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Chapter 4

Modeling Job Arrivals

In this chapter, we present how to model job arrivals with long range dependence (LRD) and burstiness\(^1\) characteristics. For network traffic, it is well-known that a Markov-modulated Poisson process (MMPP) [24, 35, 100] can capture these features. However, MMPPs are not suitable for modeling job arrivals. An MMPP-based model captures dependencies from data and introduces the dependencies into interarrival times. As we indicated in Section 3.2, job arrivals exhibit LRD under the representation of a rate process, but do not show LRD with the interarrival time representation, and so an MMPP-based model fails to produce LRD for job traffic. The limitation of MMPPs in modeling system job traffic is well studied in [46], which suggests that MMPPs should be used to capture short to middle-range autocorrelations. Li [46] indicates that LRD of job arrivals should be reliably rooted and modeled from a rate process. He introduced the multifractal wavelet model (MWM) [85] as a good choice when it comes to long-range autocorrelations. Also a detailed explanation of why MWM is trusted for modeling job traffic, instead of other models of network traffic like MMPPs, was provided. However, as MWM is applied to a rate process, capturing burstiness with the interarrival representation is an issue. In Section 4.2, we will show that MWM is not able to mimic burstiness of real data though it produces LRD well. Therefore, we modify MWM and adapt it to capture burstiness of interarrival times while it still keeps LRD in a rate process.

4.1 Theory of Multifractal Wavelet Model

This section provides a brief background of the theories on which MWM is developed. For more detailed information, we refer the reader to [85].

MWM treats a stochastic process via scaling techniques in order to decrease/increase its length to smaller/larger scales. At each scale \( j \), MWM considers the stochastic process as a series of scaling coefficients, denoted by \( \{ c_j \} \). As we will clarify later

\(^1\)The term “burstiness” used during this chapter refers to temporal burstiness for brevity.
in Section 4.3, the series of scaling coefficients at the highest scale is used as a rate process in modeling job arrivals. In addition, MWM also defines a series of wavelet coefficients at scale $j$, denoted by $\{d_j\}$, that correspond to $\{c_j\}$. The scaling and wavelet coefficients can be recursively computed by

$$c_{j,k} = \frac{1}{\sqrt{2}}(c_{j+1,2k} + c_{j+1,2k+1}), \quad (4.1)$$

$$d_{j,k} = \frac{1}{\sqrt{2}}(c_{j+1,2k} - c_{j+1,2k+1}), \quad (4.2)$$

$k = 0, \ldots, N_j - 1$, where $N_j$ is the number of scaling/wavelet coefficients at scale $j$. By rearranging Eq. (4.1) and Eq. (4.2) to

$$c_{j+1,2k} = \frac{1}{\sqrt{2}}(c_{j,k} + d_{j,k}), \quad (4.3)$$

$$c_{j+1,2k+1} = \frac{1}{\sqrt{2}}(c_{j,k} - d_{j,k}), \quad (4.4)$$

Riedi et al. [85] found a simple constraint to guarantee the positivity of the scaling coefficients: $|d_{j,k}| \leq c_{j,k}$. Positivity is a desirable feature since rate processes are inherently non-negative. To satisfy this constraint, MWM defines the following multiplicative model

$$d_{j,k} = A_{j,k} \times c_{j,k}, \text{ with } A_{j,k} \in [-1, 1], \quad (4.5)$$

where $\{A_j\}$ is considered as a series of multipliers at scale $j$. At each scale $j$, MWM uses the symmetric beta distribution [79] to fit $\{A_j\}$. The variance of a random variable $A$ of a symmetric beta distribution with the unique parameter $p$ is given by

$$var(A) = \frac{1}{2p + 1}, \quad (4.6)$$

MWM yields LRD for output processes by fixing the beta parameter at the smallest scale $p_1$ according to Eq. (4.6):

$$p_1 = \frac{1}{2\text{var}(A_1)} - \frac{1}{2}, \quad (4.7)$$

where $\{A_1\}$ is a series of multipliers at scale 1, and calculates the beta parameter at scale $j \geq 2$ recursively:
4.2 MWM and Burstiness

\[ p_j = \frac{1}{2} \left[ \frac{\text{var}(d_{j-1})}{\text{var}(d_j)} (p_j - 1) + 1 \right]. \tag{4.8} \]

For details about the mathematical demonstration of how Eq. (4.7) and Eq. (4.8) can help to produce LRD, readers are referred to [85].

