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Title: Non-decoupling of heavy scalars in cosmology
Date: 2012-06-08

CHAPTER 2

Consistent decoupling of heavy scalars and $\mathcal{N} = 1$ supergravity

The viability of theories based on extra dimensions, in particular string theory, relies on being able to stabilise and integrate out the fields (moduli) that describe the shapes and sizes of those extra dimensions, for which so far there is no observational evidence. In flux compactifications (Giddings et al., 2002) some moduli are stabilised at a high energy scale and decouple from the low energy theory. From that moment on we never see them in the effective low energy description.

Unlike in global supersymmetry, complete decoupling is of course impossible in supergravity – even in principle – because gravity couples to all fields. At low energies one is usually satisfied with gravitational strength couplings between the heavy, stabilised, fields and the low energy fields. However, such interaction terms are of order $O(G_{\text{Newton}}E^2) = O(E^2/M_P^2)$, where E is the energy scale and $M_P \approx 2.4 \times 10^{18}$GeV the reduced Planck mass. Even if they are strongly suppressed at low energy and in particle accelerators, these couplings become sizable at the energy scales relevant to the early Universe, and one must look for a more robust definition of decoupling that can be extrapolated over a wide range of energy scales. The purpose of this chapter is to provide such a definition, and a simple test of whether it holds in specific models.

There are at least two situations in which the details of decoupling are important. One is supersymmetry breaking, which will affect the heavy fields in a way that is not accounted for in the low energy effective action. Uplifting in KKLT scenarios (Kachru et al., 2003a) is a prime example. The second is inflation with moduli stabilisation, because the inflaton, which is a low energy field in this language, can have its expectation value vary over many Planck-masses.

Here we take a bottom-up approach and try to find for what types of Supergravity couplings we can be sure that the heavy moduli will not shift from their expectation values due to low energy processes. We do not require small gravitational coupling to the light(er) fields because instead we rely on supersymmetry to partially protect the expectation values of the heavy moduli.

It must be stressed that what we are proposing here, building on arguments by Choi et al. (2005), de Alwis (2005b), de Alwis (2005a), Binetruy et al. (2004) and Achúcarro and Sousa (2008), is a simple consistency test. It checks explicitly what is implicitly assumed by the very use of a low energy effective action. So it is somewhat surprising to find that the most common ansatz for decoupled fields in the literature, the standard "gravitational strength coupling" ansatz, generically fails the test. It partly explains the difficulties encountered in supergravity models of inflation with moduli stabilisation. The problem essentially disappears for consistently decoupled moduli (Davis and Postma, 2008, Achúcarro and Sousa, 2008).

2.1 Consistent decoupling of scalar fields in $\mathcal{N} = 1$ supergravity

In what follows we consider two sets of fields, heavy (H) and light (L), and assume the heavy fields are stabilised at an expectation value $H = H_0$, an extremum of the scalar potential for the heavy moduli. If the heavy field is a singlet under all low energy symmetries and its mass is large enough it will decouple from low energy phenomena and can be truncated, leaving an effective theory for the light degrees of freedom. To make this distinction, we will again use hatted quantities to indicate the full theory, including heavy and light fields, and unhatted quantities for the effective theory involving light fields only

$$S(L,\overline{L}) = \widehat{S}(H_0,\overline{H}_0,L,\overline{L}). \tag{2.1}$$

We are interested in the case in which the resulting effective theory is also described by $\mathcal{N}=1$ supergravity. In this case, there should be an effective K and W (or

G) depending only on the light fields, from which to compute the low energy action S and supersymmetry transformations

$$G[L,\overline{L}] = \widehat{G}[L,\overline{L},H_0,\overline{H}_0]\ , \tag{2.2}$$

$$\delta_\epsilon L = \hat{\delta}_\epsilon L|_{H_0} = f[L, G(L,\overline{L})]\ , \tag{2.3}$$

$$\hat{\delta}_\epsilon H|_{H_0} = 0\ . \tag{2.4}$$

Notice that the F-terms (eq. 1.45) of the heavy fields must vanish because the supersymmetry transformations read,

$$\hat{\delta}_\epsilon H \sim \chi\epsilon, \qquad \hat{\delta}_\epsilon\chi \sim \not{\partial} H\epsilon - \frac{1}{2}F\epsilon \tag{2.5}$$

and if the F-terms are non-zero a supersymmetry transformation will generate light fermions, related to the supersymmetry breaking in the heavy sector, that are not in the low energy effective action. Thus, the heavy fields cannot contribute to supersymmetry breaking, leading to

$$\partial_H \widehat{G}|_{H_0} = 0 \qquad \text{or} \qquad \widehat{\mathcal{D}}_H \widehat{W}|_{H_0} = 0\ , \tag{2.6}$$

