3.1 Introduction

Granular materials are difficult to study in three dimensions because of their opacity -- only their surface is directly visible. There is however a simple technique that gives access to the bulk: index matched scanning (IMS). This technique works by immersing transparent particles in a fluid with the same optical index of refraction. This makes the resulting suspension transparent. The bulk is then visualized by adding a fluorescent dye to the fluid: when the dye is excited with a laser sheet, only the fluid will fluoresce and the particles will appear as dark spots in a bright, well defined cross section of the suspension.

In this chapter we apply the IMS technique to the split-bottom geometry, an ideal system for this purpose due to the rich three-dimensional structure [31--34]. The IMS technique is still under development; the split-bottom flow experiments can serve as a test bed for the validation, of its three-dimensional imaging capabilities. On the other hand, many aspects of the three dimensional microstructure of granular flows in this and other geometries are not known, and we anticipate that IMS will play an important role in elucidating this.

In the following section, we describe how we employ the IMS technique to study granular flows in the split-bottom geometry. The basis of the IMS setup we used to do the flow structure experiments was developed in Wolfgang Losert’s group at the University of Maryland. Initially, the IMS setup in that
3.2 FLOW MEASUREMENTS

Group was designed to study the structure of static granular packs in three dimensions under thermal cycling [58]. Krisztian Ronaszegi adapted it to study shear reversal effects in the split-bottom geometry [59]. In close collaboration with the group, mainly during a two month visit in 2008, we modified the scanner to use it to study the flow of suspensions in the split-bottom geometry, and to do the experiments discussed in this chapter. The complete Maryland setup is described in section 3.2.

In the subsequent sections, we will show that IMS scanning is perfectly suited to study granular flows in the split bottom geometry. Our first goal is to check whether in the slow driving limit, the flow structure of our gravitational suspensions\(^1\) corresponds to the flow features established for dry granular flows in such geometries. We verify this by comparing the flow profiles of the gravitational suspensions in the split-bottom geometry to empirically determined flow profiles of dry granular materials in that same geometry; we find an excellent quantitative agreement. This validates the IMS technique for the study of slow, dry granular flows.

Our second goal is to use the interstitial fluid, necessary for the IMS technique, to our benefit, and explore the rate dependent flow structures that ensue when the driving rate \(\Omega\) is increased. As we will detail in section 3.4, the rate dependent suspension flows tend to Newtonian flows, and the crossover from the granular to the Newtonian regime is well captured by a previous scaling theory proposed by Cassar in Ref. [60]. The theory predicts that the behavior of the suspension is Newtonian near the onset of this rate-dependent regime. We verify the theory by numerically solving the steady state Navier-Stokes equation in the split-bottom geometry for a Newtonian rheology. The flow profiles obtained are then compared to the measured flow profiles, and we find a good agreement between the two.

3.2 Flow Measurements with Index Matching Scanning

The IMS setup we use to investigate suspension flows in the split-bottom geometry was developed in Wolfgang Losert’s group at the University of Maryland. This IMS was initially developed to study static granular packings [58], for which the imaging speed does not have to be fast; its imaging rate is limited to 3 images per second. That imaging rate is however sufficiently fast to probe steady state flows in two dimensional cross sections inside sheared suspensions, for flow speeds up to 2.5 cm/s. We will refer to the setup described in this chapter

\(^1\)We use the name gravitational suspensions to denote suspensions in which the density of the fluid and particles are not matched.
with setup M from now on; this will also facilitate distinguishing it from another, faster, IMS setup developed in Leiden and described in chapter 4.

3.2.1 Index Matching Setup: Version M

Figure 3.1: Setup M. (a) A schematic side view. 1: Translation stage; 2: Laser; 3: DC servo motor; 4: PMMA box; 5: Laser sheet; 6: Black curtains. (b) Shows a three dimensional representation of the flow cell, with the disk at the bottom. (c) Shows a typical image. The coordinate system used throughout the text is depicted in (d). In (e) the bottom, with the disk mounted in the recession is shown. The motor drives the disk from below; a lip seal (in green) ensures the cell is water tight.

The IMS scanner developed in Wolfgang Losert’s group at the University of Maryland is depicted schematically in Fig. 3.1. We give the details of the setup
3.2. FLOW MEASUREMENTS

below:

Laser -- The scanning laser sheet is generated by a Stocker Yale (SNF-501L-635-5-35-30) 635 nm, 25mW laser. The laser has optics mounted directly on it to generate a laser sheet and to be able to focus the laser sheet. The focussing is used to adapt the distance where the sheet is at its thinnest. Typically the laser is placed at a distance of the split-bottom cell of $\sim 35$ cm; this distance is referred to as the projection distance. The sheet is about 100 micron thick here. Laser beams display beam divergence: they are not uniform in thickness but become thicker with increasing distance to the projection distance. The distance at which the thickness is $\sqrt{2}$ thicker than at the projection distance is called the depth-of-field. At 35 cm focus this depth-of-field is at least 50 cm. This depth of field is large enough to image the whole measurement cell which measures 15 cm in diameter. The laser sheet is generated with a fixed opening (fan) angle of $30^\circ$.

Translation stage -- The laser is mounted on a translation stage, to allow imaging of different cross sections of the suspension. The translation stage (Velmex Bislide, size 23) is driven by a stepper motor and can make 1 micrometer steps, significantly smaller than the 100 micron laser sheet thickness. The stepper motor is controlled by a computer operated stepper motor driver (Velmex VXM-1). Automated measurement cycles that involve synchronizing the motion of the laser with camera operation are therefore possible; for slow flows the time between two images required for imaging the flow is of the order of minutes, and allows for acquiring cross sections at more than one imaging position; this parallel acquisition of the two dimensional flow field at different imaging positions reduces total measurement time substantially.

Digital camera -- The computer controlled digital camera used to image the cross sections illuminated by the laser, is a Sensicam 370KL. The camera is placed at 2 meters distance of the split-bottom cell. The focal depth of the camera is set before the start of the experiment. For the camera placed at that imaging distance, the different cross sections imaged are all while the laser sheet is moved.

At such a large imaging distance, little fluorescent light is available, so pixel noise in the CCD chip becomes significant. Therefore, the CCD chip of the camera is cooled to 20 degrees below ambient. This cooling reduces the dark current and increases the quantum efficiency (QE) of the camera\(^2\).

\(^2\)The dark current is the anomalous ‘detection’ of photons by the pixels on the CCD chip, even
CHAPTER 3. SUSPENSION FLOWS

The resolution of the B/W CCD is 1280x1024 pixels, with 12 bit depth. Imaging the complete cell, which measures 15 cm in diameter, corresponds to a surface area of ~ 200x200 micron per pixel. The exposure time can be adjusted between 10 microseconds to several hours. The maximum frame rate of 3 frames per second is set by the minimum amount of time it takes to transfer the images from camera memory to the computer hard drive. The lens used on the camera is a zoom lens with an aperture of 2.8.