<table>
<thead>
<tr>
<th>Trace</th>
<th>( T ) (s)</th>
<th>900</th>
<th>1800</th>
<th>3600</th>
<th>7200</th>
</tr>
</thead>
<tbody>
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<td>NOR</td>
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<td>33.14</td>
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<td>NIK</td>
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<td></td>
<td>( Z ) MWM</td>
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</tr>
</tbody>
</table>

4.2 MWM and Burstiness

We apply MWM to traffic of a real grid and a real cluster to generate synthetic job arrivals and then use the coefficient of variation \( C_v \) to quantify burstiness. The results in Table 4.1 with different time scales \( T^2 \) show that MWM is not able to mimic burstiness of the real data. To find the cause, we do a direct observation to the real job arrivals. We analyze real rate processes and see that they contain a large number of zero values as shown in Table 4.1. Each value in a rate process represents the number of job arrivals per second in a time interval. Therefore, a zero value means that there is no arrival job in the corresponding time interval. The existence of a large number of zeroes in a rate process indicates the presence of temporal burstiness in the original job arrivals.

\[ \text{In the context of parallel workloads, there is a range that should not be selected for a time scale. A too small time scale (e.g., less than 15 minutes) results in a large number of zeroes (more than 50\%) in a rate process and this will affect LRD. A too large time scale (more than 3 hours) results in a relatively short rate process that is hard to apply MWM on.} \]
Figure 4.1 gives an example to illustrate why MWM cannot produce burstiness as in real job arrivals. In this example, $R_2$, containing a large number of zeroes, is considered a real rate process with a time scale of 10 seconds. When applying MWM to $R_2$, we obtain a synthetic rate process like $R_1$ with no zero value. Because positivity is a feature of MWM, it can only produce values greater than 0 (we refer to Section 4.4 for a detailed explanation). Therefore, a rate process generated by MWM includes only nonzero values. This means that there exist job arrivals in all time intervals. In other words, long gaps with no arrivals are not found in a job arrival process generated by MWM. This is illustrated in Figure 4.1, where job arrivals 2 of the real data exhibit a long free gap and a large bursty time, but the synthetic job arrivals 1 do not. This fact contradicts the notion of burstiness where free gaps and tight bursts are required to occur. This contradiction together with the results in Table 4.1 give us the conclusion that MWM cannot capture burstiness of real job arrivals.

![Figure 4.1: An example to illustrate why MWM does not capture burstiness. Each box represents a time duration of 10 seconds and the value in the box is the number of job arrivals in that duration.](image)

### 4.3 Data Modeling and Synthesis of Standard MWM

MWM works through two fundamental procedures, so-called data modeling and synthesis. To model job traffic, the data modeling procedure will receive a real job arrival rate process as its input and train on the input to obtain a number of parameters. These parameters are then transferred to the synthesis procedure to generate a synthetic job arrival rate process.

The data modeling procedure is illustrated in Figure 4.2. This procedure represents the rate process input as a series of scaling coefficients $\{c_{j+1}\}$ at scale $j + 1$. Starting with $\{c_{j+1}\}$, MWM uses Eq. (4.1) and Eq. (4.2) to calculate a series of scaling coefficients $\{c_j\}$ and a series of wavelet coefficients $\{d_j\}$ at scale $j$. Then $\{d_j\}$ is taken into account to calculate the beta parameter $p_j$ according to Eq. (4.7) and Eq. (4.8). This procedure is repeated for smaller scales until the smallest scale ($j = 1$) is
reached. Finally, MWM produces the mean $\mu_c$ and the standard deviation $\sigma_c$ of $\{c_1\}$ as well as the vector of beta parameters $\vec{p}$ whose components are the beta parameters calculated at each scale.

