Chapter 1

Introduction

Condensed matter physics concerns the collective behaviour of a large number of particles that organize themselves into an ordered medium. It is the qualification 'ordered' that sets the field apart from the study of gases or simple liquids. Thus, the primary business of a condensed matter physicist is to discern when a system is ordered, and what sets apart one ordered medium from the other. One step further, she could investigate how one state can transform into another, for instance a liquid freezing into a solid or a paramagnet going over into a ferromagnet. This is the study of phase transitions, and in its modern incarnation is over 100 years old. The traditional way of thinking is always about obtaining a more ordered state (solid) from a less ordered state (liquid). This is accompanied by a *lowering* of the external or internal symmetry of the system.

It had been realized first in material science that metals start to degrade in their structural integrity but also their electronic properties by the presence of *defects*: aberrations in the regularity of the crystal lattice. If it is just a missing or superfluous particle, it is called an interstitial, and it will have limited effect on the overall properties of the material. Conversely, if the defects are *topological*, their influence has bearing throughout the whole system. Therefore those are usually confined in combinations whose topological effects cancel out each other.

The topological defects are sources of disorder. Letting more and more of these topological defects enter the system amounts to putting more disorder into it. It is tempting to continue this reasoning by stating that also the transitions into a more disordered phase are therefore caused by topological defects. That is indeed the principal perspective in this thesis. The defects are then the agents that *restore* symmetry in the system.

The alternative of focussing on disordering instead of ordering of matter is known as a *duality*. Each point of view is equally valid, and one can freely switch between the one or the other, in the ideal case via a mathematical isomorphism. Practically speaking, there are often advantages of preferring one approach over of the other, and it is therefore useful to develop both the traditional and the dual methods in order to maximize the size of the toolbox. Basically, the canonical formalism works well when the system is mostly ordered, the dual formalism when it is heavily disordered. But it is not just pragmatism that encourages the dual way of thinking; it also reveals deeper truths about the physical principles that dictate the effective collective behaviour in many-body systems.

This thesis fully embraces the dual side, and expands its applicability to higher dimensions where it was mostly restricted to the spatial plane. Let us now first get accustomed to dualities by some famous examples, in order to appreciate the problems we wish to address. Along the way we encounter many concepts that will be used copiously throughout this work.

1.1 Kramers–Wannier duality and its extensions

1.1.1 Kramers–Wannier duality

It is fitting that the first such duality was discovered in the simplest statistical physics problem: the Ising model on a square lattice. Kramers and Wannier noted that the partition function in terms of the variables $s_i \in \{-1,+1\}$ on lattice sites i, as a function of inverse temperature β, could be rewritten in terms of variables $\sigma_{\langle ij \rangle} \in \{-1,+1\}$ on the lattice links $\langle ij \rangle$ as a function of the dual inverse temperature $\tilde{\beta} \sim 1/\beta$ [1–3]. The Ising model maps to another Ising model, yet with a different coupling constant. As such, it is a mathematical identity; however, it hints to an alternative understanding of the physical principles.

This is illustrated in figure 1.1(a). The black, solid lines are the real lattice with on each lattice site arrows ("spins") that can point in two directions. Then on each link between two sites we can define a dual spin (blue) that points up if the neighbouring sites are parallel, and down if they are anti-

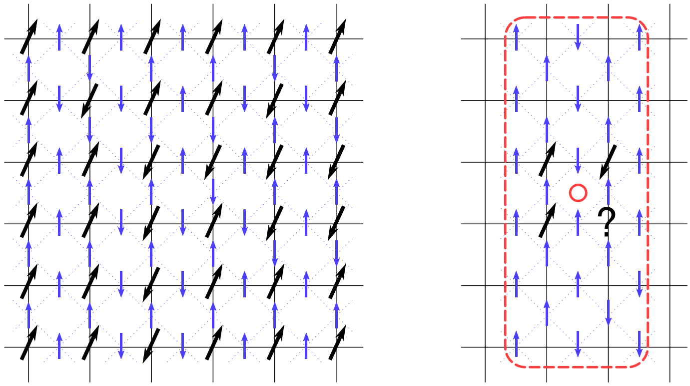


Figure 1.1: Ising spins on the square lattice (black). On each link of the lattice we can define a dual spin (blue) that points up if the two neighbouring spin are aligned and down when they are anti-aligned. The reciprocal lattice is shown in blue, dotted lines. (a) A typical configuration of spins. Note that the number of dual down spins around each plaquette is always even. (b) If we insist on having an odd number of dual down spin in the plaquette with the red circle, the original spins become frustrated. The frustration can be seen at the perimeter (dashed red), which also has an odd number of down spins.

parallel. Except for the initial condition, the dual spins contain the same amount of information as the real spins. This is the archetypical example of duality.