(see also de Alwis, 2005b) where $\widehat{\mathcal{D}}_i\widehat{W} = \partial_i\widehat{W} + (\partial_i\widehat{K})\widehat{W}$ is the Kähler covariant derivative that transforms as $\widehat{\mathcal{D}}_i\widehat{W} \to e^{-h(z)}\widehat{\mathcal{D}}_i\widehat{W}$ under Kähler transformations. Note that $\widehat{\mathcal{D}}_H\widehat{W} = 0$ is the condition used in flux compactifications (Giddings et al., 2002) and by extension in KKLT (Kachru et al., 2003a) and LARGE volume scenarios (Balasubramanian et al., 2005), where the complex structure moduli are stabilised at a supersymmetric point before uplifting.

The Kähler metric should be block diagonal in the light and heavy fields when evaluated at H_0, otherwise propagators will mix these two sets of fields. Additionally, the truncation $H = H_0$ must of course be a consistent truncation. This means that the equations of motion of the light fields derived from the effective theory are the same as the equations of motion obtained from the full theory. To zeroth order in the fluctuations of the heavy fields

$$\left.\frac{\delta\widehat{S}}{\delta L}\right|_{H_0} = \frac{\delta\widehat{S}|_{H_0}}{\delta L} = \frac{\delta S}{\delta L}\ , \tag{2.7}$$

ensuring that the fluctuations of H are not sourced by the light fields. In particular, the heavy fields should be singlets under the surviving gauge group at low energies, otherwise they remain coupled to the light fields by the gauge interaction. In what follows we will consider $f_{ab}(L)$ independent of the heavy fields. In that case they do not contribute to the D-terms, which will only involve light fields.

2.2 Analysis of the consistency conditions

The heavy fields thus need to be stabilised at an expectation value H_0, where H_0 is the solution to (eq. 2.6)

$$\left[\partial_H \widehat{W}(H,L) + \partial_H \widehat{K}(H,\overline{H},L,\overline{L})\widehat{W}(H,L)\right]\Big|_{H_0} = 0 \ . \tag{2.8}$$

which implies $\partial_H \widehat{V}|_{H_0} = 0$. The LHS is some function of both the heavy and the light fields, let us call it $\Phi(H,\overline{H},L,\overline{L})$. In general, the condition $\Phi = 0$ (together with its complex conjugate $\overline{\Phi} = 0$) relate the heavy and light fields. If we can solve for H we obtain an expression of H_0 as a function of the light fields,

$$H = H_0(L,\overline{L}) \ , \tag{2.9}$$

which can be substituted back into $\widehat{K}, \widehat{W}$ to give an effective action for the light fields

$$S(L,\overline{L}) = \widehat{S}(H_0(L,\overline{L}), \overline{H}_0(L,\overline{L}), L, \overline{L}) \ . \tag{2.10}$$

An immediate concern with the consistency of this procedure, pointed out in de Alwis (2005b), is that in general this leads to a non-holomorphic expression for the would-be effective superpotential $W = \widehat{W}(H_0(L,\overline{L}), L)$. However, this problem is easily avoided: it does not arise if $\widehat{W}$ is independent of H. The case $\widehat{W} = 0$ is obvious, so consider $\widehat{W} \neq 0$. It is always possible to perform a Kähler transformation that makes $\widehat{W}$ constant

$$\widehat{W} \to 1 \ , \tag{2.11a}$$

$$\widehat{K} \to \widehat{K} + \log \widehat{W} + \log \overline{\widehat{W}} = \widehat{G} \ . \tag{2.11b}$$

In this so called Kähler gauge, (eq. 2.8) reads

$$\partial_H \widehat{G}(H,\overline{H},L,\overline{L}) = 0 \ , \tag{2.12}$$

from which we can extract $H = H_0(L,\overline{L})$ and make the previous substitution directly into the Kähler invariant function without any inconsistency (see also Curio and Spillner, 2007):

$$G = \widehat{G}(H_0(L,\overline{L}), \overline{H}_0(L,\overline{L}), L, \overline{L}) \ . \tag{2.13}$$

In fact, the issue is not whether $H_0(L,\overline{L})$ is holomorphic but rather whether it is *a (non-trivial) function at all*. The assumption that the heavy fields are *stabilised* at

$H = H_0$ is simply the condition that $H_0(L, \overline{L}) =$ constant. Any other dependence on the light moduli would translate into a constraint on the light fields which would have to be accounted for explicitly in the low energy action (de Alwis, 2005a). This is what we have to avoid.