![Diagram of data modeling procedure of MWM](image)

**Figure 4.2:** The data modeling procedure of MWM.

Taking $\mu_c$, $\sigma_c$ and $\vec{p}$ as inputs, the synthesis procedure in Figure 4.3 starts at the smallest scale by generating a series of $N$ scaling coefficients $\{c_1\}$ as follows

$$c_{1,k} = \mu_c + \sigma_c \times \text{randn}, \text{ with } 0 \leq k \leq N - 1, \quad (4.9)$$

where $\text{randn}$ generates a random value that is normally distributed with mean 0 and standard deviation 1. Then the synthesis procedure yields $\{A_1\}$ using the beta distribution with the parameter $p_1$, the first component of $\vec{p}$. After that, a series of wavelet coefficients $\{d_1\}$ is obtained by Eq. (4.5) and is used together with $\{c_1\}$ to calculate scaling coefficients at the next higher scale $\{c_2\}$ with Eq. (4.3) and Eq. (4.4). This process is repeated for higher scales and is stopped after the last component of $\vec{p}$ is used. The final output of the synthesis procedure is a series of scaling coefficients at the highest scale $\{c_M\}^3$, which is used as a rate process in [46].

## 4.4 Modification of MWM

As shown in Section 4.2, MWM cannot capture burstiness in real job traffic because no zero value is found in its rate process. In this section we will show why MWM does not generate zeroes and how we can modify MWM to obtain zero values.

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$^3$From now on, $M$ stands for max scale, meaning the highest scale.
We answer these questions by two observations. First, we have from Eq. (4.3) and Eq. (4.4) that $c_{j+1,2k} = 0 \iff d_{j,k} = -c_{j,k}$ and $c_{j+1,2k+1} = 0 \iff d_{j,k} = c_{j,k}$. According to Eq. (4.5), these conditions are satisfied in case $A_{j,k} = -1$ or $A_{j,k} = 1$. However, we cannot obtain this with a beta distribution because its random variable is only distributed over the range $(-1, 1)$, and so all scaling coefficients $c_{j,k}$ are different from 0. This reason also explains why MWM has the strictly positive instead of non-negative feature as we desire. This shortcoming is also mentioned by the authors of MWM [85], where they note that the choice of a beta distribution for multipliers $A_{j,k}$ is not essential. Therefore, it should be dependent on the applied data to adapt the multipliers $A_{j,k}$ at each scale.

As a second observation, from Eq. (4.3), Eq. (4.4) and Eq. (4.5), we have $c_{j,k} = 0 \Rightarrow d_{j,k} = 0 \Rightarrow c_{j+1,2k} = c_{j+1,2k+1} = 0$. From this observation, we conclude that if we have, at a certain scale $j$, one scaling coefficient $c_{j,k}$ equal to 0, we will have at scale $j + n, 0 \leq n \leq M - j$, $2^n$ contiguous scaling coefficients equal to 0, as illustrated in Figure 4.4.

With this conclusion, we can control accurately the percentage of zero values in the series of scaling coefficients at the highest scale $\{c_M\}$ which is considered as a rate process. Let $m$ be the number of scaling coefficients at scale $j$ and let $m_0$ be the number of scaling coefficients at scale $j$ that are equal to 0. Because each scaling coefficient $c_{i,k}$ generates two scaling coefficients $c_{i+1,2k}$ and $c_{i+1,2k+1}$, the number of scaling coefficients is $2m$ at scale $j + 1$, $4m$ at scale $j + 2$, and so on. We conclude that the numbers of scaling coefficients at scales $j, j + 1, \ldots, M$ are $m, 2m, \ldots, 2^{M-j}m$, respectively. Similarly, because each zero scaling coefficient generates two zero scaling
coefficients at the next scale as shown in Figure 4.4, we conclude that the numbers of zero scaling coefficients at scales \( j, j + 1, \ldots, M \) are \( m_0, 2m_0, \ldots, 2^{M-j}m_0 \), respectively. Consequently, the fractions of zero values at scales \( j, j + 1, \ldots, M \) are the same and equal to \( m_0/m \). Thus, we can control the percentage of zero values in the final rate process represented by \( \{c_M\} \) via handling scaling coefficients at any scale. However, we should not do this at scale 1 because \( \{c_1\} \) is generated based on \( \mu_c \) and \( \sigma_c \) to fit the marginal distribution. Rather, we should control the percentages of zero values in \( \{c_2\}, \{c_3\}, \ldots, \{c_M\} \).