Now things get really interesting. While for the real spins is it perfectly fine to flip any one, possibly changing the energy but not violating any rules, notice that the number of dual spins that are pointing down around one plaquette is always even. Purely due to the definition in terms of the original spins, there is a constraint or conservation law for the dual spins. What happens if we try to break this law? This is pictured in figure 1.1(b). The red circle indicates a plaquette with only one dual down spin. If we try to recreate the original spins, starting bottom left, we see that there is no way to decide where to put the final spin around this plaquette. This plaquette is therefore said to be *frustrated*.

The frustrated plaquette is our first example of a topological defect: if one counts the number of down spins around the perimeter of our dual lattice, the number of dual down spins is also odd. The influence of the topological defect is felt all the way to the edge of the system.

The appearance of a constraint for the dual variables, and the ill-definedness of the original variables when violating this constraint is a very general principle, and one could say that this lies at the heart of all that will be discussed in this thesis.

Another recurring theme is that the dual coupling constant $\tilde{\beta}$ is inversely proportional to the original coupling constant β. This is therefore known as a strong/weak duality or S-duality. One often uses perturbation theory to be able to make calculations at all, and therefore the duality proves its worth in the high-temperature regime where $\tilde{\beta}$ is small, and can be used as the expansion parameter. This already indicates that the disordered state is actually dually ordered.

1.1.2 Ising gauge model

The basic duality of the square lattice Ising model can be extended in several ways. The energy of the Ising model above is invariant under flipping all spins at the same time—a global transformation—but local spin flips will in principle change the energy of the state. However, consider plaquette variables that count whether that plaquette has an even or odd number of dual spins down around it. Flipping all dual spins emerging from a lattice site will leave those plaquette variables invariant: the evenness does not change under such local spin flips. Instead of a global we have now a local or gauge symmetry. Any model built out of these plaquette variables will therefore have a gauge symmetry. This was first investigated by Wegner [2–4], and is called Ising gauge model or $\mathbb{Z}_2$ lattice gauge theory.

The Ising model on the square lattice is dual to another Ising model on the reciprocal square lattice. This *self-duality* is coincidental. Interestingly the Ising model on a three-dimensional cubic lattice is dual to an Ising gauge model on the reciprocal (cubic) lattice. This is known as a global/local duality: the global symmetry turns into a local symmetry for the dual variables. Also this phenomenon is a key ingredient of this thesis.

1.1.3 XY-model and the superfluid

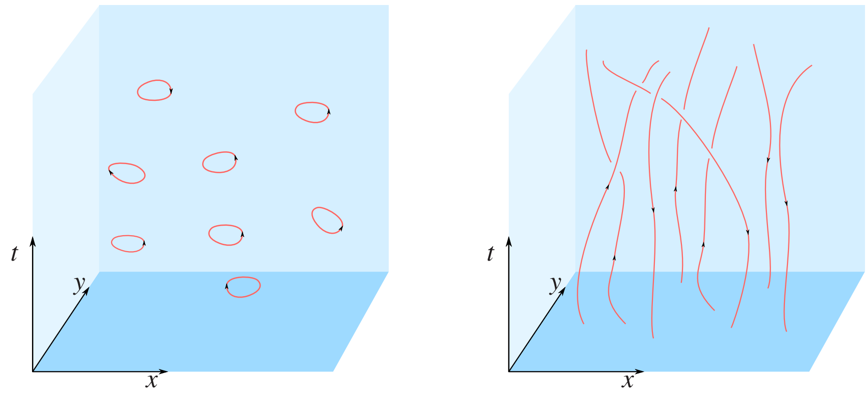
In the Ising model, the real, dual and plaquette variables take one out of two values only. This can be extended to a larger number of discrete values, but moreover to a continuous set, in particular a real or complex number. If in our picture of figure 1.1(a) the arrows on each site are of fixed length but can rotate freely in the xy-plane, then with nearest-neighbour interactions this is known as the phase-only model or XY-model. The XY-model is invariant under rotating all spins over a fixed angle α, i.e. under global $U(1)$-rotations $\mathrm{e}^{\mathrm{i}\alpha}$. An unordered XY-system has the arrows pointing in random directions whereas when their orientation is correlated over considerable length scale, it is an ordered system.

