defect is the kink in $1+1$ dimensions. It is possible to construct a model that exhibits kink solutions with a single real scalar field ϕ. The corresponding action is given by:

$$\mathcal{S} = \int dtdx \left[-\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right], \tag{1.3.2}$$

where the potential is given by $V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$. The energy functional for this action reads:

$$E = \int dx \left[\tfrac{1}{2}(\partial_t\phi)^2 + \tfrac{1}{2}(\partial_x\phi)^2 + \tfrac{1}{4}\lambda(\phi^2 - \eta^2)^2\right], \tag{1.3.3}$$

In order for the solution to have a finite energy, the field configuration ϕ describing the kink must satisfy $\phi^2 = \eta^2$ at $x \to \pm\infty$, it must be in one of the ground states of the potential $V(\phi_0) = 0$. The set of ground states of a given theory is called the *vacuum manifold* $\mathcal{V}$, and in this case it is given by the homogeneous configurations $\phi_0 = \pm\eta$. Suppose now that we choose the boundary conditions such that $\phi(-\infty) = -\eta$ and $\phi(\infty) = \eta$, then by continuity this field configuration must necessarily have a zero for some value $x = x_0 \in \mathbb{R}$. This configuration has a nonzero energy, which is expected to be concentrated around $x = x_0$, where the scalar field is out of the vacuum. The kink solution to the classical equations of motion is given by:

$$\phi(x) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}}\,\eta\,(x - x_0)\right), \tag{1.3.4}$$

which is plotted in figure 1.2. This field configuration is non-dissipative in the sense that no classical time evolution can make this object decay into the vacuum $\phi = \pm\eta$. The reason is simple: for the energy to be finite, the value of ϕ at spatial infinity must remain in the vacuum manifold at all times. Since the vacuum manifold consists of two disconnected pieces, and time evolution is continuous, it is not possible for the scalar field to evolve to a constant field configuration $\phi = \pm\eta$.

This model provides an illustrative example of spontaneous breaking of a symmetry. Its lagrangian has a $\mathbb{Z}_2$ symmetry that exchanges ϕ and $-\phi$. When the Higgs condenses and chooses one of the vacua $\phi_0 = \pm\eta$, the $\mathbb{Z}_2$ symmetry, which transforms the two vacua into each other, is spontaneously broken. In general, the vacuum manifold of a theory has a close relation with its underlying symmetry group G. Actually, since the scalar potential is invariant under the symmetry group, if ϕ_0 is a zero of the potential so is $g\phi_0$ for any transformation $g \in G$. Moreover, in the absence of accidental degeneracies, the vacuum manifold $\mathcal{V}$ can be identified with the coset space G/H, where H is the unbroken subgroup. In the present case $G = \mathbb{Z}_2$ is completely broken ($H = \{\mathbb{1}\}$), and therefore the vacuum manifold is isomorphic to the symmetry group itself $\mathcal{V} \cong \mathbb{Z}_2$. This discussion suggests that in the case of spontaneously broken GUT theories the vacuum manifold can be highly non-trivial.

In the following two sections we will review the basic features of vortex solutions in non-supersymmetric theories. We will discuss two field theory models: the global U(1) model, and the Abelian-Higgs model. For a review see [26]. We will finish this chapter commenting shortly on supersymmetric vortices, leaving the main discussion for the next chapter, after we have we have introduced $\mathcal{N} = 1$ supergravity.

1.3.1 Global vortices.

The global U(1) model is the simplest field theory that exhibits cosmic string solutions. It describes the dynamics of a single complex scalar field ϕ evolving in a scalar potential with a mexican hat form, fig. 1.3. The action reads:

$$\mathcal{S} = \int dx_4 \left[-\partial_\mu \phi \partial^\mu \bar{\phi} - V(\phi)\right], \qquad V = \lambda(|\phi|^2 - \eta^2)^2, \tag{1.3.5}$$

and its corresponding equation of motion is given by:

$$[\partial_\mu \partial^\mu - 2\lambda(|\phi|^2 - \eta^2)]\phi = 0. \tag{1.3.6}$$

This model is invariant under a global U(1) symmetry defined by the transformations:

$$\phi \to \phi\, e^{\mathrm{i}g\Lambda}, \tag{1.3.7}$$

where Λ is a real constant. The energy functional associated to our action is:

$$E = \int dx_3 \left[|\partial_t \phi|^2 + |\partial_i \bar{\phi}|^2 + \lambda(|\phi|^2 - \eta^2)^2\right]. \tag{1.3.8}$$

As in the Abelian-Higgs model, this theory has a stable set of vacua given by all the configurations satisfying $|\phi| = \eta^2$, and therefore the vacuum manifold is isomorphic to a circle:

$$\mathcal{V} = \{\phi \in \mathbb{C} \;/\; |\phi|^2 - \eta^2 = 0\} \cong S^1. \tag{1.3.9}$$

Since these vacua are not invariant under the U(1) transformations (1.2.6), the global symmetry is said to be spontaneously broken down to the trivial group $\{\mathbb{1}\}$. The mass spectrum around the vacuum consists of two types of particles, one particle has a mass $m_s = \sqrt{2\lambda}\eta$, and the other is massless, the *goldstone boson*. The original U(1) symmetry of the action prevents the goldstone boson from having a mass, since this particle corresponds to the space dependent fluctuations of the phase of the scalar field.