To summarise, the (rather obvious) mathematical condition for the heavy fields to be truncated consistently with an expectation value H_0 and to decouple from the low energy fields is that the system of equations

$$\partial_H \widehat{G} \equiv \Phi(H, \overline{H}, L, \overline{L}) = 0 \ , \tag{2.14}$$

which is the same as (eq. 2.8) defined in the Kähler gauge (eq. 2.11), admits the constant solution

$$H = H_0(L, \overline{L}) = \text{const} \ , \qquad \overline{H} = \overline{H}_0(L, \overline{L}) = \text{const} \ . \tag{2.15}$$

In spite of being obvious, this condition is not empty. For instance, we will see below that it *fails* generically for standard couplings of the form $K = K_1 + K_2$ and $W = W_1 + W_2$. However, let us first consider two specific situations in which the decoupling condition does hold.

1. The consistency condition is trivially satisfied if the function $\Phi(H, \overline{H}, L, \overline{L})$ has no explicit dependence on the light fields. In this case by integrating (eq. 2.14) one recovers the condition found in Binetruy et al. (2004)

$$\partial_H \widehat{G} = \Phi(H, \overline{H}) \rightarrow \widehat{G} = \widehat{G}_1(H, \overline{H}) + \widehat{G}_2(L, \overline{L}) \tag{2.16}$$

and it is obvious that the Kähler metric is block diagonal in this case. This ansatz has a long history which goes back to Cremmer et al. (1983a) and allows a detailed stability analysis of the heavy fields (Achúcarro and Sousa, 2008 and chapter 3), in particular in the context of F-term uplifting of flux compactifications.

2. On the other hand, this requirement is too restrictive. It is sufficient if the function $\Phi(H, \overline{H}, L, \overline{L})$ factorises,

$$\Phi(H, \overline{H}, L, \overline{L}) = \Phi_1(H, \overline{H}, L, \overline{L})\, \Phi_2(H, \overline{H}) = 0 \ , \tag{2.17}$$

in which case we just solve $\Phi_2 = \overline{\Phi}_2 = 0$ to get constant $H_0, \overline{H}_0$. We cannot give the general form of $\widehat{G}$ for which this factorisation occurs, but it will certainly hold if $\widehat{G}$ has the following functional form:

$$\widehat{G} = f(L, \overline{L}, g(H, \overline{H})) \ , \tag{2.18}$$

since in that case (eq. 2.6) is replaced by

$$\partial_H g(H,\overline{H}) = 0\ . \tag{2.19}$$

The first situation, (eq. 2.16), is a special case of (eq. 2.19), with Φ_1 constant. In both cases, the same condition that makes $\widehat{G}_H|_{H_0} = 0$ also implies that the Kähler metric and the Hessian of V are block diagonal for any Φ_1. Indeed, from (eq. 2.19) we find that

$$\widehat{G}_{LH}|_{H_0} = \partial_L \partial_g f(L,\overline{L},g(H,\overline{H}))\partial_H g(H,\overline{H})|_{H_0} = 0 \tag{2.20}$$

and further all mixed derivatives with only one derivative with respect to the heavy field vanish. As V_{LH} always contains terms $\propto \widehat{G}_H$ or $\propto (\partial_L)^n \widehat{G}_H$, which vanish at H_0, the Hessian of V is block diagonal.[1]

