Perceptual Mapping
Based on Three-Way Binary Data

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Perceptual Mapping - 1

- **Perceptual mapping:** Graphical display summarizing consumers’ perceptions of multi-attribute objects.

- **Example:** Displaying brands in a product class together with their attributes e.g. brands for treating stomach problems.

- **Brunswick’s (1955) Lens Model:**
  Theoretical foundation for understanding the importance of perceptions in consumer purchases

  \[ \text{Perceptions} \rightarrow \text{Preferences} \rightarrow \text{Choice} \]
Perceptual Mapping - 2

• **Goals perceptual mapping**
  – Aid for strategic marketing decisions
  – *Summarizing nature and degree of competition among a set brands via key product attributes.*

• **Common application areas**
  – product positioning
  – *identification of market gaps for new product development.*
Perceptual Mapping - 3

• **Basic data**
  – Brands are scored on a number of attributes by several individuals
  – Scores averaged over individuals
  – Result: Brand by Attribute matrix

• **Common data analysis techniques**
  – correspondence analysis
  – principal component analysis,
  – multidimensional analysis
  – discriminant analysis
  – factor analysis
Perceptual Mapping - 4

• Basic data
  – Doctor thinks a brand possesses an attribute => score = 1, if not: score = 0
  – Three-way binary data: Brands × Attributes × Doctors
  – Why average over doctors?
  – Different doctors may be sensitive to different attributes

• Three-way data analysis techniques
  – Three-mode binary hierarchical cluster analysis
  – Three-mode principal component analysis (numerical)
The Binary Data Cube

$i=1,...,I$
Objects (Brands)

$j=1,...,J$
Variables (Attributes)

$k=1,...,K$
Subjects (Doctors)

Fibers
Slices

011001
100101
111000
Stacked Two-Way Data

Columns: Attributes
1 through J

Rows: Brands 1 through I

Doctor 1 ($k=1$)

Doctor 2 ($k=2$)

Doctor $K$ ($k=K$)

010001
101101
111010

010001
010101
110001

011001
100101
111000

Rows: Brands 1 through I

Rows: Brands 1 through I

Rows: Brands 1 through I
HICLAS3: Algebraic Representation
(Tucker3-HICLAS)

• **Hiclas3 model** (uses Boolean algebra)

\[ \hat{\chi}_{ijk} = m_{ijk} = \bigoplus_{p=1}^{P} \bigoplus_{q=1}^{Q} \bigoplus_{r=1}^{R} \tilde{a}_{ip} \tilde{b}_{jq} \tilde{c}_{kr} \tilde{g}_{pqr} \]

• \( m_{ijk} = 1 \) iff \( \tilde{a}_{ip} = 1 \) and \( \tilde{b}_{jq} = 1 \) and \( \tilde{c}_{kr} = 1 \) and \( \tilde{g}_{pqr} = 1 \) for at least one combination of \( p, q, \) and \( r; \)

• \( \tilde{a}_{ip}, \tilde{b}_{jq}, \tilde{c}_{kr} \): elements binary component matrices A, B, and C, respectively (brands, attributes, doctors).

• \( \tilde{g}_{pqr} \): element of the \( P \times Q \times R \) three-way binary core array \( \tilde{G} \), indicates links between binary components of the three modes.
### HICLAS3 – Pictorial Representation

#### Tables:

<table>
<thead>
<tr>
<th>brands</th>
<th>attributes</th>
<th>doctors</th>
<th>core array</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1 1</td>
<td>1 0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 1</td>
<td>0 1</td>
<td>1 0</td>
</tr>
<tr>
<td>3</td>
<td>0 1</td>
<td>0 1</td>
<td>1 1</td>
</tr>
<tr>
<td>4</td>
<td>1 0</td>
<td>1 0</td>
<td>1 1</td>
</tr>
<tr>
<td>5</td>
<td>0 1</td>
<td>0 1</td>
<td>1 0</td>
</tr>
<tr>
<td>6</td>
<td>0 0</td>
<td>1 0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1 1</td>
<td>1 0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Core Array:

- $G_1$
- $G_2$

#### Equations:

$m_{211} = 1$ as $a_{22}b_{12}c_{11}g_{221} = 1 \times 1 \times 1 \times 1$ (all other 7 combinations contain a zero)
Three-Mode Component Analysis

- **Tucker3 model** (numerical)

\[ \hat{x}_{ijk} = m_{ijk} = \sum_{p=1}^{P} \sum_{q=1}^{Q} \sum_{r=1}^{R} a_{ip} b_{jq} c_{kr} g_{pqr} \]