Since we are now dealing with continuous variables, we are equipped with the concept of smoothness, which shall turn out to be an essential property. Even in the ordered system, there will now be small fluctuations in the direction of the arrows around their equilibrium position, which were unavailable in the discrete systems above. Similarly, when we disturb the ordered system from the outside, this disturbance will propagate through the ordered system as the equivalent of a sound wave. This is called a Nambu–Goldstone mode, and it communicates the rigidity of the order. Goldstone modes are present in any ordered system of continuous variables—this is the famous Goldstone theorem [5–7]. Using a similar duality transformation as above, the Goldstone modes are expressed as dual gauge field, so it is a global/local duality. Here we have the natural interpretation of gauge fields are force carriers (cf. a photon), conveying the rigidity of the order parameter.

The XY-model in the continuum limit is the simplest model that describes the freely propagating zero-sound mode in a superfluid, where the arrows represent the superfluid phase variable. A superfluid in a rotating vessel will show the formation of vortices, which are in fact the topological defects. In the XY-model a vortex is a configuration where the direction of the phase changes by a multiple of 2π when traversing a closed contour. Therefore the vortices are the cause of the disordering of the phase rigidity. Surely when the external angular momentum gets too large, the superfluid will be destroyed entirely by the induced vortices.

1.1.4 Vortex unbinding transitions

But in the duality viewpoint, also thermal (or quantum) fluctuations cause spontaneous formations of small vortex loops. These loops grow with rising temperature, and then the thermal phase transition is also understood as



(a) small vortex loops (b) vortex blowout

Figure 1.2: The phase transition in terms of vortex world lines. (a) In the ordered phase, the dual coupling constant (the vortex line tension) is large, such that it is very costly to form vortex lines. In spacetime they only appear as small loops of creation and annihilation of vortex–anti-vortex pairs. (b) Increasing disorder amounts to lowering the dual coupling constant, so that the vortex loops grow. Across the phase transition the loops have grown to the system size, and the proliferate throughout the whole system. This picture should always be kept in mind when reading this thesis.

the demise of order due to vortices.

This is best understood pictorially. At low temperatures, the formation of vortex pairs will be heavily suppressed, and only small spacetime loops of vortex–anti-vortex pairs will appear (Fig. 1.2(a)). But as temperature rises, it becomes entropically favourable to let the vortex lines grow—this is the dual equivalent of the increasing population of excited states with phase orientations different than the purely ordered ground state. At the critical temperature, these loops grow to be of the system size, and energetically the vortex lines can now permeate the system freely (Fig. 1.2(b)). The phase (the arrows) is completely disordered. This is referred to as the “vortex blowout” or the “tangle of vortex world lines” and the phase transition is the “vortex unbinding transition”.

In principle, the discrete model like the Ising models also undergo a defect-unbinding transition, but the effect is more striking in the continuous models: in 1966 Mermin and Wagner showed that a two-dimensional magnetic system will always disorder due to thermal fluctuations; this actually holds for any two-dimensional system, and is known as the Mermin–Wagner–Hohenberg–Coleman theorem [8–10]. Therefore it came as a surprise when Kosterlitz and Thouless, and independently Berenzinskii, showed that there is a vortex unbinding transition in the two-dimensional XY-model [11, 12]. This is commonly explained as: "this phase transition is not a order–disorder phase transition". I will have some comments on this issue in the conclusions, chapter 7. This theme was expanded to external (spatial) symmetries by Nelson, Halperin and Young, which had the most impact for classical liquid crystals [13, 14]. In this context one speaks of defect-unbinding transitions or defect-mediated melting.

