A flavour
of family symmetries
in a family
of flavour models

Proefschrift

ter verkrijging van
de graad van Doctor aan de Universiteit Leiden,
op gezag van de Rector Magnificus prof. mr. P.F. van der Heijden,
volgens besluit van het College van Promoties
te verdedigen op dinsdag 21 februari 2012
klokke 15.00 uur

door

Reinier de Adelhart Toorop

Geboren te Amsterdam in 1984
Promotiecommissie
Promotor Prof. dr. Jan-Willem van Holten (Nikhef en Universiteit Leiden)
Co-promotor Dr. Federica Bazzocchi (SISSA, Trieste, Italië)
Overige leden Prof. dr. Daniël Boer (Rijksuniversiteit Groningen)
Dr. Alexey Boyarski (Universiteit Leiden)
Prof. dr. Éric Elie (Universiteit Leiden)
Prof. dr. Ferruccio Feruglio (Universiteit van Padua, Italië)

Het werk beschreven in dit proefschrift is onderdeel van het onderzoeksprogramma van de stichting Fundamenteel Onderzoek der Materie. Deze stichting wordt financieel ondersteund door de Nederlandse Organisatie voor Wetenschappelijk Onderzoek.

Omslagontwerp Rozan Vroman
Casimir PhD series Delft-Leiden 2012-3
Contents

Table of Contents i

List of publications iii

1 Introduction 1

1.1 The Standard Model ........................................ 2
1.2 Reasons why the Standard Model is incomplete .............. 4
1.3 Theoretical reasons to extend the Standard Model .......... 9
1.4 Flavour symmetries ........................................ 14
1.5 Outlook of this thesis ..................................... 18

2 Fermion masses in the Standard Model and beyond 19

2.1 The one family Standard Model ............................. 19
2.2 The three family Standard Model ............................ 28
2.3 Fermion masses in family symmetric models ................. 36
2.4 The Altarelli–Feruglio model ............................... 40
2.5 Conclusions of the chapter ................................. 47

3 Mixing patterns of finite modular groups 49

3.1 Introduction ............................................... 49
3.2 Finite modular groups and their representations .............. 50
3.3 Lepton mixing patterns from $\Gamma_N$ ........................ 56
3.4 Four interesting mixing patterns ............................ 66
3.5 Conclusions of the chapter ................................ 68

Appendices to chapter 3 69

3.A The alternating group $A_4$ ................................. 69
3.B The symmetric group $S_4$ ................................ 74
3.C Tables of Abelian subgroups for $A_5$, $PSL(2,7)$, $\Delta(96)$ and $\Delta(384)$ .............. 77

4 The interplay between GUT and flavour symmetries in a Pati–Salam $\times S_4$ model 83

4.1 Introduction ............................................... 83
4.2 A detailed look on patterns in the elementary fermion masses .... 84
4.3 Bimaximal versus tribimaximal mixing ........................ 87
4.4 The Grand Unified Theory of Pati and Salam .................. 91
4.5 The flavour model building ................................ 93
4.6 Fermion mass matrices at leading order ...................... 96
4.7 Fermion mass matrices at higher orders ..................... 101
4.8 The flavon scalar potential ................................ 106
4.9 Higgs scalar potential .................................... 108
4.10 Running of the Yukawa couplings ........................... 113
4.11 Neutrino Phenomenological Analysis ......................... 118
4.12 Conclusions of the chapter ................................ 120
List of publications

1. Reinier de Adelhart Toorop, Ferruccio Feruglio and Claudia Hagendorn
   *Discrete Flavour Symmetries in Light of T2K*
   (Chapter 3)

2. Reinier de Adelhart Toorop, Ferruccio Feruglio and Claudia Hagendorn
   *Finite modular groups and lepton mixing*
   (Chapter 3)

3. Reinier de Adelhart Toorop, Federica Bazzocchi and Luca Merlo
   *The Interplay Between GUT and Flavour Symmetries in a Pati-Salam x S4 Model.*
   (Chapter 4)

4. Reinier de Adelhart Toorop
   *Family physics with S4 and Pati-Salam*
   Proceedings of the Erice School of Nuclear Physics 2009

5. Reinier de Adelhart Toorop
   *The interplay between grand unified and flavour symmetries in a Pati-Salam x S4 model*
   Proceedings of Pascos 2010

6. Reinier de Adelhart Toorop, Federica Bazzocchi, Luca Merlo and Alessio Paris
   *Constraining Flavour Symmetries At The EW Scale I: The A4 Higgs Potential*
   (Chapter 5)

7. Reinier de Adelhart Toorop, Federica Bazzocchi, Luca Merlo and Alessio Paris
   *Constraining Flavour Symmetries At The EW Scale II: The Fermion Processes*
   (Chapter 5 and subsection 2.2.5)

8. Reinier de Adelhart Toorop
   *Flavour symmetries at the EW scale*
   Proceedings of Discrete 2010

9. Reinier de Adelhart Toorop, Federica Bazzocchi and Stefano Morisi
   *Quark mixing in the discrete dark matter model*
   (Subsections 5.10.4 and 5.12.4)