This model admits cosmic string solutions, which are also known as *global vortices*. We will now discuss static cylindrically symmetric cosmic solutions laying along the z-axes. An appropriate ansatz for this configuration is given by:

$$\phi = \eta f(r) e^{in\theta}, \tag{1.3.10}$$

where we have used cylindrical coordinates $\{r, \theta, z\}$. In order for the string configuration to have finite energy, the scalar field ϕ must approach the vacuum manifold asymptotically at spatial infinity, and thus the profile function $f(r)$ must satisfy the boundary condition $f(r \to \infty) = 1$. Therefore at spatial infinity the field configuration is

$$\phi = e^{in\theta}, \tag{1.3.11}$$

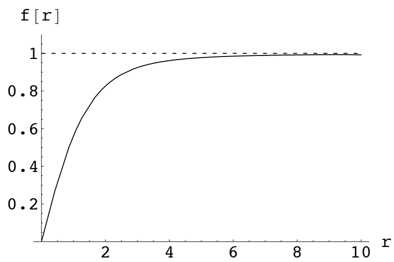

Figure 1.3 – *Profile function (1.3.10) $f(r) = |\phi(r)|$ characterizing the field configuration of a global string for $n = \lambda = \eta = 1$.*

which represents a mapping between the the circle at infinity in real space and a circle in field space, the vacuum manifold. The parameter n, called the *winding number*, is an integer which counts the number of times the phase of the scalar field winds around the vacuum manifold when we go once around the circle in field space.

Since the field ϕ has to be single valued we have to require the profile function $f(r)$ to vanish at the origin $r = 0$, thus ϕ leaves the vacuum manifold which implies that the field configuration has nonzero energy. Following the same argument we used in the case of the kink, we can see that there is no continuous way to deform the configuration 1.3.11 into a homogeneous solution of the form $\phi = e^{i\alpha}$, and in particular, since time evolution is continuous, the cosmic string solution cannot decay into the vacuum.

Introducing the ansatz (1.3.10) into the equation of motion we obtain a second order ordinary differential equation for the profile function $f(r)$:

$$f'' + \tfrac{1}{r^2} f' - \tfrac{n^2}{r^2} f + 2\lambda\eta^2(1 - f^2)f = 0. \qquad (1.3.12)$$

The numerical solution of this equation is displayed in fig 2. The profile function $f(r)$ has the following asymptotic behavior:

$$f(r) \approx C_n\, r^n \quad \text{for} \quad r \to 0, \qquad (1.3.13)$$

$$f(r) \approx 1 - \frac{n^2}{4\lambda\eta^2\, r^2} \quad \text{for} \quad r \to \infty. \qquad (1.3.14)$$

This indicates that the string core has a width of the order of the Compton wavelength of the massive scalar $\delta \sim 1/(\sqrt{2\lambda}\eta) = m_s^{-1}$; for r equal to a few δ the Higgs is approximately in the vacuum manifold $|\phi| \approx \eta$ (see Fig. 1.3).

Strings of this type have an infinite energy per unit length. This can be checked inserting the asymptotic value of the Higgs, $\phi(r \to \infty) = \eta\, e^{in\theta}$, in the energy functional (1.3.8). Including a small contribution from the core we obtain the following energy per unit length μ:

$$\mu \approx \eta^2 + 2\pi n^2\eta^2 \int_\delta^R \frac{dr}{r} \approx 2\pi n^2\eta^2 \log R/\delta, \tag{1.3.15}$$

which is logarithmically divergent for large values of the cutoff R. In a cosmological context this cutoff would be given by the distance to the nearest string with the opposite winding number. A characteristic property of these strings is that they interact with long range forces, which decay as R^{-1} with the distance R between them. Actually, due to the repulsion between the strings, those with winding number $|n| > 1$ are unstable to decay into single $|n| = 1$ strings, which are completely stable [28].

1.3.2 Abrikosov-Nielsen-Olesen vortex.

Let us discuss again the Abelian-Higgs model presented in section 1.2.2. In this case the internal U(1) symmetry has been promoted to a local one. In our previous discussion of the Abelian-Higgs model we mentioned that the ground states of the system, up to gauge transformations, were given by the configurations satisfying $A_\mu = 0$ and $\phi = \eta e^{i\alpha}$, with α some real constant. Thus, as in the global U(1) model, the vacuum manifold is isomorphic to S^1, implying the existence of cosmic string solutions.