The setup -- See Fig. 3.1a. The setup is placed on a large optical table. The optical pathway from the camera to the measurement cell is about two meters and therefore relatively large. To accommodate this large optical pathway within the confined space of a laboratory room and optical table, a mirror is used. The whole setup is shielded from external light with black curtains.

Flow geometry -- The split-bottom geometry (standard type -- see section 2.5) in which the flow takes place, is a square box of 15 cm width, and 20 cm height. The walls of the box are made of transparent PMMA. The driving disk is mounted in a recession in the bottom of the flow cell. The disk radius $R_s$ is 4.5 cm. The disk is driven from below by a DC servo motor (type) via a 1/25 gearbox. Driving rates are $\Omega = 8.3 \times 10^{-5}$ to $8.3 \times 10^{-2}$ rotations per second. The hole through which the disk is connected to the motor is sealed by a lip seal, to make the box watertight; see Fig. 3.1e. To the bottom of the box, as well as to the disk, a disordered, low density arrangement of 1/8" (~ 3.1 mm) PMMA particle is glued with an epoxy glue, to make the surface rough and provide a no-slip boundary condition for particles up to about 6 mm in diameter.

Particles -- We use 3/16" (~ 4.6 mm) PMMA particles (Engineering Labs). They are spherical, monodisperse, polished transparent particles with an index of refraction $n_D$ of 1.48-1.50, depending on the production batch. Their density is $1.18 \times 10^3$ kg/m$^3$.

Index matching fluid -- As an index matching fluid we use Triton X-100, mixed with water and zinc chloride, based on Ref. [61]. The density of the fluid is around $1.08 \times 10^3$ kg/m$^3$. The density difference $\rho_p - \rho_f$ between the fluid is always positive and in the range of 10-110 kg/m$^3$; the particles will therefore always settle when inside the fluid. The viscosity of the fluid varies slightly between

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1 PolyMethyl MethAcrylate, also known as acryl or Plexiglas

if no light reaches the photosensitive parts. The QE is the number of photons needed to release a single electron in the photosensitive part of each pixel.
0.2 and 0.5 Pa-s, depending on the preparation of the fluid and the amount of water evaporated. We cover 3 decades in driving rates, so this variation in the viscosity will not affect our conclusions significantly. See section 3.2.2 for more details regarding the preparation of index matching fluids.

**Fluorescent dye** -- In the index matching fluid we dissolve a small amount of fluorescent dye. We use Nile Blue 690; see section 3.2.2 for more details regarding the use of the dye.

**Laboratory environment** -- All experiments are carried out in a laboratory with climate control, hence under ambient yet constant temperature and humidity. The temperature control is relevant, since the temperature affects the viscosity of the index-matching fluid used. Evaporation of water from the solution has not been prevented, so over the course of days the density and viscosity change slightly. See section 3.2.2 for more details.

**Operation** -- The suspension in the split-bottom cell is imaged by placing the laser sheet parallel to the bottom of the box, at imaging position $z$. See Fig. 3.1b for a schematic picture of this imaging method. Since the dye inside the fluid fluoresces when excited by the laser light, the particles appear as dark spots in a bright background -- Fig. 3.1 shows a typical image. To image the flow at a particular height $z$, we take a ‘movie’ of the flow with a fixed number of images, usually 1000, with several seconds between the acquisition of two images. This is done while the disk is driving the suspension. To image the flow at different heights, the laser sheet is moved to a different height, and another sequence of images is acquired.

**Analysis of data** -- From the cross sections of the suspension, we obtain the velocity profiles with PIV. This method is described above in appendix 8.1

**Limitations** -- The flow rates that can be studied with this setup are limited from above by camera acquisition speed: the upper limit in flow speed is 2.5 cm/s, or equivalently: 5 rotations per minute $= 8.3 \times 10^{-2}$ rotations per second. The lower limit on the flow rates is the constraint of having a reasonable measurement time. However, the fluid stability is sometimes a limiting factor as well; see section 3.2.2. The zinc chloride precipitates in the form of small white crystals after a few days. This precipitate scatters light and makes the laser sheet disperse, whence imaging becomes progressively more difficult. Replacement or mechanical mixing of the fluid is then necessary. The laser dye also photobleaches, but this happens on longer timescales. Considering all of
the above, the lowest rotation rate we were able to attain was roughly 1 rotation per 3 hours, or $8.3 \times 10^{-5}$ rotations per second.

### 3.2.2 Making Index Matched Suspensions

In this section we list techniques, tricks and caveats to keep in mind to accomplish index matching for Triton and PMMA particles. Matching the index of refraction of a fluid to that of a solid, as it turns out, is much more of an art than a science. We will therefore describe the methods mostly as recipes, without much supporting evidence as to why these methods work. We index matched PMMA particles with a mixture of the following components: Triton X-100 (a polymer fluid), ZnCl$_2$ and H$_2$O.

**Composition of the fluid** -- Ref. [61] states the following relative quantities that can be used for index matching: 77.9% Triton, 13% H$_2$O, 9% ZnCl$_2$, all fraction by weight. However, this mixture has several disadvantages: it is almost density matched$^4$, so the particles take a long time to settle. The mixture is cloudy, which inhibits transmission of laser light. The fluid is also very viscous$^5$. In Ref. [62], we find a slightly different composition of Triton, water and zinc chloride used to index match PMMA: 92% Triton, 4.67% H$_2$O, 3.22% ZnCl$_2$ and 0.1% 12M HCl. Considering the literature, the precise recipe to achieve index matching is therefore ambiguous. There are two however two more considerations that make creating the right composition of constituents difficult: absorption of the fluid by the PMMA particles, and precipitation of the zinc chloride. We explain these two phenomena below.

**Absorption** -- Absorption of the polymer fluid can change the refractive index of the beads slightly$^6$. We observe that washing and drying the PMMA particles after having submersed them in Triton often makes them develop cracks. This evidences that absorption and desorption occurs in our PMMA

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$^4$Ref. [61] states that the recipe is index and density matched with PMMA. However, we find that a 3 mm PMMA sphere settles 47 mm in 526 seconds at 22$^\circ$C. The particles used in Ref. [61] were about 0.1 mm in diameter, so most likely made with a different process, and therefore probably had a slightly different density.

$^5$The viscosity of a Triton-water mixture for 10% water is more viscous than pure Triton itself, by at least 10%.

$^6$In Ref. [63] it was observed that $n_D$ could change by 0.0025 by letting a PMMA film dry in vacuum. This change was attributed to solvents degassing under vacuum, stated as follows in Ref. [63]: "Any variations of the residual water content and solvents in the polymer material can induce an index change [...]".
3.2. FLOW MEASUREMENTS

particles; these processes most likely develop stresses in the PMMA\(^7\).

**Zinc Chloride** -- Zinc chloride is added to improve the miscibility of water and Triton, and to change the density of the fluid. Zinc chloride however seems to precipitate out of the mixture\(^8\). Using less zinc chloride ensures that less will precipitate. Note that in Ref. [62] it was mentioned that adding HCl to the fluid diminishes the formation of this precipitate; we have confirmed that this is indeed the case: the turbidity of the mixture disappeared completely after adding 2 ml of a 37 % HCl solution to 1.5 liter of the Triton mixture.