Vortices are pointlike in two spatial dimensions, and the mnemonic for the BKT transition is, also in 2+1 dimensions, the picture of Fig. 1.2. But a vortex is a line in three spatial dimensions. Still the phase transition cannot be anything different than the disordering of the phase variable. The question arises if one can generalize the vortex blowout when the vortices are not points but extended objects. We will show in chapter 3 that that is indeed the case.

1.1.5 Phase transitions with gauge fields

If one were to promote the global $U(1)$-symmetry of the superfluid to a local or gauge symmetry, this necessitates the introduction of a vector-valued gauge field. This is precisely the situation in the superconductor, where the massless photon field A_μ, a vector field with gauge symmetry, couples to the superconducting phase, the Goldstone modes. The gauge field then undergoes the famous Anderson–Higgs mechanism [15], and becomes massive. As a result, the photon field is expelled from the superconductor, and there are only massive, gapped excitations in the medium. Also the interactions between vortices in the superconductor become short-ranged, which shows in the correlation functions of the *dual* variables. The simplest model that features the Higgs mechanism is the Abelian-Higgs model, and in 2+1 dimensions this is precisely how the vortex unbinding transition works. Therefore, vortex duality often goes by the name of Abelian-Higgs duality, and the disordered XY-phase is in this context a "dual superconductor".

Now a field with a local symmetry cannot undergo a phase transition (spontaneous symmetry breaking) by itself, this is Elitzur's theorem [16].

Therefore it seems natural to argue that, in the superconductor, first the 'superfluid' order is established, and secondarily the gauge field follows the symmetry breaking by coupling to the Goldstone mode. That is indeed the point of view we will take in this thesis, and will even prove to be more than just an equivalent description when identifying the massive modes in the 3+1 dimensional disordered superfluid and superconductor (chapters 3 and 5).

1.1.6 Going quantum

In recent years there has been increasing interest in phase transitions due to the disordering effect of quantum fluctuations instead of thermal fluctuations. Such phenomena are called *quantum phase transitions* [17].

It has long been noted (e.g. by Feynman [18]) that the quantum mechanical weight factors in the path integral are just like Boltzmann factors if one transforms to imaginary time $t \to \mathrm{i}\tau$. As such, as a dynamical quantum field theory in D dimensions is easily mapped to a statistical mechanics problem in $D+1$ dimensions, where the role of time is played by temperature. This correspondence was originally used to carry over knowledge from thermal physics to quantum many-body systems; for instance the textbook by Mahan carries out many calculations at a finite temperature, to let temperature go to zero at the very end [19].

In quantum phase transitions this is taken one step further. It is not just equilibrium physics, but also phase transitions that are closely mimicked. One now has a coupling constant that represents the strength of quantum fluctuations, and which is therefore the analogue of the temperature. For instance, in high-temperature superconductors it is the number of free charge carriers that plays this role (see §5.1.2). Despite the numerous similarities, quantum phase transitions are more intricate and eventually richer than thermal ones.

This is most prominently seen by the phenomenon of spontaneous symmetry breaking. In second-order phase transitions, the system spontaneously chooses one of many ground states, for instance one particular direction of the $U(1)$-spins. It will cost a lot of energy to change this order: it is rigid. In classical, thermal systems, only one direction can be chosen. But in quantum systems, any superposition of ground states is just as valid. Therefore, the quantum system allows for much more interesting ordering patterns. Also the excitation spectrum is affected in a similar way.

In much of what follows, the quantum nature of the phase transition is not really emphasized. One needs to keep in mind though, that the system under investigation are inherently quantum mechanical in nature, and they are dominated by the Goldstone modes arising from quantum rigidity. Only in chapter 6 will be make a sharp distinction between classical and quantum systems, and discussion on the classicalness of quantum system takes place in §7.2.4.

1.1.7 Other dualities

Up to know we have only discussed the simplest dualities: strong/weak and local/global dualities in the Ising model and in $U(1)$-symmetry, which is the simplest continuous symmetry, and is Abelian, i.e. two consecutive symmetry transformations commute.

Higher symmetry groups, especially non-Abelian groups such as $SU(2)$ for spins, are much more complicated; in particular, the "braiding" of vortex (world) lines follows the symmetry structure, and may also be non-Abelian. While this opens up many interesting avenues such as in the fractional quantum Hall effect and topological quantum computation, it leads to ambiguities in defining the tangle of vortex world lines as the disordered state. There has be some progress on the mathematical side using quantum groups or Hopf algebras [20–26].