The cosmic strings in the Abelian-Higgs model have different properties from those in the global U(1) model. In particular it is possible to find solutions with finite energy per unit length. For the energy to be finite we have to require, as in the case of global vortices, the Higgs to lay in the vacuum manifold at spatial infinity:

$$\lim_{|\vec{x}|\to\infty} |\phi|^2 = \eta^2. \tag{1.3.16}$$

Moreover, the covariant derivatives $D_\mu\phi$ must vanish asymptotically. In order to achieve this the gauge fields have to satisfy the condition:

$$\lim_{|\vec{x}|\to\infty} A_\mu = \frac{1}{ig}\partial_\mu \log\phi, \tag{1.3.17}$$

which also implies that the field strength $F_{\mu\nu}$ vanishes at spatial infinity, since $[D_\mu, D_\nu]\phi = -igF_{\mu\nu}\phi$.

As in the previous section we now consider static cylindrically symmetric cosmic string solutions along the z-axis. We will use in cylindrical coordinates $\{r, \theta, z\}$, and we will impose the gauge $A_0 = A_r = 0$. A cosmic string with a winding number n can be described by the ansatz:

$$\phi = \eta f(r) e^{in\theta} \qquad A_\theta = \tfrac{n}{g} v(r). \tag{1.3.18}$$

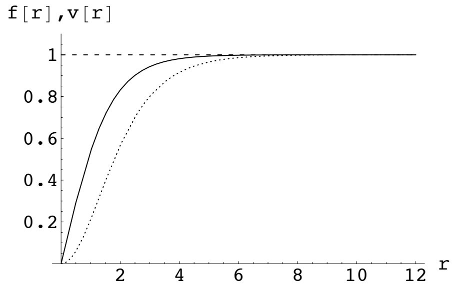

Figure 1.4 *– Profile functions (1.3.18) $f(r)$ (solid line) and $v(r)$ (dotted line) characterizing the field configuration of a local string saturating the BPS bound, $2\lambda = g^2$, with $n = g = 2\eta = 1$.*

where $f(r)$ and $v(r)$ are real functions satisfying the following boundary conditions:

$$f(r), v(r) \to 1 \quad \text{for} \quad r \to \infty, \qquad \text{and} \qquad f(r), v(r) \to 0 \quad \text{for} \quad r \to 0. \quad (1.3.19)$$

Introducing this ansatz in the equations of motion (1.2.3) we find the equations for the profile functions $f(r)$ and $v(r)$:

$$\begin{aligned} f''(r) &+ \tfrac{1}{r}f'(r) - \tfrac{n^2}{r^2}[1-v(r)]^2 + 2\lambda\eta^2(1-f(r)^2)f(r) = 0, & (1.3.20)\\ v''(r) &- \tfrac{1}{r}v'(r) + g^2\eta^2[1-v(r)] = 0. & (1.3.21)\end{aligned}$$

This type of vortices have a tube of magnetic flux in their core which is quantized by the winding number n. Integrating the magnetic flux over the whole plane perpendicular to the string gives:

$$\Phi_m = \int_{R^2} dx^2 B = \int_{S^1_\infty} d\vec{l}\vec{A} = \frac{2\pi n}{g}. \quad (1.3.22)$$

Here we have applied Stokes' theorem with S^1_∞ being the circle at spatial infinity containing the string, and used the the asymptotic behavior of the gauge field and the Higgs. The energy per unit length of these strings is finite and has a dependence on the parameters of the form [29]:

$$\mu = \pi\eta^2\epsilon(2\lambda/g^2). \quad (1.3.23)$$

This result can be obtained analytically for the particular choice of couplings $2\lambda = g$, which is called the *BPS limit* (Bogomonlyi-Prasad-Sommerfield), borrowing the name from magnetic monopoles [30], [31]. The energy per unit length for a static cosmic string parallel to the z-axis can be written in the following way:

$$\mu = \int dx^2 \left[|(D_1 \pm \mathrm{i}D_2)\phi|^2 + \tfrac{1}{2}[B \pm g(|\phi|^2 - \eta^2)]^2 + \tfrac{1}{2}(2\lambda - g^2)(|\phi|^2 - \eta^2)^2\right]$$
$$\pm g\eta^2 \int d_2xB \geq 0. \quad (1.3.24)$$

This expression can be obtained from (1.2.5), using the identities:

$$|(D_1 \pm \mathrm{i}D_2)\phi|^2 = |\vec{D}\phi|^2 \mp \mathrm{i}\bar{\phi}[D_1, D_2]\phi \pm \vec{\nabla} \wedge \vec{J}, \quad (1.3.25)$$
$$[D_1, D_2]\,\phi = -\mathrm{i}gB\phi, \quad (1.3.26)$$

and discarding the boundary term associated to the curl of the current $\vec{J} = \mathrm{i}\bar{\phi}\,\vec{\nabla}\phi$. Note that in the limit $2\lambda = g^2$ the first line in (1.3.24) is a sum of positive definite terms, and thus the energy per unit length μ is bounded below[6]:

$$\mu \geq \pm g\eta^2 \int d_2xB = 2\pi\eta^2|n|. \quad (1.3.27)$$

Any field configuration saturating this bound must also be a solution to the static equations of motion, since it extremizes the energy functional. In order to saturate the bound the cosmic string background must satisfy the following first order differential equations:

$$(D_1 \pm \mathrm{i}D_2)\phi = 0, \qquad B \pm g(|\phi|^2 - \eta^2) = 0, \quad (1.3.28)$$

which are know as the BPS equations. Inserting the ansatz for a cylindrically symmetric straight string (1.3.18) we obtain the following system of equations for the profile functions:

$$f'(r) + \tfrac{|n|}{r}(v(r) - 1)f(r) = 0 \qquad |n|v'(r) + g^2\eta^2 r(f(r)^2 - 1) = 0. \quad (1.3.29)$$