**A new protocol** -- The considerations outlined above have led us to develop a new protocol to achieve index matching. This protocol is stated below. Note that we did not use the HCl to prevent the formation of precipitate; we use such small quantities of ZnCl\(_2\) that this is not necessary. The protocol does not use a fixed amount of water/ZnCl\(_2\), but instead adapts this amount to achieve best index matching:

1. Soak the particles in pure Triton for a few days.
2. Premix water and zinc chloride; we found 10 grams of zinc chloride per 100 ml of water to be sufficient.
3. Remove the particles from the Triton they were soaking in, and add the particles in the measurement system. Do not wash all Triton from the particles at this step.
4. Add enough pure Triton in the measurement system to submerse the particles -- make a layer of 1 cm of pure Triton above the surface of the suspension. This layer is to ensure when the granular suspension expands due to shear, it will not experience the surface tension of the fluid layer above it.
5. Add small amounts (0.1% of the total liquid volume in the box) of the H\(_2\)O/ZnCl\(_2\)-mix repeatedly until index matching is achieved. Below we describe a way to determine this.
6. Once the PMMA particles have been in contact with the Triton based fluid, store them submersed in pure Triton to prevent drying of the particles.

\(^7\)Although according to the manufacturer, PMMA is supposed to absorb only 0.3% of its volume on water per day, and therefore even less for a far larger molecule like Triton X-100.

\(^8\)Although the water/zinc chloride mix is not saturated: water can dissolve about 400 gram of zinc chloride per liter.
Note that the density and viscosity of this mixture have to be measured separately, since they are not a priori known. This recipe can therefore not be used to achieve density matching.

**Achieving index matching** -- Step 5, the index matching step, is best done inside the measurement system itself, using the laser light that is to be used also for illumination. The index of refraction depends both on the temperature and the wavelength of the light. Refractometers typically use the $D$ lines of sodium (589 nm) in the determination of the index of refraction, and if the laser light used in the experiment is not of the same wavelength, good index matching based on the $n_D$ values might prove to be insufficient in the experimental setup.

An effective way to check for index matching is to see whether one can read a text printed in a large font through the suspension inside the 15 cm wide box. We have found the readability of such a text to be a sufficient criterion for achieving images that are good enough for at PIV techniques.

**Index of refraction** -- The index of refraction of most materials show modest temperature dependence. Generally speaking, the refractive index of liquids and solids goes down with temperature. Typical temperature coefficients are in the range of $0.0001-0.0005 \, K^{-1}$ [64,65].

**Laser dye** -- The dye we use is Nile Blue 690. The dye is available in the form of small crystals, which do not dissolve easily in either Triton or water. Therefore, we dissolve 1 mg of Nile Blue 690 crystals in 1 liter of ethanol, in which the dye dissolves easily. One milliliter of the ethanol-dye solution is added to every 100 ml of index matching fluid. The ethanol affects the index of refraction of the fluid, but this effect is limited, due to the small amounts of ethanol added. Moreover, the ethanol evaporates from the index matched fluid. Naturally, the more dye one adds to the index matched suspension, the larger the contrast one can get in the images. However, the dye also absorbs the light, and the more dye is added, the less light is available for imaging deeper inside the bulk of the suspension. See Fig. 3.2 for an illustration of a high dye concentration (a), and a low dye concentration (b) fluorescence gradient. The precise quantity of dye that is to be added therefore depends on how many particle layers one wants to or can image, the desired location within the suspension where one wants to image, and the imaging rate.

**Nile Blue spectrum** -- Nile Blue 690 perchlorate has a different absorption and emission spectrum, depending on the liquid it is dissolved in [66]. We observed that *pure* Triton with Nile Blue added to it becomes pink. Adding small
3.3. COMPARISON TO DRY FLOWS

amounts of water gradually shifts the color from pink to dark blue, which is the color Nile Blue obtains when dissolved in ethanol. This implies that one always has to add a certain amount of water to the Triton in order to tune the spectrum of the dye. For Triton-based index matching fluids, the amount of water necessary to achieve index matching is usually also enough to make the spectrum of Nile Blue 690 compatible with the laser light used in our experiments. In combination with other index matching liquids, this amount of water is not necessary: in section 4.4.1 we will describe a dimethyl-sulfoxide (DMSO) based index matching liquid, in which the amount of water necessary to achieve index matching is not sufficient to change the spectrum of Nile Blue 690 enough. In that index matching liquid, we therefore use another fluorescent dye. Note that adding milliliter quantities of an HCl solution to Triton is also sufficient to shift the spectrum of Nile Blue to the desired wavelength; this effect can also be used with DMSO, to avoid the necessity of changing the dye.

Photobleaching -- The laser dye loses its ability to fluoresce over the course of weeks. This process is called bleaching, and is enhanced under the influence of light (photobleaching). Therefore, it is advisable to protect the dye in the fluid or in storage from light at all times. The laser used in the experiments can actually be triggered, such that it is only on during the exposure time of the CCD. Using this feature ensures that the laser beam is not bleaching the dye unnecessarily. Under standard laboratory lighting conditions, a prepared batch of index matched fluid with the dye added was found to lose its blue color, which presumably is the sign of photobleaching, over the course of 2-3 weeks. This makes photobleaching seem to be not such an important issue. Yet, since the optimal contrast and gradients in the acquired images depend exponentially on the dye concentration, it is still recommended to avoid this whenever possible.

3.3 Comparison to Dry flows

We verify experimentally that the split-bottom suspension flows at slow driving indeed correspond very well with flow profiles measured for dry flows in the same geometry. To do this, we measure the flow profiles of 4.6 mm PMMA particles suspended in an index matched Triton-based fluid, based on the recipe from Ref. [61]. We measure the flow profiles at a rotation rate of the disk $\Omega$ of $8.3 \times 10^{-5}$. We perform all the experiments in setup M described above.

We measure these profiles for different filling heights $H$: 22, 32 and 45 mm. We will refer to these filling heights as low, medium and high respectively. The
radius of the disk $R_s$ is 45 mm. $H/R_s$ is then: 0.5, 0.7 and 1.0, all ±0.05. From the estimate that the measured filling height $H$ has an error bar of $\delta H \pm d/2$, we it follows that $\delta H/R_s \pm 0.05$.

We image the flow in several horizontal slices at position $z < H$. The imaging positions are as follows: for the low filling height, $z = 3, 7, 10, 13, 17$ mm. For medium, we use $z = 3, 9, 15, 21, 27$ mm, and for the largest filling height we use $z = 4.5, 9.5, 14.5, 19.5, 24.5, 29.5, 34.5$ mm. For all positions $z$, the error bar is ±1.5 mm from the uncertainty in determining where the rough bottom starts.

