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**Title:** Application of evolutionary strategies to industrial forming simulations for the identification and validation of constitutive laws  
**Date:** 2012-05-29
Appendix A

Additional figures

Figure A.1: Production process of a side panel.
Figure A.2: YLIT-Experiments; Left: Shape of the specimen of the YLIT-3-BMW after forming; Right: Shape of the specimen of the YLIT-4-BMW after forming.

Figure A.3: Bending experiment; The shape of the specimen after the bending operation.

Figure A.4: Left: Hole extrusion experiment; The shape of the specimen after the forming operation; Right: U-Profile experiment; The shape of the specimen after the forming operation.
Figure A.5: Bending experiment; Field of the accumulated plastic strain on the upper surface.

Figure A.6: U-Profile experiment; Field of the accumulated plastic strain on the upper surface.
Figure A.7: Hole extrusion experiment; Comparison of the predicted and measured strain field.
Appendix B

Tensor analysis

The subsequently presented symbols and tensors are taken from [60], [62] and [116].

B.1 Symbols

\[ a \cdot b \] Contraction of the inner indices; Example: \( a \) and \( b \) are vectors \( a_i, b_j \); The contraction of the inner indices is identical with the scalar product \( a_i b_i \).

\[ a : b \] Double contraction of the inner indices; Example: \( a \) and \( b \) are second-order tensors \( (a_{ij}, b_{kl}) \); The double contraction of the inner indices is given by \( a_{ij} b_{ij} \).

\[ a \otimes b \] Dyadic product; \( a \otimes b := a_i b_j \)

\[ tr(\bullet) \quad tr(a) \] Trace of a tensor; Example: For a second order tensor \( a \) the trace is given by \( tr(a) = a_{ii} \).

\[ \|\bullet\| \quad \|a\| \] \( L_2 \) Norm of a vector, which is the Euclidean distance and defined by \( \|a\| = (\sum_{i=1}^{n} a_i^2)^{\frac{1}{2}} \).
B.2 Tensors

\[(1)_{ij} = \delta_{ij}\]  
Second-order identity tensor; Example: For a vector \( \mathbf{a} \), the following equation applies: \( \mathbf{1a} = \mathbf{a} \).

\[(I)_{ijkl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})\]  
Fourth-order symmetric identity; Example: For a second-order tensor \( \mathbf{a} \), the relation \( \mathbf{I : a = a : I = sym(a)} \) is valid.

\[(1 \otimes 1)_{ijkl} = \delta_{ij} \delta_{kl}\]  
For a second-order tensor \( \mathbf{a} \), the identity satisfies \( (1 \otimes 1) : \mathbf{a} = \text{tr} (\mathbf{a}) \mathbf{1} \).
Appendix C

Shape functions

As an example, the shape functions of an eight-node hexahedral element are given below [60]. Figure C.1 summarizes the constants of expression (C.1) and shows a visualization of the element.

\[ N_I(\xi) = N_I(\xi, \eta, \zeta) = \frac{1}{8} (1 + \xi_I \xi) (1 + \eta_I \eta) (1 + \zeta_I \zeta) ; \quad I = 1, \ldots, 8 \quad (C.1) \]

Figure C.1: Left: Visualization of an eight-node hexahedral element; Right: Definition of the constants of the shape functions.
Appendix D

Numerical parameters

LS-Dyna

All forming and springback simulations of this thesis are performed with the simulation system LS-Dyna. Table D.1 summarizes the choice of important numerical parameters.

Table D.1: Choice of numerical parameters.

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<th>Name Parameter</th>
<th>Value</th>
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