Dualities are prevalent in string theory, in fact they are one the appealing mathematical features of that framework. In this context, the strong/weak duality is called S-duality. In several instances it relates one string theory to another. The underlying principle is the same: local variables one side turn into extended or topological objects on the other side, which unbind as the dual coupling constant grows smaller.

Almost all of what follows focussed on the Abelian $U(1)$-symmetry. Only in chapter 6 we will passingly address the space groups of general relativity and of elasticity. Trouble is avoided by focussing on the translations subgroup, which is Abelian.

1.2 The road to higher-dimensional vortex duality

The application of the Abelian-Higgs duality to many-body physics had been identified and studied for over three decades [27–36]. Even though it is still unfamiliar to many researchers in the field, once the basic concept has been grasped, the framework is quite simple and rather elegant. One reason why it remains to reside in relative obscurity may be that it has not really led to new predictions, but had been confined to placing known results in a different light.

Furthermore, vortex duality has been mostly restricted to 2+1 dimensions. The reason, which we shall discuss extensively in §2.2, is that in that case the vortices act just as point particles do: in spacetime they trace out world lines, and we capture those in a regular quantum field theory. In higher dimensions, the vortices becomes extended objects like lines or surfaces. As long as they are distant from each other (strong coupling limit of the vortices), the duality works fine: dual gauge fields mediate interactions between individual vortex sources. The dual gauge field is just the Hodge dual of the Goldstone scalar field, i.e. a free $d-2$-form field, and the dynamics of such a free tensor field is well known (see e.g. [37]).

However trying to effect the phase transition is really difficult. One wishes to form a condensate of the extended vortex world sheets, in which their number is no longer conserved. In other words: we are looking for a quantum field theory of extended objects. This is the subject of string field theory, and its progress has been severely limited [38, 39]. This was recognized for instance in Ref. [40, §2.5], and therefore not pursued any longer.

It is amusing to trace back how this work was initiated originally. The correspondence between spacetime deformations of general relativity (GR) and elasticity in crystals has been noted by many authors. In recent years, the mathematical physicist Hagen Kleinert has explored this relation in depth by imagining a "world crystal" deformed by topological defects [41, 42]. Then the defects are like sources of curvature and the stress tensor corresponds to the Einstein tensor. He also recognized that the dynamics is slightly off, leading to wrong correlation functions, and tried to solve this with a "floppy world crystal", which is in a sense "looser" than an ordinary crystal. The deeper reason is that even in the continuum limit the crystal remembers that both translational and rotational symmetry are broken, while GR is practically translationally invariant (see §6.2).

A different proposal was put forward by Kleinert and Zaanen [43] that GR does not correspond to a crystal, but to a quantum liquid crystal. In such a material, part of the spatial symmetry is restored by reviving translational invariance. The route to this symmetry restoration is precisely via the unbinding of topological defects, in this case the crystals dislocations. While the claim was made that this should hold for any dimension, the calculation was done in 2+1 dimensions only. However, gravity in 2+1 dimensions is simple, or boring, in the sense that there are no propagating modes—no gravitons. The real magic happens in four spacetime dimensions, where GR predicts two graviton polarizations as massless spin-2 modes. Gravitons have not been detected directly, and a huge effort is currently invested to find them in the form of gravitational waves [44].

Thus, I set out to identify the hydrodynamic modes of quantum liquid crystals in 3+1 dimensions that should correspond to gravitons, building upon the work done by Cvetkovic and Zaanen in 2+1 dimensions [31, 40, 45]. Many parts are readily generalized to higher dimensions, but soon we bumped into the obstacle mentioned above: the unbinding transition of extended topological defects in higher dimensions. Therefore it was necessary to take a few steps back, to really comprehend where the difficulties in the vortex duality lie.

It was fortunate that at about that time, Marcel Franz just published a work on this topic [46], continuing an idea by Rey [47] into the realm of condensed matter physics. Some research had in fact been done, starting with Marshall & Ramond in the context of string theory [48]. These attempts take the dual gauge field as the central object and start from there. Then it is logical to extrapolate the Anderson–Higgs mechanism from vector fields to tensor fields and suggest that the vortex condensate will turn the higher-form gauge field massive. Another approach was taken in Ref. [49].