We use PIV methods to extract the velocity profile; see appendix 8.1 for details. For each imaging position we acquire 1000 frames, with an interframe time of 40 seconds. Total measurement time is then 11 hours. At $\Omega$ of $8.3 \times 10^{-5}$ rps, this interframe time limits the maximum displacement per image pair approximately 5 pixels. This is less than the ideal strain mentioned in appendix 8.1, so in the PIV we use $n=10$. For small $r$, we cannot reliably obtain $v(r)$; we place the cutoff at $r \sim 18$ mm. The profile is smoothed by subdividing the $r$-coordinate range into 20 bins of $\sim d/2$, with $d$ the particle diameter, and averaging the values for $\omega(r, z)$ in those bins.
3.3. COMPARISON TO DRY FLOWS

Figure 3.3: Normalized angular velocity profiles \( \omega(r) = v(r)/(2\pi r \Omega) \) for the \( H/R_s = 0.5 \) suspension flow, for \( \Omega = 8.3 \times 10^{-5} \) rps. (a) Shows the data on linear scale, with all the curves shifted upwards for clarity, (b) shows the same data on logarithmic scale -- here the curves are shifted horizontally as well as vertically. The range on the abscissa corresponds to the width of the cell. The line style indicates the imaging position \( z \), given in the legend, and the solid red line is an error function fit.

3.3.1 Qualitative Comparison: Low Filling Height

In Fig. 3.3 we plot the individual profiles measured for the low filling height, \( H/R_s = 0.5 \). For all imaging positions \( z \) we see that the \( \omega(r, z) \) data qualitatively resemble an error function. This is evidenced by the red line in Fig. 3.3, which is an error function fit. This is the same function Eq. 1.3 as used to fit dry granular flow profiles in the split-bottom geometry; we reproduce this equation here:

\[
\omega(r, z) = \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{R_c(z) - r}{W(z)} \right).
\]

This suggests that the suspension flow behaves as a dry granular flow at this flow rate. Also, the trends in the smaller details of the angular velocity profiles for increasing imaging position \( z \) give more evidence for dry granular flow behavior in the suspension. We some trends that are very similar to what is observed in dry granular flows: the point \( R_c(z) \), the center of the shearband where \( \omega(r, z) = 0.5 \), moves inwards for increasing \( z \). The error functions also clearly broaden with \( z \). This same behavior is also found for dry flows, see Eq. 1.8:
\[ z = H - R_c(z) \left[ 1 - R_c(z)/R(s)(1 - H/R(s))^{2.5} \right]^{1/2.5}, \]

and Eq. 1.5 respectively:

\[ W(z) = W(z = H) \sqrt{1 - (1 - z/H)^2}. \]

In Fig. 3.4a the individual normalized angular velocity profiles from Fig. 3.3 are combined and plotted as a contour plot. \( \omega(r, z) \) here is interpolated and color-graded in the two dimensional vertical cross section \((r, z)\); red means \( \omega = 1 \), so there the material co-rotates with the driving disk. Blue/black shows that no motion is present in that region. In the white areas, flow is also absent. The trends observable in Fig. 3.3, such as the widening of the shear zone and the inward movement of the center of the shear band, are now clearly observable. It is also obvious that the ‘trumpet’ - shaped part of the suspension just above the disk is co-moving with the rotating disk; the angular velocity in the region \( r < 30 \text{ mm} \) is clearly very close to one.

To compare the contour plot of the suspension flow, we create the same contour plot for what a dry granular flow would have been in the split-bottom flow cell with the same dimensions. We use the equations from Chapter 1, that were reiterated above. These empirical equations have only one fit parameter: \( W(z = H) \). \( W \) encodes for the fact that particles with a different roughness, aspect ratio or diameter create different overall widths of the shear bands. We determined the best fit for \( W(z = H) = 14 \), by overlaying contour plots of the suspension flow data \( \omega_S(r, z) \) and the predictions based on the equations \( \omega_D(r, z) \).

This best fit is shown in Fig. 3.4b. The contour plot in Fig. 3.4b is remarkably similar to the plot in (a), even though only one fit parameter was used to match the two profiles, instead of three per individual profile. This gives further evidence for the similarity of suspension flow profiles to dry flows.

### 3.3.2 Quantitative Comparison: Low Filling Height

The strictest, quantitative test we can apply to compare the suspension flow profiles to the dry flow profiles is the following. We make a scatter plot of \( \omega_S(r, z) \), the set of angular velocities measured in the suspensions at different locations in the cell, as a function of \( \omega_D(r, z) \), the set of the angular velocities at those same locations, extracted from the empirically obtained dry granular flow profiles. This plotting is shown in Fig. 3.4c on a linear scale and in (d) on a logarithmic scale. If all data points were on the straight line, it would indicate that the two profiles are identical. Even in this strict quantitative test, it is clear the the flow profile for the suspension at \( \Omega = 8.3 \times 10^{-5} \text{ rps} \) matches almost perfectly with
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Figure 3.4: The normalized angular velocity profile measured at $\Omega = 8.3 \times 10^{-5}$ (a), and the empirical dry granular flow profiles (b, see text for explanation). In (c) all $\omega_S(r,z)$ values measured in the suspension flow are plotted against the expected $\omega_D(r,z)$ at that location. (d) Shows the same data as (c), but on double logarithmic scale. The straight line would indicate a perfect agreement between the profiles in (a) and (b).

the dry granular flow profiles. The deviations visible in Fig. 3.4d are most likely attributable to systematic experimental inaccuracies.

We also used this scatter plot to optimize the fitting parameter $W(z = H)$. By reducing $W(z = H)$ from 14 to 13, we observe that the tail of the logarithmic plot improves its linearity substantially, whereas the scattered data on the linear plot Fig. 3.4(c) then starts to deviate more from the ideal straight line, especially for $\omega_D(r,z) \sim 1$. For $W(z = H) = 15$, this trend is reversed: linearity on the linear plot is improved, but curvature in the scatter plot on the logarithmic scale is increased. These trends also show that $W(z = H) = 14$ is the best fitting parameter, with which we can get excellent agreement between the established dry granular flow profiles and the measured suspension flow structure.
Figure 3.5: Contour plots for the $\Omega = 8.3 \times 10^{-5}$ rps flows measured in the suspension. (a) to (c) show the angular velocity profiles $\omega(r, z)$ for three different filling heights: $H/R_s = 0.5, 0.7$ and 1.0. The color indicates the normalized angular velocity. The dashed line shows where the surface at height $H$ is.

### 3.3.3 Comparison for Different Filling Heights

To further test the similarity between the suspension flow profiles and the dry granular flow profiles, we can also see whether the trends observed in dry flows in which the filling height was increased, are visible for the suspensions.

For dry flows in the split-bottom geometry, the flow field depends strongly on the filling height $H/R_s$. This phenomenology is described in section 1.4.4, we will briefly mention it again here: in the dry flows, for low filling heights with $H/R_s \lesssim 0.6$ there is a ‘trumpet’-shaped shear zone propagating through the granular medium from the edge of the disk to the surface of the granular bed; for large filling heights $H/R_s \gtrsim 0.7$ the shear zone is entirely confined to the bulk, in a ‘dome’-like structure.