**Figure 4.4:** Generating zeros with modified MWM.

In addition to controlling the percentage of zero values in \( \{c_M\} \), it is also important to control the way these zero values occur in \( \{c_M\} \). For example, if real data contain 100 zero values that are consecutive, we should also produce 100 consecutive zero values. If we produce 100 zero values that are not consecutive, the idle period with no job arrival of synthetic data will be different from that of the real data and hence the produced burstiness is also different. In our study, this kind of distribution of zero values is controlled by calculating carefully the number of isolated zero values to be produced at each scale \( j = 2, \ldots, M \). Each isolated zero value at a scale will expand by a factor of 2 at the next scale to a string of zeroes of the required length at \( \{c_M\} \).

In summary, we obtain \( z\% \) zero values in the final rate process by generating \( z_2\% \) isolated zero values in \( \{c_2\} \), \( z_3\% \) isolated zero values in \( \{c_3\} \), \ldots, and \( z_M\% \) isolated zero values in \( \{c_M\} \) in such a way that \( z = \sum_{j=2}^{M} z_j \). Now, we need to answer the question of how to calculate \( z_j \) for \( j = 2, \ldots, M \) so that we achieve a distribution of zero values in \( \{c_M\} \) similar as in real data. As we already found above, if at a certain scale \( j \) we have \( c_{j,k} = 0 \), we will at scale \( j + n \) have \( 2^n \) contiguous scaling coefficients with value 0. In other words, for each contiguous series of \( r \) zero value scaling coefficients in \( \{c_M\} \), we deduce that there is an isolated zero value in \( \{c_j\} \), where \( j = M - \log_2 r \). Based on this idea, we propose the following steps to calculate \( z_j \):

1. Calculate the zero percentage \( z \) in the real job rate process \( \{data\} \).

2. Compute the lengths of the maximal contiguous sequences of zeroes in \( \{data\} \) to obtain a series \( \{r\} \). For example, if \( \{data\} = \{0.01, 0.02, 0, 0, 0, 0.09, 0, 0, 0, 0.02, 0.03, 0\} \), then \( \{r\} = \{4, 3, 1\} \).
3. \( \forall r_k \in \{r\} \), let \( s_k = M - \log_2 r_k \) to obtain a series of scales \( \{s\} \).

4. For each \( j = 2 \) to \( M \), let \( z_j = f_j \times z \), where \( f_j \) is the fraction of occurrences of \( j \) in \( \{s\} \).

Now we have determined the fraction of zero values to be generated at each scale, but there is still the question of how to do the generation. In order to obtain \( z_j \% \) zero values in \( \{c_j\} \), we will generate \( \{A_{j-1}\} \) with \( \text{length}(\{c_j\}) \times z_j \) values 1 or −1: if \( A_{j-1,k} = 1 \Rightarrow c_{j,2k+1} = 0 \), otherwise \( A_{j-1,k} = -1 \Rightarrow c_{j,2k} = 0 \).

Although our method in theory can assure the percentage of zero values in a synthetic workload to be exactly the same as in a real trace, they can be slightly different in practice for the following reason. If we have \( \text{length}(\{c_1\}) = m \), the length of the synthetic workload is \( m \times 2^{M-1} \). In practice, the length of the real trace does not have a so-called “power-of-two-like” shape. Rather, its length is cut off in the data modeling procedure as well as in the first of the four steps above to get a “power-of-two-like” shape. For example, if the length of the real trace is 52, it is cut off to 48 = 3 \( \times \) 2^4.

![Figure 4.5](image)

**Figure 4.5:** The autocorrelation functions of real and synthetic rate processes in case of LAN with different time scales.