2.3 Consistent decoupling compared to gravitational coupling in rigid supersymmetry

Finally, we stress that the condition derived here has no direct relation to the condition usually associated with gravitational strength coupling, see also the discussion in section 1.3.2. In fact, the ansatz

$$\widehat{K} = K_1(H,\overline{H}) + K_2(L,\bar{L})\ , \tag{2.21a}$$
$$\widehat{W} = W_1(H) + W_2(L) \tag{2.21b}$$

does not satisfy the decoupling condition in general. Suppose (eq. 2.6) admits a constant solution $H = H_0$. Then

$$0 = \partial_H W_1|_{H_0} + \partial_H K_1|_{H_0}[W_1(H_0) + W_2(L)]\ , \tag{2.22}$$

which only holds if

$$\begin{aligned} \partial_H K_1|_{H_0} = 0 \ &\Rightarrow\ \partial_H W_1|_{H_0} = 0 \quad \text{or} \\ \partial_H K_1|_{H_0} \neq 0 \ &\Rightarrow\ W_2(L) = -\frac{\partial_H W_1|_{H_0}}{\partial_H K_1|_{H_0}} - W_1(H_0) \\ &\qquad\qquad = \ \text{const.} \end{aligned} \tag{2.23}$$

[1] Note that it is always possible to block-diagonalise the Kähler metric or the Hessian of V at one point, but it is not necessarily the case that both block-diagonalisations are compatible, as we have here.

Another way to see this: since $\widehat{\mathcal{D}}_H \widehat{W} = 0$ does not factorise, the (Kähler-gauge covariant) requirement that it is independent of the light fields is (see also Ben-Dayan et al., 2008)

$$\widehat{\mathcal{D}}_L(\widehat{\mathcal{D}}_H \widehat{W}) = 0 \; . \tag{2.24}$$

Inserting the ansatz (eq. 2.21) then gives

$$\partial_H K_1|_{H_0} \partial_L W_2 = 0 \; . \tag{2.25}$$

Unless $K_1(H, \overline{H})$ has no linear terms or $W_2(L) = $ constant, the condition will not be met. However, if $W_2(L) = $ constant (e.g. no scale models, such as presented in Cremmer et al., 1983b, Giddings et al., 2002) then equation (eq. 2.16) holds and $\widehat{W}$ is trivially a product. On the other hand, we can always expand $K_1(H, \overline{H})$ around H_0 and remove the linear terms by a Kähler transformation (eq. 1.36), but this spoils the separability of the superpotential (eq. 2.21b).

In other words, if two sets of fields are described by a separable Kähler function $K = K_1(\text{heavy}) + K_2(\text{light})$, the addition of their superpotentials does not respect the decoupling condition except in special cases (and, incidentally, neither does it guarantee gravitational strength couplings if $K_1(\textit{heavy}) = O(M_p^2)$, as is usual for moduli).

2.4 Discussion

In this chapter we have studied how to truncate heavy scalars and moduli and their superpartners in $\mathcal{N} = 1$ supergravity, subject to two *explicit* requirements. First, the expectation values of the heavy fields should be unaffected by low energy phenomena, in particular supersymmetry breaking. Second, the low energy effective action should be described by $\mathcal{N} = 1$ supergravity. This is what we call *consistent decoupling*.

If the heavy fields are stabilised at a critical point of the potential, integration of the whole superfield requires that the F-terms should be zero (Binetruy et al., 2004). The criterion for consistent decoupling is that the expectation value of the heavy scalars H should not depend on the light fields L (de Alwis, 2005a). Our main result is a class of Kähler invariant functions that satisfy the condition, given in (eq. 2.18):

$$\widehat{G} = f(L, \overline{L}, g(H, \overline{H})) \; .$$

This functional form guarantees that the Kähler metric and Hessian of V are simultaneously block diagonal in the heavy and light fields. It also allows the embedding of BPS solutions of the low energy effective theory into the full theory without destroying their BPS character (if the F-terms of the heavy fields are zero and in the absence of constant Fayet-Iliopoulos terms, the supersymmetric transformation of the gravitino depends only on the light fields). We would expect the BPS character to survive quantum corrections – now in the full theory –. Thus, at least in this special case it would seem possible to "screen" the heavy, decoupled fields from the effects of (partial) supersymmetry breaking in the low energy sector.

We only have experimental access to G, the effective low energy theory, and there is a large class of supergravity models (read a landscape of compactifications), characterised by $\widehat{G}$, in which the low energy theory could be embedded. Here, $\widehat{G}$ includes all stringy, perturbative and non-perturbative effects. The decoupling condition restricts the allowed functional form of $\widehat{G}$ and therefore the class of models that are consistent with the assumption of decoupling that is implicit in our use of G. From the point of view of model building, it provides a simple test that has not been considered before. There are string compactifications which approximately satisfy the decoupling condition in the form (eq. 2.16), such as some LARGE volume scenarios (LVS) (Balasubramanian et al., 2005, Conlon et al., 2005, 2007, Conlon, 2008).