These equations only admit solutions with the correct boundary conditions (1.3.19) provided we choose the signs so that $n = \pm|n|$. The functions $f(r)$ and $v(r)$ which solve this set of equations are represented in figure 1.4. These cosmic strings have an energy per unit length $\mu = \pi\eta^2|n|$, which satisfies the expression (1.3.23) with $\epsilon = 1$.

For generic values of the couplings, λ and g, parallel cosmic strings interact with short-range forces. For instance, if $2\lambda > g^2$ the cosmic strings repel, and if $2\lambda < g^2$ they attract [30]. When the Abelian-Higgs model satisfies the BPS limit $2\lambda = g^2$, it is possible to find static multivortex solutions of the equations of motion [32]. In consequence, string configurations with windings $|n| > 1$ are only stable provided that $2\lambda \leq g^2$, otherwise the strings decay into n stable vortices with a unit of magnetic flux [28]. The vortex solutions of the AH model satisfying the BPS limit are especially interesting because their low energy dynamics can be studied with high accuracy using the moduli space approximation [33, 34, 35].

[6] In order for the energy per unit length to be positive definite, as required by (1.2.5), we must choose the signs so that $n = \pm|n|$.

1.4 Topological defects in cosmology.

1.4.1 Defect formation.

The process of formation of topological defects in cosmology in known as the *Kibble mechanism* [36] (for a review see [26]). As the universe undergoes a phase transition, in general the Higgs will condense into different vacua at different points in space time. For example in the model we just discussed, as the two vacua $\phi = \pm\eta$ are completely equivalent from the point of view of the equations of motion, the Higgs could condense into any of them with equal probability. This leads to the formation of separate domains, or regions of the universe where the Higgs has the same value. As the universe cools down, these domains expand and eventually coalesce. In the boundary of the domains the scalar field interpolates between different vacua, as occurs in the previous example. These field configurations interpolating between different vacua at spatial infinity are the defects. The formation of these domains is unavoidable, because the vacuum where the Higgs condenses can only be correlated over finite distances smaller than the size of the horizon (see section 1.1), and therefore it will be different at points of space-time which are not in causal contact.

The type of topological defects that are formed during a phase transition is determined by the topology of the vacuum manifold. The value of the Higgs in a m-sphere S^m at spatial infinity defines a continuous map h between S^m and $\mathcal{V}$:

$$h : S^m \longrightarrow \mathcal{V}. \tag{1.4.1}$$

Since the Higgs has to remain in the vacuum manifold at spatial infinity at all times, and time evolution is a continuous operation, then it is clear that the existence of stable defects can be determined studying the set of continuous deformations of the map h. In particular, non-dissipative solutions are associated with non-contractible m-spheres in the vacuum manifold. Saying that the image of the the map h cannot be contracted to a point, is the same as stating that the field cannot evolve to a homogeneous configuration. Note that if the field configuration "wraps around" one of these non-contractible m-spheres of the vacuum manifold, then continuity implies that somewhere in real space the Higgs must leave the vacuum manifold restoring the old (symmetric) phase. As in the case of the kink, the energy density is expected to peak in the region of space where the Higgs leaves the vacuum. These concentrations of energy signal the positions of the defects.

The non-contractible m-spheres of the vacuum manifold are classified by the elements of the m-th homotopy group[7] $\pi_m(\mathcal{V})$. In particular, if the vacuum manifold is disconnected, $\pi_0(\mathcal{V}) \neq \mathbb{1}$, sheet-like structures are formed: topological *domain walls*. When the first homotopy group of the vacuum manifold is

[7] The set $\pi_m(\mathcal{V})$ has a group structure only for $m \geq 1$.

non-trivial then one-dimensional defects form during the transition: topological *cosmic strings*. If $\pi_2(\mathcal{V})$ is non trivial then the phase transition will lead to the formation of point defects, topological *monopoles*.

For example, in the case of vortices (both global and local), the gauge group $G = \mathrm{U}(1)$ is completely broken by the vacuum, i.e. the unbroken subgroup $H = \mathbb{1}$. Therefore the vacuum manifold has a non-trivial first homotopy group $\pi_1(\mathrm{U}(1)/\mathbb{1}) = \pi_1(S^1) = \mathbb{Z}$, implying that the action should admit cosmic string solutions labeled by an integer $n \in \mathbb{Z}$, which is precisely the winding number of the string.

1.4.2 Cosmological implications.

The formation of defects has many cosmological implications. In particular, in four dimensions, they contribute to the energy density of the universe, affecting its geometry and the evolution of the scale factor. They might source density perturbations in the CMB in addition to those seeded by inflation, and they can have an impact in the amount of observed dark matter. The predictions of cosmological models related to the formation of defects, when contrasted with astronomical data, can help to constrain their parameter space, or even rule them out completely.