### 4.5 Experimental Results

We will present in this section our experiments to evaluate the quality of our job arrival model. The evaluation is done by comparing the synthetic job arrivals generated by the model with those in real traces. Workload features including long range dependence and temporal burstiness as well as the marginal distribution of job interarrival times are taken into evaluation.
4.5. Experimental Results

4.5.1 Long Range Dependence

Since we represent job arrivals by a rate process which depends on a time scale $T$, we consider several time scales in this experiment to compare our model with the data of a grid (NOR) and a cluster (LAN). The LRD of a stochastic process can be visually observed via its autocorrelation function (ACF). We draw in Figure 4.5 the ACFs of the rate processes of LAN and the synthetic rate processes generated by our model. Furthermore, we also quantify LRD by estimating the Hurst parameter with the estimate approach described in Section 3.2 and show results in Table 4.2. From Figure 4.5 and Table 4.2, we see that our job arrival model gives a good result because LRD is controlled well by Eq. (4.7) and Eq. (4.8). Moreover, the model is also stable over different time scales.

**Table 4.2: LRD of job arrivals via estimating the Hurst parameter.**

<table>
<thead>
<tr>
<th>T (s)</th>
<th>900</th>
<th>1800</th>
<th>3600</th>
<th>7200</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOR</td>
<td>0.86 ± 0.07</td>
<td>0.89 ± 0.07</td>
<td>0.88 ± 0.08</td>
<td>0.84 ± 0.08</td>
</tr>
<tr>
<td>Model</td>
<td>0.79 ± 0.06</td>
<td>0.87 ± 0.14</td>
<td>0.85 ± 0.14</td>
<td>0.81 ± 0.11</td>
</tr>
<tr>
<td>LAN</td>
<td>0.80 ± 0.03</td>
<td>0.81 ± 0.03</td>
<td>0.78 ± 0.01</td>
<td>0.81 ± 0.02</td>
</tr>
<tr>
<td>Model</td>
<td>0.81 ± 0.05</td>
<td>0.87 ± 0.16</td>
<td>0.83 ± 0.11</td>
<td>0.78 ± 0.12</td>
</tr>
</tbody>
</table>

**Table 4.3: The coefficient of variation of interarrival processes.**

<table>
<thead>
<tr>
<th>Time scale (s)</th>
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<th>1800</th>
<th>3600</th>
<th>7200</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOR</td>
<td>33.14</td>
<td>33.14</td>
<td>33.14</td>
<td>33.14</td>
</tr>
<tr>
<td>MWM</td>
<td>5.09</td>
<td>4.56</td>
<td>5.28</td>
<td>4.60</td>
</tr>
<tr>
<td>Model</td>
<td>24.65</td>
<td>25.81</td>
<td>25.77</td>
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<tr>
<td>LAN</td>
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<td>MWM</td>
<td>2.93</td>
<td>2.48</td>
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<tr>
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<td>13.32</td>
<td>13.79</td>
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</tr>
</tbody>
</table>

4.5.2 Temporal Burstiness

We measure temporal burstiness based on calculating the coefficient of variation of an interarrival process, which is converted from a rate process using the integrate-and-fire algorithm [107]. Calculated results in Table 4.3 confirm that our model controls
temporal burstiness better than MWM. In addition, we also show the complementary cumulative distribution functions (CCDFs) of the real and synthetic interarrival processes. As we can see from Figure 4.6, our job arrival model is able to capture temporal burstiness well since its interarrival times are fitted nicely to those of the real data.

![Figure 4.6: The complementary cumulative distribution functions of the interarrival times of NOR, LAN and the model.](image)

4.6 Summary

Job traffic plays a crucial part in workloads of parallel systems and grids. Therefore, modeling job arrivals is essential to provide realistic workloads for study on performance evaluation of scheduling. Towards this end, this chapter introduced a new job arrival model that can capture two observed features of real job traffic, namely long range dependence and temporal burstiness. The model was developed based on the multifractal wavelet model [46, 85]. Experimental results showed that the developed job arrival model can accurately control the temporal burstiness degree and can produce long range dependence well. Moreover, the synthetic job arrival process generated by the model also fits the marginal distribution nicely. However, the daily cycle feature is a shortcoming of this job arrival model which should be investigated in future work.