To see this, note first of all that the tree level or GKP limit (Giddings et al., 2002) of $\widehat{G}$ satisfies (eq. 2.16) with the complex structure moduli and the dilaton S playing the role of the heavy fields. Assume the usual form for the leading non-perturbative and α' corrections, $\widehat{W} = W_{\text{GKP}}(H) + W_{\text{np}}(L)$, $\delta\widehat{K} \sim 2(S+\bar{S})^{3/2}/\mathcal{V}$, with $\mathcal{V}$ is the volume modulus of the compact manifold. Ignoring for a moment the dilaton dependence of $\delta\widehat{K}$, we find for the complex structure moduli

$$\partial_H \widehat{G} = \partial_H K_{\text{heavy}}(H) + \frac{\partial_H W_{\text{GKP}}(H)}{W_{\text{GKP}}(H)}\Big[1 + \delta(L,H)\Big]^{-1} , \tag{2.26}$$

where $\delta = W_{\text{np}}(L)/W_{\text{GKP}}(H)$. Including dilaton effects adds a correction $\delta \sim (S + \bar{S})^{3/2}/\text{vol}$ (whichever is larger). The condition of consistent decoupling is violated by the L-dependence of δ.

A stabilised LVS or KKLT vacuum, with nonperturbative corrections, is obtained from a Kähler potential and superpotential of the form

$$K = K_{cs} - 2\log\left[e^{-\frac{3\phi_0}{2}}\mathcal{V} + \frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2}\right] , \tag{2.27}$$

$$W = W_0 + \sum_n A_n e^{i a_n \rho_n} . \tag{2.28}$$

Here, τ represents the axio-dilaton field and $\xi = -\zeta(3)\chi(M)/(16\pi^3)$, where $\chi(M)$ is the Euler characteristic. Furthermore, ρ_i are the complexified Kähler moduli which in this specific case correspond to 4-cycles. Furthermore, $a_i = 2\pi/K$, with $K \in \mathbb{Z}_+$. From the Kähler potential K and superpotential W one obtains a scalar potential

$$\begin{aligned} V = e^K \Big[& G^{\rho_j \bar{\rho}_k} \left(a_j A_j a_k \bar{A}_k e^{i\left(a_j \rho_j - a_k \bar{\rho}_k\right)} + i \left(a_j A_j e^{i a_j \rho_j} \bar{W} \partial_{\bar{\rho}_k} K - a_k \bar{A}_k e^{-i a_k \bar{\rho}_k} W \partial_{\rho_j} K \right) \right) \\ & + 3\xi \frac{\left(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2\right)}{(\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} |W|^2 \Big] \\ & \equiv V_{np1} + V_{np2} + V_{\alpha'} \ . \end{aligned} \tag{2.29}$$

It can be shown that for Calabi-Yau manifolds with a negative Euler characteristic the potential is positive at $\mathcal{V} = 0$, zero at $\mathcal{V} \to \infty$ and negative for large $\mathcal{V}$. This construction can thus be used to generate a potential that allows for a vacuum solution at an exponentially large volume. As this solution is obtained after truncating the complex structure, it might be shifted as given by (eq. 2.26). In the case of an LVS vacuum with parameters $A \sim 1$, $W_{\mathrm{GKP}}(H_0) \sim 10$, $\mathcal{V} \sim 10^{10}$, $Ae^{-a_4\tau_4} \sim 1/\mathcal{V}$ (see Balasubramanian et al., 2005) it is negligible, $\delta \sim O(10^{-10})$, as the correction scales inversely with the volume.[2] In the mirror mediation scenarios (Conlon, 2008) δ is even smaller.

Another well known solution to (eq. 2.29) is the KKLT solution (Kachru et al., 2003a). This solution is characterised by parameters $A \sim O(1)$, $W_{\mathrm{GKP}}(H_0) \sim O(10^{-4})$, $aL \sim O(10)$ and leads to a significant change of the vacuum obtained in the truncated theory as $\delta \sim O(1)$.

[2]Berg et al. (2007) and Cicoli et al. (2008) suggest that string loop corrections to $\hat{K}$ scale as $(\mathcal{V})^{-2/3}$ and would lead to $\delta < 10^{-6}$. We thank M. Cicoli for this remark.