Domain walls and magnetic monopoles are very constrained from the observations. Let's consider first the case of a domain wall network. Although the expansion of the universe tends to dilute the density of the network, at the same time these sheet-like defects are stretched. Since their surface energy density remains constant, the energy density of the network evolves with the scale factor as $\rho_{dw} \sim a^{-1}$ rapidly dominating any other contribution, which is in contradiction with the observations (1.1.6). The production of magnetic monopoles during the GUT transition leads to a similar problem. According to these theories magnetic monopoles would be created with high abundances and, moreover, since they are extremely massive, $m_{monop} \sim 10^{16}$ GeV, they would behave as non-relativistic matter $\rho_{monop} \sim a^{-3}$. Although, in principle, monopoles and anti-monopoles can annihilate, the process is not effective enough to reduce their abundance significantly. Therefore in a standard Big-Bang cosmological scenario their formation would lead to an overclosed universe, and would also disrupt the Big-Bang nucleosynthesis. As we saw in section 1.1, this discrepancy with the astronomical data can be cured invoking a period of inflation after the GUT transition. After inflation monopoles become extremely rare, with only one per Hubble volume, and do not have significant cosmological implications.

Local cosmic strings do not suffer from these problems. The main reason is that the string network has a very efficient energy loss mechanism, the production of small loops[8]. The numerical simulations of cosmic string formation show that generically the networks consist of a small number of infinite strings (~ 10) crossing the Horizon, together with a scale invariant distribution of small loops. These loops are continuously formed during the self-interesctions of the long strings. Due to the tension of the string, the loops oscillate and shrink emitting gravitational waves. The network has an additional energy loss mechanism, namely the emission of massive radiation. Full field theory simulations indicate that the later one is the main energy loss mechanism [37, 38], while Nambu-Goto simulations seem to favor loop production [39]. According to a recent work by Blanco-Pillado *et al.* [40] the origin of the discrepancies arises because most loops form in scales too large to fit in the simulation box[9] of field theory calculations, while Nambu-Goto simulations do not describe the emission of massive radiation.

Eventually, no matter which is the main energy mechanism, the network reaches a scaling regime, where all its relevant length scales grow in proportion with the size of the Horizon. It follows that during the scaling regime the energy density of the network evolves as $\rho_{string} \sim \mu t^{-2}$, with μ the string tension, similarly to the critical density $\rho_c \sim G^{-1}t^{-2}$. Therefore their quotient remains a constant, which for GUT strings is of the order of $\Omega_{string} \sim G\mu \sim 10^{-6}$, and does not contradict the observations.

1.5 Cosmic Strings.

The formation of a cosmic string network after inflation is a common prediction of many promising cosmological models (see [41, 42, 43]). In particular it was shown in [27] that all supersymmetric GUT scenarios below a certain complexity, (based in gauge groups with rank $r \leq 8$), lead to the formation of cosmic strings. More recently, in the context of superstrings, string networks have been proven to appear at the end of brane inflation[10], which is a particular realization of hybrid inflationary models in the context of superstrings.

Cosmic strings were originally proposed in the 80's as a possible candidate to explain the primordial perturbations observed in the CMB. Nowadays inflation, due to its remarkable prediction of the CMB spectrum, has been established as the main mechanism to seed the perturbations, but the CMB data is still compatible with a 4.8% contribution from cosmic strings[11] [44].

[8] Global string networks lose energy in a very efficient way too. Due to the coupling to the goldstone boson the strings can radiate their energy into massless particles.

[9] The size of the simulation box is limited by the computational resources available and the complexity of the problem.

[10] In brane inflationary models the brane positions along the compact directions play the role of the inflaton.

[11] This bound was obtained in [44] using field theory simulations of the string network. Other works which use the Nambu-Goto approximation to simulate the network suggest a slightly

Cosmic strings could be detected in several ways. Despite being a subdominant component of the temperature power spectrum, it might be the main contribution to certain modes (*B-modes*) of its polarization power spectrum [46, 47, 48]. Since the energy of the strings eventually is radiated in form of gravitational waves, they also produce a gravitational wave background. This signature could be detected by measurements of pulsar timing [49], or directly with the LIGO and LISA gravitational wave detectors.

It is also possible for a cosmic string to be detected directly. The space-time surrounding an infinite straight string is deformed due to its tension, μ. Although the space-time background of the string is asymptotically flat, it has the topology of a cone. Physically this means that a circle of radius R centered in the string has a length $L = (2\pi - \Delta)R$, which is shorter than in a euclidean background $2\pi R$. The quantity $\Delta = 8\pi G\mu > 0$, known as the *deficit angle*, only depends on the tension of the string. As a result of the conical geometry around it, the cosmic strings act as a gravitational lens producing double images of the objets behind them, such as a galaxies [26]. Due to this lensing effect, strings moving respect to the CMB background would also produce step-like discontinuities in the CMB temperature [50].