We measure the normalized angular velocity $\omega(r, z) \equiv v_\theta(r, z)/r\Omega$ for $\Omega = \ldots$
3.4 Beyond Slow Flows

For increased driving rates, viscous drag forces will start to play a role in the dynamics of the particle. In this section we will show that this has a profound effect on the velocity profiles of suspensions in the split-bottom geometry – by increasing the driving rate by three orders of magnitude, we go from the rate independent flow regime to the rate dependent regime.

We apply the inertial number theory described for dry flows in section 1.5.1 to our suspension flow in this faster flow regime. This theory will explain the observed change in the flow profile as follows. We will show that the stresses in suspension flows in the rate dependent regime are predicted to be dominated by a term \( \sim \dot{\gamma} \), which makes the suspension Newtonian in this limit. We verify this prediction by numerically calculating the flow profiles for a Newtonian fluid in the split-bottom geometry. The Newtonian flow profiles are indeed strikingly similar to the profiles observed in the suspension.

3.4.1 Measured Flow Profiles

We measure the flow profiles of the PMMA-Triton suspension for three values of \( \Omega = 8.3 \times 10^{-4}, 8.3 \times 10^{-3} \) and \( 8.3 \times 10^{-2} \) rps, for the low, medium and large filling height. For the low filling height, we show the three velocity profiles measured at these \( \Omega \) in Fig. 3.6b-d. We also include the velocity profile measured in the slow flow limit, at \( \Omega = 8.3 \times 10^{-5} \) rps in Fig. 3.6a. The change in the flow profiles
Figure 3.6: Contour plots for the normalized angular velocities for $H/R_s = 0.5$ (a) Through (d) show data for $\Omega = 8.3 \times 10^{-5}, 8.3 \times 10^{-4}, 8.3 \times 10^{-3}$ and $8.3 \times 10^{-2}$ rps. Color code is identical to color code in Fig. 3.5.

is striking: the solid-like core that co-rotates with the disk at $\Omega = 8.3 \times 10^{-5}$ rps vanishes completely with increasing driving rate -- the shearbands broaden substantially with $\Omega$.

**Slip** -- At the largest $\Omega$, the normalized angular velocity profile does not reach 1 anywhere in the suspension, not even close to the bottom. This suggests slip is present. At this $\Omega = 8.3 \times 10^{-2}$ rps, we push the imaging capacities of the IMS system to its limits: motion blur is clearly visible in the images, so the presence of some imaging artifacts cannot be ruled out. However, since the flow profiles are still smooth some slip is certainly present.

For $\Omega = 8.3 \times 10^{-2}$ rps, we show contour plots of the flow profiles for the three different filling heights used in Fig. 3.7. The broadening of the shearzone with increased $\Omega$ is also apparent for $H/R_s = 0.5$ and 0.7, although it is less evident for these filling heights. Slip is present to the same extent for all filling heights.
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Figure 3.7: Contour plots for the $\Omega = 8.3 \times 10^{-2}$ rps flows measured in the suspension. (a) to (c) show the angular velocity profiles $\omega(r,z)$ for three different filling heights: $H/R_s = 0.5$, 0.7 and 1.0. The dashed line shows where the surface at height $H$ is.

3.4.2 Theory: Rearrangement Timescales

We want to establish a mapping between the inertial number theory described in section 1.5.1, established for dry flows, and the submerged flows we study here. Such a mapping will give insight on how the rheology of the suspension depends on the driving rate. This in turn will tell us more about the rate dependence of the structure of the flows. We will show that the theory gives a reasonable bound for which $\Omega$ the rate independent regime should end. Moreover, we will show that in the rate dependent regime, the theory gives remarkably accurate predictions for the structure of the flow of gravitational suspensions.

In the inertial number theory the behavior of dry granular flows is described with the so-called inertial number. As detailed in section 1.5.1, the inertial number is the dimensionless product of the shearrate and the typical rearrangement timescale of a particle. For dry flows, the rearrangement timescale of a particle is
simply \( d/\sqrt{P/\rho_p} \). For suspension flows, the viscosity of the interstitial fluid plays a role in the rearrangement process of the particle, and hence the particulate rearrangement timescale in suspensions will be different.

To estimate the relevant timescales in the dynamics of the particles in the suspension at these driving rates, it is necessary to write down an approximation for their equation of motion. In this analysis, we follow Courrech du Pont et al. and Cassar et al. [60, 67].

The equation of motion for a particle in a fluid can be stated as follows:

\[
\frac{\pi}{6} \rho_d d^3 \frac{dv_p}{dt} = \frac{\pi}{4} P d^2 - F_d, 
\]

(3.1)

Here \( \rho_p \) is the bulk density of the material the particle is made of, \( d \) their diameter, \( v_p \) their velocity, \( P \) is the confining pressure due to the presence of other particles, and \( F_d \) the drag force experienced by the particle.

For a particle moving in a fluid, the drag force can have a different functional dependence on the velocity of the particle, depending on the particulate Reynolds number. This dimensionless number is the ratio between inertial and viscous forces acting on a particle:

\[
Re = \frac{\rho v_p d}{\eta_f} 
\]

(3.2)

For \( Re \ll 1 \), the particle experiences viscous or Stokes drag: \( F_d \sim v_p \). For \( Re \gg 1 \) the particle experiences inertial drag forces: \( F_d \sim v^2 \). We can estimate the Reynolds number by assuming that \( v_p \leq 2\pi \Omega R_s \). We then find that even for the largest rotation rates possible in our system, \( 8.3 \times 10^{-2} \) rps, that \( Re \leq 1 \). Therefore we can assume the drag forces on the PMMA particles in our Triton suspension are purely viscous.

The proportionality factor in the relation \( F_d \sim v_p \) can be estimated as follows: the movement of a grain inside a packing requires the displacement of an equal volume of liquid. The resistance that this liquid experiences by having to flow through the packed bed is equal to the drag force on the particle. So if we know the resistance the fluid experiences, we also know the drag force on the particle. We calculate the resistance as follows. The pressure difference required for a fluid to flow through a medium of porosity \( k = \alpha d^2 \) with velocity \( v_p \) is given by Darcy’s law [68]:

\[
\Delta P_{\text{fluid}} = -\frac{v_p L \eta_f}{sk},
\]

(3.3)

Here \( L \) is the length over which the pressure gradient exists - in this case the volume inside the packing in which the particle moves. Using \( L = d \), we arrive at
the resulting drag force on the particle:

\[ F_d = \frac{n}{4} d^2 \Delta \rho_{\text{fluid}} = \frac{v_p d \eta_f}{-\alpha}, \tag{3.4} \]

The parameter \( \alpha \) is given by the Carman-Kozeny equation\(^9\), and depends on the size distribution of the particles, their shape and also on the density of the granular bed\([70]\)\(^10\).

\[ \alpha = \frac{\Phi_2 \epsilon^3}{150(1 - \epsilon)^2}, \tag{3.5} \]

with \( \epsilon \) the porosity, equivalent to \( 1 - \Phi \) where \( \Phi \) is the packing fraction. \( \Phi_2 \) is the sphericity\([70]\) of the particles, which equals 1 for spheres. For our suspensions, we assume \( \Phi = 0.59 \), the typical value of a settled granular bed. This gives \( \alpha = 0.001 \).