- \( i=1,...,I \) (brands); \( j=1,...,J \) (attributes); \( k=1,...,K \) (doctors);
- \( m_{ijk} \) is the model matrix or structural image
- \( a_{ip}, b_{jq}, c_{kr} \) : elements loading matrices \( A, B, \) and \( C \), respectively (brands, attributes, doctors).
- \( g_{pqr} \) : element of the \( P \times Q \times R \) three-way core array \( G \); indicates strength of the link between the components of the three modes
Three-Mode Binary Analysis in Action

Perceptions of Medical Doctors w.r.t.
Gastro-Intestinal Drugs
Perceptions of Medical Doctors

Gastro-Intestinal Drugs

- Tagamet
- Zantac
- Pepcid
- Axicid
- Sulcrate
- Cytotec
- Losec
Attributes

Adjectives  [Binary answers– no (0) or yes (1)]

• Relieves Pain  RelPain
• Does not have serious side effects  NoSideEf
• Relatively safe w.r.t. potential interactions with other drugs  Safe
• Flexible in terms of dosage  FlexDose
• Not too costly for the patient  LowCost
• Relieves symptoms  RelSymptoms
• Promotes healing  Heals
• Prophylactic  Prophylactic
Data: Brands × Attributes × Doctors
(7×8×283)

- Variables (Attributes): $j=1,...,8$
- Subjects (Doctors): $k=1,...,283$
- Objects (Brands): $i=1,...,7$

**MODE A**

**MODE B**

**MODE C**
Perceptions of Medical Doctors

Central questions

• What is the position of brands w.r.t. each other?

• Which attributes are related to this positioning?

• Do doctors differ in their perceptions in which brands have which attributes?
HiClas3 Model

Tucker3 hierarchical classes model

Basic elements
• Binary components for all three modes (doctors, brands and attributes)
• Plus linkage information about the components

Basic literature
• Papers by Ceulemans, Van Mechelen in *Psychometrika* (Catholic University Leuven, Belgium)
HiClas3 – Choosing a Model

Brands × Attributes × Doctors

Model complexity: \((3,3,2) = (\text{Brands} = 3 \text{ components} ; \text{Attr} = 3; \text{Docs} = 2)\)

Discrepancy: Data have a 1, model matrix a 0 and vice versa
Binary Component Matrices  
(brands; attributes)

<table>
<thead>
<tr>
<th>Brand</th>
<th>Discrepancies</th>
<th>Fit</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sulcrate</td>
<td>659</td>
<td>0.626</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Cytotec</td>
<td>645</td>
<td>0.564</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Zantac</td>
<td>488</td>
<td>0.709</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Pepcid</td>
<td>388</td>
<td>0.743</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Axid</td>
<td>467</td>
<td>0.691</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Losec</td>
<td>499</td>
<td>0.665</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Tagamet</td>
<td>627</td>
<td>0.589</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Discrepancies</th>
<th>Fit</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relieves Pain</td>
<td>369</td>
<td>0.79</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Relieves Symptoms</td>
<td>330</td>
<td>0.82</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Promotes Health</td>
<td>406</td>
<td>0.77</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>No Side Effects</td>
<td>517</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Relatively Safe</td>
<td>542</td>
<td>0.57</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Flexible Dose</td>
<td>632</td>
<td>0.52</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Prophylactic</td>
<td>500</td>
<td>0.61</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Low Cost</td>
<td>477</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

B1 = Cytoprotective agent  
B2 = Tagamet (Oldest)  
B3 = Histamines; H-2 blocker  
A1 = Primary medical  
A2 = Use in practice  
A3 = Secondary medical  

Low Cost had no relations with other attributes
## Binary Component Matrices (doctors)

<table>
<thead>
<tr>
<th>Doctors</th>
<th>MD1</th>
<th>MD2</th>
<th>f</th>
<th>Prop. 1s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctor Type 1</td>
<td>1</td>
<td>1</td>
<td>70</td>
<td>.73</td>
</tr>
<tr>
<td>Doctor Type 2</td>
<td>1</td>
<td>0</td>
<td>69</td>
<td>.50</td>
</tr>
<tr>
<td>Doctor Type 3</td>
<td>0</td>
<td>1</td>
<td>98</td>
<td>.61</td>
</tr>
<tr>
<td>Doctor Type 4</td>
<td>0</td>
<td>0</td>
<td>46</td>
<td>.28</td>
</tr>
</tbody>
</table>

Average sd = .09

Doctor Type 4 (0,0) has no links with other doctors
### Binary Core Array

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>B1 (Cytoprotective)</th>
<th>1</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2 (Tagamet)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B3 (Histamines)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Dr3** (0 1)

<table>
<thead>
<tr>
<th>B1 (Cytoprotective)</th>
<th>1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2 (Tagamet)</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B3 (Histamines)</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Dr1** = Dr2 + Dr3

1: a link exists between components of the three modes
Doctor Type 2

- Sulcrate
  - Zantac, Avid, Pepcid, Losec
  - No side effects
  - Safe
  - Relieves pain and symptoms, Promotes health