However, we soon noted that there was a flaw in this argument, which leads to an overcounting in the number of degrees of freedom. In condensed matter physics we often have the advantage that the systems under consideration are accessible in the laboratory, in computer simulations, and by several theoretical approximations. As such, we knew that the vortex duality in the continuum XY-model should eventually reproduce the results of a firmly established lattice model, namely the Bose-Hubbard model. This model has been realized almost perfectly in cold atom experiments [50].

Our guiding principle was therefore to obtain the characteristics of the Mott insulating state (the strong-coupling limit of the Bose-Hubbard model) from the vortex condensate, in particular a doublet of degenerate gapped modes in any dimension. In other words: we needed to generalize those properties of the vortex duality from 2+1 to higher dimensions that carry the information of these massive propagating modes. Naively Higgsing the dual tensor gauge field will not do this for you. We were finally able to perceive that one should focus on the conserved currents rather than on the dual gauge fields, and this enabled a comprehensive generalization of the vortex duality which should hold for any order–disorder transition in condensed-matter systems in any dimension higher than two (chapter 3).

As we struggled through unexplored territory, it became clear that the vortex lines as spacetime world sheets interacting via dual gauge fields contain a huge amount of information that can be extracted directly in the dual language. In condensed matter physics, most work on vortices is related to laboratory-based setups in superconductors and superfluids, and the mathematical niceties of extended defects that play a large role in for instance cosmology and string theory are glossed over or not even acknowledged. Conversely the fact that a vast body of knowledge on vortex lines has been collected does not reflect back on the high-energy community, which is demonstrated by the unwillingness to admit what are called Nielsen–Olesen strings are just relativistic Abrikosov lines, and what is called the Abelian-Higgs model is just relativistic Ginzburg–Landau theory.

In this light, even on the weakly-coupled side of the phase transition, the electrodynamics of Abriksov vortices turns out to be completely contained in the dual, relativistic description of the vortex world sheet. By incorporating the time direction on even footing, the well-known magnetic equations are directly generalized into similar equations for the electric field. It turns out that all basic effects of vortex electrodynamics are captured in a single equation, which is the subject of chapter 4.

The most interesting aspect of the duality is that it is truly dual: the vortex condensate supports vortices of its own, and when these condense we are back to the original weak-coupling phase (see §§2.4.6, 3.4.5). This is not just an enjoyable gimmick, but moreover a true physical prediction. Already present in neutral 2+1 dimensional systems, it is most striking in charged 3+1 dimensional systems. Where the defects in superconductors are Abrikosov vortex lines of magnetic flux, the duality suggests vortex lines of electric current in Bose-Mott insulators. This unexpected result may be directly accessible for experimentalists to find in underdoped cuprate superconductors, and will be investigated in chapter 5.

Now that the vortex duality has be generalized to higher dimensions for the simple $U(1)$-symmetry superfluids and superconductors, we can finally direct it to the original problem of gravitons in quantum liquid crystals. Unfortunately the full calculation is not yet completed to include in this thesis. However, the wisdom acquired in chapter 3 demotes that calculation in terms of gauge fields to be of secondary importance, at least in the relativistic limit. The conserved currents (the stress tensor c.q. Einstein tensor) dictate the physical content of the model, and symmetry considerations do the rest. In fact, the local conservation law of the currents and the emergence of a conserved quantity in the vortex condensate, i.e. the density of the vortex liquid, are in harmony and closely connected. This general principle follows from the duality construction, and is established in condensed matter physics under the name of emergent gauge invariance. In the final chapter 6 we will show that the two gauge principles are indeed opposite sides of the same coin. Then we come full circle by illustrating these emergence phenomena in quantum liquid crystals, and show that a quantum nematic liquid crystal has to correspond to the linearized approximation of gravity, containing the two graviton modes.

We shall start off with one chapter of preliminary material (ch. 2) that collects known results on which the rest of the work is built. In the concluding part (ch. 7) we summarize all obtained results, try to contextualize their impact on condensed matter physics and beyond, and present open questions and new waypoints as directions of research.