With the equation of motion for our particles, we can now estimate the rearrangement timescale for a particle in the fluid. With this estimate of the rearrangement timescale for particles in a fluid, we can recalculate the inertial number for these flows, and then apply the inertial number framework from dry flows to suspension flows.

In a static packing the hydrostatic pressure on a single particle is balanced. Due to rearrangements, this balance may disappear. The particle will then accelerate when it has space to do so. We assume that when the balance disappears, the confining pressure from the mass of the particles above the moving particle will act on it as force \( P_g d^2 \). When the particle accelerates, it will start to experience a drag force. The limiting velocity \( v_{\text{inf}} \) at which the accelerating force is equal to the viscous force is then

\[ v_{\text{inf}} = \frac{P_g d}{\eta_f} \tag{3.6} \]

The particle will reach this velocity in a characteristic time \( t_a \):

\[ t_a = \frac{2 \rho_p d^2}{3 \eta_f} \tag{3.7} \]

---

\(^9\)The Carman-Kozeny (see e.g. Ref [69] and references therein) equation is an extension of Darcy’s law and gives a prediction for the proportionality constant in Darcy’s law. The Carman-Kozeny equation in its turn is the low \( Re \) limit of the Ergun equation, the more general equation for the pressure drop in flow across a porous medium. The Ergun equation also accounts for inertial dissipation in the fluid, that occurs at higher \( Re \). Since we work at low \( Re \), the full Ergun equation does not have to be used here.

\(^{10}\)Note that this only applies to homogeneous porous media; for heterogeneous media the determination of the prefactor is more involved; see e.g. Ref. [71].
For our suspension, \( t_a < 1 \) ms. Comparing this to the shear rate, which even for our fastest driving rates is maximally \( \sim 2\pi\Omega R_s/H \sim 2\pi\Omega \), we can assume that during the complete rearrangement process, the particle travels with \( v_{inf} \). Therefore the typical rearrangement timescale for the particles in the suspension becomes \( d/v_{inf} = \eta_f/\rho g a \). With this the inertial number for suspension flows becomes

\[
I_s = \frac{\dot{\gamma}\eta_f}{\rho g a}
\]  

(3.8)

### 3.4.3 Theory: An Upper Bound for Slow Flows

With the inertial number scalings presented, we first want to establish where the theory predicts that the rate independent regime crosses over into a rate dependent regime -- this we can readily check with the data presented in section 3.4.

For \( I_s \ll 0.01 \), submersed granular flows should behave as dry granular flows [23]. Estimating that in the split-bottom geometry, \( \dot{\gamma} \sim 2\pi\Omega R_s/H \) and given that \( H \sim R_s \), \( I_s \) gives an estimate for which rotation rates we do not expect viscous interactions to be important:

\[
\frac{2\pi\eta_f\Omega}{a\Delta\rho g R_s} \ll 1 \rightarrow \frac{a\Delta\rho g R_s}{2\pi\eta_f} \gg \Omega
\]

(3.9)

For the PMMA/Triton suspension, \( I_s = 0.01 \) at \( \Omega \sim 0.002 \). We estimate that that for driving rates less than about \( 10^{-3} - 10^{-4} \) rotations per second, a rate independent regime should be observed. The approximations made above and the uncertainty in \( \eta_f \) and \( \Delta\rho \) mean that we cannot precisely determine where fluid drag becomes negligible. However, it is already clear that the inertial number scaling gives the right estimate for the order of magnitude where rate dependence should start to be observed. This is apparent in Fig. 3.6.

### 3.4.4 Theory: Prediction for Faster Flows

Knowing the inertial number for suspension flows, we also can try to use the inertial number theory established for dry flows [2] in suspensions. We write for the stress \( \tau \):

\[
\tau = \mu(I)\rho,
\]

(3.10)

with \( \mu(I) \) the an empirical function. For low inertial numbers, \( \mu(I) \) can be approximated by \( \mu(I) = \mu_0 + \mu_1 I \) (Ref. [60], Fig. 13), with \( \mu_0 \) and \( \mu_1 \) empirical values,
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as was also done in Ref. [23]. Using this \( \mu(l) \) in Eq. 3.10 together with Eq. 3.8, we arrive at:

\[
\tau = \mu_0 P + \mu_1 \frac{\eta \dot{\gamma}}{\alpha}
\]  

(3.11)

In other words: the local stress in a suspension is a linear combination of a frictional stress and a purely viscous stress. This also means that if we increase the driving rate \( \Omega \) in the split-bottom cell, we should see less and less of the frictional nature of the interactions between the particles in the suspension.

**Consequences for the flow** -- For large driving rates and small pressures, the stress \( \tau \) in Eq. 3.11 will essentially be given by \( \tau = \mu_1 \frac{\eta \dot{\gamma}}{\alpha} \): the suspension should become Newtonian, with an effective viscosity \( \mu_1 \eta / \alpha \). Indeed, in the high rotation rate limit, the rheological data that we will discuss in the next chapter, indicates that the behavior of the suspension becomes approximately Newtonian. To verify that we are in a regime where the inertial theory becomes applicable, we calculate the spatial distribution of the inertial number \( \tilde{l}(r, z) \) for the flow profiles measured in the suspensions. For each flow profiles measured at a different \( \Omega \), we then obtain a set of inertial numbers found in that flow profile. These sets for different \( \Omega \) are plotted against \( \Omega \) in Fig. 3.8.

**Newtonian Flows** -- For Newtonian fluids, we can numerically predict what the flow profile should be in the split-bottom geometry. We can then compare the flow profiles of the suspension to numerical predictions of Newtonian flows in the split-bottom geometry.

We use a Finite Element Method (FEM) software package (COMSOL) to compute the stead state Navier-Stokes flow equations for a Newtonian incompressible fluid in the split-bottom geometry. We assume rotational symmetry to reduce the problem to a two-dimensional configuration; we use no-slip boundary conditions for the disk and the static bottom and the walls, a slip condition for the surface, and drive the flow with the condition \( v_\theta = 2 \pi \Omega r \) at the disk, where \( v_\theta \) is the velocity in the azimuthal direction. For details regarding the COMSOL calculation, we refer to appendix 3.6.1.

We solve the Navier-Stokes equations for the three different filling heights for which we also measured the suspension flow profiles: \( H/R_s = 0.5, 0.7, 1.0 \). The results are shown in Fig. 3.9a-c. We observe that in the solutions, a change from trumpet-like shearbands to dome-like shearbands is also visible, when changing the filling height from low to high. However, the shearbands for the low filling height are broad; most notably, the solid like inner core, that for dry flows co-rotates with the disk, is absent in the Newtonian flow solution. Note
that in the steady state Newtonian flow approximation, the rotation rate $\Omega$ only enters into $\omega(r, z)$ as a trivial prefactor as long as centrifugal forces are small. See Appendix 3.6A for a more detailed treatment of the influence of centrifugal forces on the flow profile of Newtonian flows in the split-bottom geometry.