Note that all these effects are gravitational, and thus they depend on the string tension μ, which only appears through the dimensionless combination $G\mu$. The current bound on this quantity coming from CMB measurements is of the order of $G\mu < 0.5 \times 10^{-6}$ [44, 45]. Although the measurements on pulsar timing imply a more severe constraint $G\mu < 2{\times}10^{-7}$ [51], the production of gravitational waves by the network is not completely understood yet, and thus this bound is less reliable than the previous one.

1.5.1 Supersymmetric cosmic strings.

During the last decade there has been a lot of interest in understanding cosmic string solutions in both globally and locally supersymmetric theories. This is partly motivated by the large number of cosmological scenarios based on supersymmetric theories, such as supersymmetric extensions of Grand Unified Theories and hybrid inflationary models derived from superstrings [52, 53] (see also [41, 42]). Topological defects arise naturally in most of these models, and among them, cosmic strings are the defects which are most consistent with astronomical observations, therefore it is interesting to search for cosmic string solutions in cosmological models based on supersymmetric theories.

more restrictive bounds [45].

Cosmic strings in supersymmetric theories can have interesting new properties, such as fermionic zero modes from partial supersymmetry breaking which are confined to the core of the string [54, 55]. These zero modes result in the string carrying a massless current, which can stabilize string loops (vortons). Vorton formation leads to severe constraints for the cosmological models, since they tend to contribute to the dark matter density causing a cosmological evolution which is in contradiction with observed astrophysical data [56]. Although most authors agree that these modes are not present in supergravity theories [57, 58, 59], there are studies which indicate that some zero modes survive the coupling to gravity [60].

The BPS limit discussed earlier has a close relationship with supersymmetry: when the coupling constants of the Abelian-Higgs model satisfy the BPS limit then the model can be identified as the bosonic sector of a supersymmetric theory, and the cosmic string solutions preserve half of the supersymmetries [54]. Actually, in a supersymmetric theory, solitons saturating a BPS-type of bound typically leave unbroken a fraction of the supersymmetries. These solitons constitute interesting probes of the high energy regime of the theory as they are often protected from quantum corrections by the unbroken supersymmetries.

The interest in *supersymmetric cosmic strings* solutions in $\mathcal{N} = 1$ supergravity models increased after the authors of [61] conjectured that they could represent the low energy manifestation of fundamental objects in string theory called *D-strings*, which are D-branes with one non-compact spatial dimension.[12] In [62] it was shown fundamental strings and D-branes of cosmic size could be formed after inflation. For example, this occurs in the brane anti-brane inflationary model [64, 65], which predicts the formation of a network of fundamental strings and D-strings at the end of inflation. In order to determine precisely the cosmological implications of such a network it is necessary to find effective field theory models that describe the formation and evolution of these objects, and the conjecture proposed in [61] provided a way to construct the corresponding low energy effective actions. If these defects have observable effects on the evolution of the early universe we could obtain information about superstrings and M-theory from cosmological data (see [42, 41]). Since [61] was published several string theory analyses have appeared in the literature that support the conjecture [66, 67, 68, 69, 70], however they are also indications of limitation of the conjecture[13].

[12] For a review of cosmic strings in superstring theory, see [62, 49, 63].

[13] The conjecture was based on the observation that D-term strings were the only BPS saturated strings available in $\mathcal{N} = 1$ supergravity. By now, other BPS strings have been obtained with different stability behaviors. For example semilocal strings [71, 66] and axionic D-term strings [59, 72] have a core radius that can vary in size, a property that is not generally expected for D-strings. Moreover, D-term strings are not expected to reproduce the scattering properties of D-strings [62, 49].

The study of supersymmetric cosmic strings has some advantages with respect to their non-supersymmetric counterparts. Usually cosmological models describe the evolution of a large number of particles, (both bosons and fermions), especially those based on superstring theories. Thus the full set of equations of motion can be extremelly involved, making the search of cosmic string solutions particularlly difficult. However, supersymmetric cosmic strings solutions are simpler to study than the rest, since they obey first order differential equations similar to (1.3.28), in contrast with regular cosmic strings solutions which are obtained solving second order differential equations such as (1.2.3), i.e. the full set of equations of motion. Some aspects of the dynamics of these supersymmetric vortices are also simpler to study than in the general case.

In general cosmological models based on supergravity are $\mathcal{N} = 1$ theories, such as the one discussed in [61], since they are chiral and therefore more suitable for phenomenology than higher $\mathcal{N}$ models. However, in order to understand better the connection between $\mathcal{N} = 1$ supergravity half-BPS solitonic solutions and superstring models it is useful to work in $\mathcal{N} = 2$ supergravity, since many string theory compactifications admit a low energy description in terms of $\mathcal{N} = 2$ supergravity in 4 space-time dimensions. The first known example of a half-BPS cosmic string in $\mathcal{N} = 2$ supergravity was constructed in [73], however the main purpose of this paper was to prove the possibility of obtaining this kind of solutions in $\mathcal{N} = 2$ supergravity, and thus it involved only the minimal necessary matter content. In chapter 6 we present a work which is intended to enlarge the family of $\mathcal{N} = 2$ supergravity models admitting half-BPS cosmic string solutions. The model we discuss there is related to many type-IIB superstring and heterotic string compactifications, and in particular to a well-known brane inflationary model, the D3-D7 model [74].