- Cytotec
  - Prophylactic

- Tagamet
  - Flexible dose
  - Low cost

$n = 69$
Doctor Type 3

- **Sulcrate**
  - Zantac, Axiv, Pepcid, Losec
  - Cytotec
  - Tagamet

- **Relieves pain and symptoms, Promotes health**
- **No side effects**
- **Safe**
- **Prophylactic**
- **Flexible dose**
- **Low cost**

$n = 98$
Doctor Type 1

Sulcrate

Zantac, Axic, Pepcid, Losec

Cytotec

Tagamet

No side effects
Safe

Prophylactic

Flexible dose

Relieves pain and symptoms, Promotes health

Low cost

n = 70
Doctor Types

**Sulcrate**
- Zantac, Avid, Pepcid, Losec
- No side effects
- Safe
- Relieves pain and symptoms, Promotes health

**Cytotec**
- Prophylactic

**Tagamet**
- Flexible dose
- Low cost

Dr1
Dr2
Dr3
## Characterisation of Doctor Types

<table>
<thead>
<tr>
<th>Brand Name</th>
<th>Dr 2 ((n=69))</th>
<th>Dr 3 ((n=98))</th>
<th>Dr 1 ((n=70))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zantac</td>
<td>a0, a1</td>
<td>a0, a2, a3</td>
<td>a0, a1, a2, a3</td>
</tr>
<tr>
<td>Axic</td>
<td>a0, a1</td>
<td>a0, a2, a3</td>
<td>a0, a1, a2, a3</td>
</tr>
<tr>
<td>Pepcid</td>
<td>a0, a1</td>
<td>a0, a2, a3</td>
<td>a0, a1, a2, a3</td>
</tr>
<tr>
<td>Losec</td>
<td>a0, a1</td>
<td>a0, a2, a3</td>
<td>a0, a1, a2, a3</td>
</tr>
<tr>
<td>Cytotec</td>
<td>a0, a1</td>
<td>a0, a1, a3</td>
<td>a0, a1, XX, a3</td>
</tr>
<tr>
<td>Sulcrate</td>
<td>a0, a1</td>
<td>a0, a1, a2, a3</td>
<td>a0, a1, a2, a3</td>
</tr>
<tr>
<td>Tagamet</td>
<td>a0, a1, a2</td>
<td>a0, a2</td>
<td>a0, a1, a2, XX</td>
</tr>
</tbody>
</table>

- \(a0=\{\text{Relieves Pain, Relieves Symptoms, Promotes Healing}\}\)
- \(a1=\{\text{Prophylactic}\}, \ a2=\{\text{Flexible Dosage}\}, \ a3=\{\text{No Side Effects, Safe}\}\)
- Doctor Type 4 has no links; Low Cost has no links
Further Considerations

• No information on Low Cost (more complex HiClas models can model a separate component for Low Cost)

• Tagamet is relatively inexpensive, while the others are not

• Don’t the doctors see this?

• HiClas3 suggest they do not.
## Proportions of Ones across Doctors

<table>
<thead>
<tr>
<th></th>
<th>Tagamet</th>
<th>Zantac</th>
<th>PepCid</th>
<th>Axid</th>
<th>Losec</th>
<th>Sulcrate</th>
<th>Cytotec</th>
</tr>
</thead>
<tbody>
<tr>
<td>RelievePain</td>
<td>.8</td>
<td>.9</td>
<td>.8</td>
<td>.7</td>
<td>.8</td>
<td>.7</td>
<td>.5</td>
</tr>
<tr>
<td>RelieveSymptoms</td>
<td>.9</td>
<td>.9</td>
<td>.8</td>
<td>.8</td>
<td>.9</td>
<td>.7</td>
<td>.6</td>
</tr>
<tr>
<td>PromotesHealth</td>
<td>.7</td>
<td>.8</td>
<td>.7</td>
<td>.8</td>
<td>.8</td>
<td>.7</td>
<td>.6</td>
</tr>
<tr>
<td>NoSideEffect</td>
<td>.3</td>
<td>.7</td>
<td>.6</td>
<td>.6</td>
<td>.4</td>
<td>.8</td>
<td>.4</td>
</tr>
<tr>
<td>RelativeSafe</td>
<td>.2</td>
<td>.6</td>
<td>.5</td>
<td>.5</td>
<td>.4</td>
<td>.7</td>
<td>.4</td>
</tr>
<tr>
<td>FlexbileDose</td>
<td>.7</td>
<td>.7</td>
<td>.5</td>
<td>.4</td>
<td>.3</td>
<td>.4</td>
<td>.3</td>
</tr>
<tr>
<td>Prophylactic</td>
<td>.4</td>
<td>.5</td>
<td>.4</td>
<td>.3</td>
<td>.2</td>
<td>.6</td>
<td>.7</td>
</tr>
<tr>
<td>LowCost</td>
<td>.7</td>
<td>.2</td>
<td>.2</td>
<td>.2</td>
<td>.0</td>
<td>.3</td>
<td>.1</td>
</tr>
</tbody>
</table>
Further Considerations

• **Surprise**
  Tagamet is the only relatively inexpensive brand

• **Possible reason:**
  Doctors from all groups say Tagamet is not expensive.
  Thus unrelated to the present groups.