### 3.4.5 Validation of the Inertial Number Theory

We can compare the Stokes flow prediction shown in Fig. 3.9 to the flow profiles measured at $\Omega = 8.3 \times 10^{-2}$ rps shown in Fig. 3.7. The qualitative similarity between the contour plots is strong. For both Newtonian and suspension velocity profiles, we see the same trends with increasing filling heights: consider the band for which $\omega = 0.5$, which is the light blue band. For the lowest filling height, this band still reaches the surface in the Newtonian flow profile, and the data shown for the suspension flows also strongly suggests that this band reaches the surface of the suspension, although at a slightly smaller $r$. For the larger filling heights, the $\omega = 0.5$ band is confined to the bulk. However, the suspension flows seem to be more confined to the bulk, than the Newtonian flow profiles. We can again also quantitatively compare the predictions for the Newtonian flow profiles with the suspension data in a scatter plot, as was done.
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Figure 3.9: Stokes flow solutions for the split-bottom for three different filling heights. From (a) to (c): \( H/R_s = 0.5, 0.7 \) and 1.0. The contour plot shows the normalized angular velocity, \( \Omega \) is arbitrary in this approximation, see text. The dashed line shows where the surface at height \( H \) is.

In section 3.3 when we compared the dry granular flow profiles to the slowly driven suspension flow profile.

In Fig. 3.10, we plot \( \omega_S(r, z) \), the set of angular velocities measured for \( \Omega = 8.3 \times 10^{-2} \) rps in the suspensions at different locations in the cell, as a function of \( \omega_N(r, z) \), the set of the angular velocities at those same locations, extracted from the numerically calculated Newtonian flow profiles. We do this for the three different filling heights in Fig. 3.10a-c. Although the matching is quite good, especially for Fig. 3.10a, evidently the match is not as good as the match in Fig. 3.4. There are two possible effects that can cause this discrepancy. First of all there is 30% slip at the driving disk; the suspension flow profiles nowhere reach \( \omega > 0.7 \). This effect can be taken into account by comparing the data points to the line \( \omega_S(r, z) = 0.7 \times \omega_N(r, z) \). This line is also plotted in Fig. 3.10a-c. Clearly the data lies closer to that line, but the finite curvature in
Figure 3.10: Comparison for three different filling heights of the Stokes flow solution to the velocity profiles found in the suspensions. \( H/R_s \): (a),(d) = 0.5, (b),(e) = 0.7 and (c),(f) = 1.0. The upper row shows only the comparison between Newtonian and the \( \Omega = 8.3 \times 10^{-2} \) rps suspension data. The line indicates equivalence between Newtonian and suspension flows, assuming a slip of 30\%.

The second effect that has not been considered is the fact that the suspension rheology is not Newtonian throughout the whole flow. In regions in the flow where the local strain rate \( \dot{\gamma} \) is small the local rheology given by Eq.3.11 might still be dominated by the frictional stresses \( \mu_0 P \). In steady state flow, local stress balance has to hold. For small stresses of the order of \( \mu_0 P \), and for the rheology of Eq. 3.11, this stress balance can be achieved locally with very small \( \dot{\gamma} \) correction on the already present \( \mu_0 P \) term. A Newtonian rheology can only achieve stress balance by adjusting \( \dot{\gamma} \), and is therefore inclined to have larger \( \dot{\gamma} \) where the local stresses become of the order of \( \mu_0 P \). The concave shape of the scattered data is
3.5 CONCLUSIONS

precisely an indication of this overestimation of $\dot{y}$.

3.5 Conclusions

Figure 3.11: For $H/R_5 = 0.5$: (a-d) The velocity profiles at driving rates $\Omega = 8.3 \times 10^{-5}$ rps (a) to $\Omega = 8.3 \times 10^{-2}$ rps (d), as in Fig. 3.3. In (e) the profile for dry granular flows; compare this to (a). In (f) the profile for Newtonian flow is plotted; compare this to (d). (f) has a scatter plot comparison between the dry flow profiles $\omega_D(r, z)$ and the measured suspension flow profile $\omega_S(r, z)$ for the driving rates shown in (a-d); darker blue is faster driving. (f) has a scatter plot comparison of $\omega_S(r, z)$ to the Newtonian profile $\omega_N(r, z)$.

In this chapter, we described the application of the IMS technique to split-bottom flows. We showed that there is an excellent quantitative agreement between the empirically established flow structure equations for dry split-bottom flows, and the measured flow fields for slowly driven suspensions -- see Fig. 3.11a-e. This means that for slow driving rates, suspension flows in the split-bottom geometry behave as dry granular fluids -- the interstitial fluid does not affect the flow field.

At increasingly larger driving rates, we observe a change in the flow profiles of the suspensions, as shown in Fig. 3.11a-d. We apply the inertial number
theory for suspensions to the regime of flow rates and find that the theory predicts Newtonian behavior for faster driving rates. Comparing the measured flow profiles to that of Newtonian flow in the split-bottom geometry as done in Fig.3.11d-f, we see that this inertial number theory captures the flow behavior of the suspensions in the fast driving rate limit very well.

3.6 Appendices

3.6.1 A: Details of the COMSOL Calculations

Figure 3.12: (a) The boundary conditions of a two dimensional cross section of the split-bottom cell. Red: no-slip boundary conditions. Green: slip. Blue: no-slip with the condition \( v_\theta = 2\pi r \Omega \). \( R_s \) is equal to that of the setup used: 45 mm. (b) A typical mesh with nodes with which the flow equations are solved numerically. (c) The flow field for pure Triton at 1 rpm driving rate. The contour plot shows the angular velocity \( \omega \), the arrows indicate the magnitude of the local in-plane flow field \( v_r \) and \( v_z \). (d) The flow field for water driven at 1 rpm. The lengths of the vectors for water are reduced by a factor 40. Units on abscissa and ordinates are all in [m].
We used the Finite Element Package (FEM) COMSOL to solve the steady state Navier-Stokes equations [72] in the split-bottom geometry. In this section, we give the details of how the calculations were carried out. The rotational symmetry of the cell allowed us to reduce computation time by solving the equations on a two-dimensional cross section in the split-bottom geometry. This cross section is shown in Fig. 3.12. Solving the steady state Navier-Stokes equations for this geometry in three dimensions essentially gives the same results, but is more CPU and memory intensive [73].

**Boundary conditions** -- The boundaries are shown in Fig. 3.12. The surface is fixed and horizontal. We verified experimentally that the surface of the suspended particles is flat up to $\sim 1.5 \times 10^{-1}$ rps, so for the range of rotation rates encountered in this section, the flat surface assumption is appropriate. The disk, the outer wall and static part of the bottom are treated as no-slip boundaries. The disk has the velocity condition $v_\theta = 2\pi r \Omega$, where $r$ is the radial coordinate.