1.6 Overview

The chapters of this thesis can be arranged in two different groups according to the type of problem that is considered. Chapters from 2 to 5 are mainly concerned with the problem of consistent truncation of heavy moduli in $\mathcal{N} = 1$ supergravity theories and their stability after supersymmetry breaking in the surviving sector, and chapters 6 and 7, together with appendix B, are dedicated to the study of Cosmic Strings in extended $\mathcal{N} = 2$ supergravity and in $\mathcal{N} = 1$ globally supersymmetric models respectively.

Chapter 2.

In this chapter we will introduce the essential elements in $\mathcal{N} = 1$ supergravity theories. In section 2.2 we present the supermultiplets, the supersymmetry transformations, and we discuss the most important features of the action following the review in [75].

In section 2.4 we introduce the analytical technique used in chapter 4 to study the stability of the truncated scalar fields in a supersymmetric reduction of an $\mathcal{N} = 1$ supergravity model. As an application we derive the well know result that all supersymmetric critical points are perturbatively stable and, in addition, we find a relation between the curvatures of the scalar potential and the Kähler function at the critical point. Indeed we show that there is a one to one correspondence between the minima of the Kähler function and the supersymmetric AdS maxima of the scalar potential.

The chapter ends with section 2.5, which contains a short review of cosmic string solutions in $\mathcal{N} = 1$ supergravity theories based on the article [61]. The solutions discussed here leave unbroken the supersymmetries of the system, and provide an intermediate step between the Abrikosov-Nielsen-Olesen vortices and the cosmic string solutions discussed in the last chapter of this thesis.

Chapter 3.

An important problem present in cosmological models based on compactifications of superstring theories is the prediction of a large number of scalar fields, named moduli fields, which are not observed in nature. In flux compactifications a fraction of these fields are stabilized at a high energy scale while, for phenomenological reasons, it is assumed that the stabilization leaves supersymmetry unbroken (see section 1.2.5).

In supergravity it is not trivial to decouple a heavy sector of the theory from the low energy fields in a supersymmetric way. Indeed the interactions between the heavy fields, and those surviving the truncation have to satisfy certain constraints. This issue is especially relevant for inflationary models, where the slow roll conditions can be easily spoilt by the interactions between the inflaton and the heavy fields. Moreover, even when these couplings are consistent with an inflationary period, they might produce characteristic features in the CMB spectrum which would give valuable information about the high energy regime. Thus, it is important to have a precise characterization of the interactions between the inflationary and heavy sectors.

This chapter is dedicated to the problem of consistent supersymmetric decoupling of heavy scalars in supergravity theories. Here we will derive a set of necessary and sufficient conditions for truncating heavy fields in $\mathcal{N} = 1$ supergravity theories subject to explicit requirements. First, that the truncated fields should not be sourced due to the interactions with the surviving fields, and second the low energy effective action should be described by $\mathcal{N} = 1$

supergravity. These conditions can be expressed as constraints on the couplings between the truncated fields and those surviving in the reduced theory. After solving these constraints we will present the most general class of models which are compatible with the supersymmetric truncation of the heavy fields. To the best of our knowledge, this analysis has not been published yet.

In view of these results, in section 3.3 we discuss various approaches followed in the literature in order to truncate a heavy sector in supergravity models while preserving supersymmetry, and to obtain the corresponding low energy action. Finally in sections 3.5 and 3.6 we analyze some explicit examples which illustrate the differences between models which allow for the supersymmetric decoupling of the heavy sector, and those where such a decoupling is not possible. Part of the work presented in the last sections of this chapter can be found in the article [76].

Chapter 4.

Since supersymmetry is not a symmetry of nature, any viable cosmological model based on a supersymmetric theory should include a mechanism of supersymmetry breaking. Moreover, in $\mathcal{N} = 1$ supergravity, supersymmetric vacua can not have a positive cosmological constant, and thus their are not suitable to describe the present acceleration of the universe. Therefore, in supergravity models such as those studied in chapter 3, where part of the field content is truncated in a supersymmetric way, the sector surviving the reduction must necessarily break supersymmetry.

The decoupling conditions found in chapter 3 ensure that the truncated fields are fixed at a critical point of the scalar potential, but once supersymmetry is broken in the surviving sector there is nothing to guarantee that the critical point is a minimum of the scalar potential, and thus the perturbative stability of the truncated fields needs to be studied.

In chapter 4 will we discuss the case where supersymmetry is broken in the surviving sector due to the interactions among the chiral fields ($F-$term mechanism). We will make a detailed stability analysis for the simplest class of models satisfying the decoupling conditions derived in the previous chapter, that is when the Kähler function is separable in the truncated and the surviving sectors. We will show that in these models the supersymmetric configurations of the truncated fields given by the minima of the Kähler function remain stable for arbitrarily high scales of supersymmetry breaking. Interestingly this result can be extended to the most general case provided that we impose a mild restriction on the field dependence of the fermionic masses. The work presented in this chapter is based on the results we published in [77, 78].