1.3 Conventions

vortex duality I shall use the term "vortex–boson duality" throughout this thesis, or "vortex duality" *tout court*. This exactly describes what is happening, and is completely unambiguous. Alternative names are "XY-duality" or "Abelian-Higgs duality". The latter is only applicable in 2+1 dimensions, but quite common in the literature. Furthermore here we dualize the Goldstone boson of the Abelian-Higgs model, whereas in high-energy physics often the

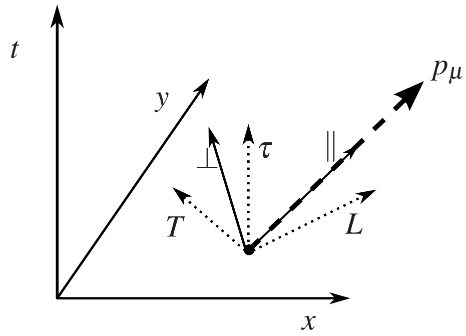


Figure 1.3: We often use two coordinate systems related to the momentum p_μ of the gauge particle. In the (τ, L, T)-system (dotted lines), the temporal direction is preserved, and the spatial ones are separated in longitudinal and transversal. This system is useful in the Coulomb gauge and when Lorentz invariance is broken. In a relativistic context, more appropriate is the $(\|, \perp, T)$-system (solid lines), where the τ and L-directions are rotated so that one is parallel to the spacetime momentum p_μ. This direction $\|$ is also called longitudinal. The spatial-transversal directions are the same as in the previous system. In higher dimensions $D+1$, there are simply more spatial-transversal directions $T_1, \ldots, T_{D-1}$.

gauge field is dualized.

metric In relativistic expressions we use the "spacelike convention" for the Minkowski metric: $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$. The reason is that the spatial parts will carry the same sign as the quantities in common static, non-relativistic expressions, such as the Hamiltonian. We will often work in imaginary time $t \to \mathrm{i}\tau$, with Euclidean metric $\delta_{\mu\nu} = \text{diag}(1,1,1,1)$. Then the integrand in the path integral reads $\mathrm{e}^{\mathrm{i}S/\hbar} \to \mathrm{e}^{-S_\mathrm{E}/\hbar}$ and looks like a Boltzmann factor. For the momentum $\mathrm{i}\partial_\mu \to p_\mu$ we use $p_\mu = (p_\tau, \mathbf{q}) = (\frac{1}{c}\omega, \mathbf{q})$. In imaginary time the frequency here is strictly speaking a Matsubara frequency ω_n, but unless there is room for confusion, we suppress the label n.

Fourier components It is often useful to use coordinate systems related to Fourier components, as shown in figure 1.3.

units Wherever dimensionful quantities are present we express them in SI-units, for which the Ampère–Maxwell law reads,

$$\nabla \times \mathbf{B} - \frac{1}{c}\partial_t \mathbf{E} = \mu_0 \mathbf{J}. \tag{1.1}$$

This is to be compared to this relation in the quite common Gaussian cgs-units,

$$\nabla \times \mathbf{B} - \frac{1}{c}\partial_t \mathbf{E} = \frac{4\pi}{c} \mathbf{J}. \tag{1.2}$$

The reason for this choice is that in relation to experiments it is easier to refer to Ampères than to statcoulombs per second. Additionally it will turn out to be quite useful to keep around the magnetic constant μ_0, as it signals contributions from the Maxwell electromagnetic field as opposed to electric current due to moving charges.

current There will repeatedly appear two kinds of sources or currents in this thesis: the electromagnetic current (density) and the vortex current. Since they do both act as current/sources in the equations, both are represented by some form of the conventional letter J. For clarity, the vortex current will always carry a superscript label $^{\mathrm{V}}$ to distinguish is from the material current in superconductors and Mott insulators. Vortex currents in the superfluid/superconductor are denoted by the Roman symbol J^{V}, and in the Mott insulator by the script symbol $\mathscr{J}^{\mathrm{V}}$.

spacetime dimensions A capital letter "D" will be used when referring to exclusively spatial dimensions, and a small letter "d" when referring to spacetime dimensions. Thus a 2D particle traces out a world line in 2+1d spacetime.