**Centrifugal forces** -- In the split-bottoms geometry, centrifugal forces set up a secondary flow in the fluid. This can be shown easily with the FEM calculations. The magnitude of this secondary flow, however, depends on the rotation rate and the viscosity of the fluid. This can be seen in Fig. 3.12c-d. Flow fields for both Triton and water are shown as contour plots. Also, the in-plane velocity fields are plotted as vectors. The magnitude of the vector in the $x$-direction is given by $v_x$, the magnitude of the local velocity component in the $x$ direction. Mutatis mutandis for the component in the $z$-direction, we obtain an indicator for the secondary flow profile, set up by the centrifugal force. Note that the vectors in the plots for both Triton and water are rescaled to have similar length. The maximum in-plane flow velocity for Triton is $1.9 \times 10^{-3}$ m/s, for water this is $7.7 \times 10^{-4}$ m/s, whereas the driving rate is 0.1 m/s. We conclude that for Triton, with a viscosity of 0.2 Pa·s, the secondary flow strength in the range of rotation rates reported in this chapter is not significant. So in the limit of $\Omega \to 0$, all the flow profiles are as in Fig. 3.12c. However, for low viscosity liquids such as water, with a viscosity of 1 mPa·s, at $8.3 \times 10^{-2}$ rps, a significant secondary flow develops at the highest rotation rate seen in the suspension experiments.

**Strain rate above split** -- In the COMSOL calculations, there is an infinite strain rate precisely at the location of the gap. We verify here that this divergence does not affect the flow field predictions for Triton. We compare a

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11Many thanks to support engineer Anders Ekeroth from COMSOL for help with the software package.
rate independent flow profile for $H/R_s = 1$, shown in Fig. 3.13a, to a flow profile calculated with slightly different boundary conditions: we add a 2 mm wide gap with a slip boundary condition next to the rotating disk. This slip region represents the small gap between the rotating disk and the static bottom. Note that 2 mm is a strong overestimation of the size of the gap; in actual experiments the gap is never larger than 1 mm. The flow profile predicted for this geometry is shown in Fig. 3.13b. The difference between the two flow profiles in that figure is very small and only notable near the disk edge region. In Fig. 3.13c, we plot the logarithm of this difference, normalized by $v_\theta(R_s) = 2nR_s\Omega$. It is obvious that in the numerics, removing the divergent strain rate near the edge by adding a slip region is not necessary -- it does not affect the results appreciably.

### 3.6.2 B: IMS and Other Visualization Techniques

There are more experimental techniques to study granular flows in three dimensions. Each has its own advantages; to highlight under which circumstances
the IMS technique is beneficial, we briefly describe the other techniques, their typical use, and advantages and disadvantages in this appendix.

The IMS approach is a relatively young technique, and has so far only been applied to silo flows [74] and flows in plane shear [18], albeit only in preliminary form. The first study in which the benefit of IMS has been used to its full extent, is in the analysis of the static three dimensional structure of granular packings exposed to thermally induced expansion and contraction cycles [58].

The index matching scanning technique offers some benefits over other methods to study granular materials in three dimensions. The most common other techniques are X-ray tomography, Magnetic Resonance Imaging (MRI), and confocal imaging; we give a short overview of the existing techniques.

MRI techniques have been used to study flow profiles and density profiles inside sheared granulates [34, 40, 75]. MRI techniques have a submillimeter accuracy in macroscopic volumes, but only at a slow scanning rate. They are very sensitive to density differences, but can only track small numbers of tracer particles, and are limited to tracking materials that contain a high density of hydrogen atoms\(^\text{12}\), like organic materials. The use of magnetic components in any MRI setup is impossible.

X-ray tomography has been used to reconstruct the three dimensional stacking structure of granular piles [76, 77], and has been used to investigate the overall density of vibrated granular beds [78]. Also X-ray tomography can achieve very high resolution, but the scanning time can be up to a few hours, although recent development has decreased that scanning time to minutes. It requires substantial CPU time to analyze experimental data, partly due to the fact that direct imaging of cross-sections, as is done in MRI scanners, is not possible. Due to the ionizing energies of X-rays, also personal safety is a concern. MRI and X-ray based scanners can be used for both dry granular flows and suspensions.

Confocal imaging can nowadays image quickly, and with high resolution, both simple cross-sections and complete volumes. It can only be used for index matched\(^\text{13}\) suspensions, since its imaging technique uses visible light to access the interior. It has however been developed to study small systems, with particle sizes ranging from hundreds of nanometers to a few microns, and scaling this technique up to systems with millimeter-sized particles poses significant challenges.

The essential advantages of IMS can now be summarized: the single slice illumination technique allows one to image a full cross section within only one

\(^{12}\)Strictly speaking any non-magnetic material with a non-zero nuclear spin will suffice.

\(^{13}\)Although the index matching does not have to be very well tuned, since the scatter of visible light with $\sim$ micrometer sized particles is not very much affected by interfaces with slightly different indices of refraction.
exposure time of a digital camera. The system sizes that IMS can image are only limited from below by the in-plane resolution. This resolution is set by the optics used, and can therefore be on the order of a few micron. The imaging speed is independent of the resolution, and set by the amount of fluorescent light available. In Chap. 4, we will describe an IMS we developed at Leiden University, in which exposure times of 10 milliseconds can be reached and still give sufficient contrast, putting the imaging rate of IMS on par with the fastest confocal scanners nowadays available. Scanning three dimensional volumes can be done by moving the sheet through the suspension, while taking pictures with a camera. Moving the sheet can be done at practically arbitrary rates\textsuperscript{14}, so three dimensional samples consisting of hundreds of cross-sections can be imaged within seconds.

Another advantage of the IMS technique is that it is cheap compared to the other techniques mentioned. The technology required is a standard workstation, a laser sheet on a translation stage, and a digital camera, so cost are about 10-20 k\text{}$\textsuperscript{15}. In comparison, an MRI scanner costs on the order of 1 M\text{}$, X-ray tomography scanners cost several 100k\$, and a fast confocal 100-200k\$. These scanning techniques are also usually bought as stand-alone units, and cannot easily be adapted to the requirements of the specific experiment; IMS, with its simple components, can be almost freely adapted to any suspension and flow geometry.

There are also a few disadvantages to the IMS technique: only suspension flows can be images, since the particles always have to be studied in an index matched fluid. Voxel\textsuperscript{16} resolutions are limited by the thickness of the laser sheet used, which is usually on the order of a few tens to a few hundred microns. The technique also requires very good matching of the refractive index of the particles and the fluid, otherwise light scatter will limit visibility to only a few layers in the bulk. We have so far been able to visualize 20-30 layers with relative ease.

\textsuperscript{14}Mechanically, by moving the laser, or optically, by illuminating a cross-section via a rotatable mirror.

\textsuperscript{15}The cost of software are not taken into account.

\textsuperscript{16}A voxel is the three dimensional generalization of the concept of a pixel, a unit of surface.