This analysis is complementary to the study by Covi *et al.* [79], who provided a necessary condition in order to have a (meta)stable de Sitter vacua in supergravity theories. This condition can be expressed as a constraint on the geometry of the Kähler manifold characterizing the supergravity model, but it only applies to the sector of the theory where supersymmetry is broken. Therefore it can not be used constraint the interactions on the sector that is truncated in a supersymmetric way, which is the precisely the aim of the work presented in chapter 4.

As we mentioned above, the idea of supersymmetric decoupling is especially relevant to inflationary models, indeed this stability analysis has already been used to study the viability of *sgoldstino inflation* [80], where the inflaton is precisely the scalar partner of the goldstino.

Chapter 5.

Supersymmetry breaking can also be induced in the surviving sector by gauge interactions ($D-$term mechanism). An important ingredient of cosmological models which implement the $D-$term supersymmetry breaking are Fayet-Ilipoulos terms, which are related to the charge of the gravitino under abelian gauge interactions. Fayet-Iliopoulos terms can also be used to describe the present accelerated expansion of the universe, cosmic inflation and to construct cosmic string solutions which partially preserve the supersymmetries of the system.

Recently there has been some discussion in the literature about the viability of constructing consistent string theory models which are described at low energies by a supersymmetric effective action containing a Fayet-Iliopoulos term (see for example [81] and [82]). These discussions have led to a better understanding of the properties of Fayet-Iliopoulos terms, and in particular they have helped to clarify the issue of identifying these terms in non-linear sigma model with general gauge couplings.

In chapter 5 we use these new results to discuss the possibility of producing these terms by integrating out the heavy fields in a $\mathcal{N} = 1$ supergravity theory. We find that the Fayet-Iliopoulos terms cannot be generated during the supersymmetric truncation of a sector of the theory. This conclusion generalizes previous results presented by Binetruy *et al.* in [75], which only apply to models with a particular choice of gauge couplings.

This chapter provides a bridge between the first part of the thesis, which focuses on supersymmetric truncations in $\mathcal{N} = 1$ supergravity, and the second part which discusses supersymmetric cosmic string solutions in $\mathcal{N} = 2$ supergravity, where Fayet-Iliopoulos terms play crucial röle.

Indeed, in the last section of chapter 5, we consider the properties of supersymmetric cosmic strings in the context of consistent truncations, and we show that if a cosmic string solution leaves unbroken part of the supersymmetries of the system in the reduced theory, the same supersymmetries have to be preserved in the full parent theory. Moreover we will present a $\mathcal{N} = 1$ supergravity model containing a Fayet-Iliopoulos term which, as shown in chapter 6 admits an embedding in $\mathcal{N} = 2$, and can be used to construct supersymmetric cosmic string solutions in extended $\mathcal{N} = 2$ supergravity.

Chapter 6.

In order to provide the basis needed to discuss the supersymmetric cosmic string solutions presented in chapter 7, here we give a short review of $\mathcal{N} = 2$ supergravity theories. We discuss the bosonic sector of the action, and the supersymmetry transformations.

We also explain the difficulties encountered when trying to embed supersymmetric cosmic string solutions in $\mathcal{N} = 2$ supergravity, associated to the strong constraints to the presence of Fayet-Iliopulos terms in these theories. The solution to these problems, presented by Achúcarro *et al.* in [73], consists in considering the embedding of $\mathcal{N} = 1$ supergravity models with a Fayet-Iliopoulos term in $\mathcal{N} = 2$.

This technique relies on the idea of consistent reductions of a $\mathcal{N} = 2$ supergravity model down to $\mathcal{N} = 1$, reviewed in section 6.8, and is closely related to the supersymmetric truncations from $\mathcal{N} = 1$ to $\mathcal{N} = 1$ discussed in chapter 3.

Chapter 7.

In the final chapter of this thesis we construct an explicit example of a local supersymmetric cosmic string solution in $\mathcal{N} = 2$ supergravity. To the best of our knowledge, this is one of the only two known cosmic string solutions in four dimensional $\mathcal{N} = 2$ supergravity. In the model we discuss here the self coupling of the scalar and vector fields is characteristic of many compactifications of string theory, in particular it is related to type-IIB superstrings compactified in $K3 \times T^2/\mathbb{Z}_2$, which is the framework of one of the best studied inflationary models in string theory, the D3-D7 model [74].

The solution has interesting supersymmetric properties. In particular the cosmic string ansatz is compatible with a consistent truncation of supergravity down to $\mathcal{N} = 1$, and the string background leaves unbroken half of the $\mathcal{N} = 2$ supersymmetry transformations. The string configuration contains a zero mode connecting solutions with different radii and equal energy. In [72] we proved that a similar class of cosmic strings which contain such a zero mode do not usually survive in a cosmological context, since the excitation of the zero mode typically makes the string grow in width until it disappears. However we argue that in this case the zero mode requires an infinite energy to be excited in an infinite volume, and therefore is not dynamical. This study can be found in the article [83].