• **Possible solution:**
  More groups for attributes
  (we are working on this)

• **Question**
  Other variability not present in HiClas solution?
Further Analyses

• Treat the binary data as numerical and analyse with Tucker3.

• Handle the data such that emphasis is on:
  – relative differences between brands
  – relative differences between attributes.
Three-Mode Component Analysis

• Concentrate on consensus and individual differences between doctors in the relationships between brands and attributes.

• Absolute differences between brand and between attributes are ignored.

\[
\begin{array}{cc}
0 & \begin{array}{cc}
M & 5 \\
A & B \\
C & D \\
E & 5
\end{array} \\
5 & \begin{array}{cc}
M & 10 \\
A & B \\
C & D \\
E & 5
\end{array} \\
-2 & \begin{array}{cc}
M & +2 \\
A & B \\
C & D \\
E & -2
\end{array}
\end{array}
\]
Component Scores

Consensus among doctors

Individual differences between doctors

Tucker3_332 Subject Component 1

Tucker3_332 Component 2
Joint Biplot
(Consensus among doctors - Mean)

Mean of each brand and each attribute
Joint Biplot
(Individual differences between doctors - Deviations from mean)
HiClas model

• Given the data are binary, the binary hierarchical classes model is an obvious analysis method and has a relatively straightforward interpretation.
• Effective graphics to display results
• Many components might be necessary to model all systematic variability present.
Conclusions - 2

Tucker3 model

- By using a numerical model variance can be portrayed in a different and also insightful manner
- Differential weighting may simplify model description
- Enlightening graphics are available (joint biplots), but it requires some training to understand them
Conclusions - 3

Substantive conclusions concern the perceptual mappings of the brands with respect to the attributes as seen by the doctors.

The main patterns have been discussed during the presentation and will not be repeated.
Thank You.

01100100 10010101 11100011

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Tucker3 Model in Matrix Notation

\[ X = AG (B' \otimes C') + \varepsilon \]

A, \( (I \times P) \) loadings matrix for \textit{brands}
B, \( (J \times Q) \) loadings matrix for \textit{attributes}
C, \( (K \times R) \) loadings matrix for \textit{subjects}
G, \( (P \times Q \times R) \) core array with \textit{links} between the components
PARAFAC/CANDECOMP Model:

\[ \hat{x}_{ijk} = m_{ijk} = \sum_{s=1}^{S} g_{sss} a_{is} b_{js} c_{ks} \]

- \(m_{ijk}\) is the **model matrix** or **structural image**
- \(A\) is the \((I \times S)\) loadings matrix for **brands**
- \(B\) is the \((J \times S)\) loadings matrix for **attributes**
- \(C\) is the \((K \times S)\) loadings matrix for **subjects**
- \(G\) is the \((S \times S \times S)\) superdiagonal core array
  - exclusive **links** between the components \(s\) of the three modes

- Based on the principle of Parallel Proportional Profiles (Cattell 1944).
## Three-Mode Components Analysis: Model Comparison

<table>
<thead>
<tr>
<th>MODELS</th>
<th>NUMBER OF COMPONENTS</th>
<th>STANDARDIZED SS</th>
<th>Number of Param.</th>
<th>St. Fit/#Param (x1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TUCKALS2</td>
<td>3 3 ---</td>
<td>.49</td>
<td>2754</td>
<td>.19</td>
</tr>
<tr>
<td>TUCKALS2</td>
<td>2 3 ---</td>
<td>.40</td>
<td>1723</td>
<td>.23</td>
</tr>
<tr>
<td>TUCKALS3</td>
<td>3 3 5</td>
<td>.40</td>
<td>1462</td>
<td>.27</td>
</tr>
<tr>
<td>TUCKALS2</td>
<td>2 2 ---</td>
<td>.31</td>
<td>1154</td>
<td>.27</td>
</tr>
<tr>
<td>TUCKALS3</td>
<td>2 2 4</td>
<td>.31</td>
<td>1154</td>
<td>.27</td>
</tr>
<tr>
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<td>1165</td>
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<td>.37</td>
<td>1179</td>
<td>.37</td>
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<tr>
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<td>.32</td>
<td>888</td>
<td>.36</td>
</tr>
<tr>
<td>TRILIN</td>
<td>3 3 3</td>
<td>.32</td>
<td>883</td>
<td>.36</td>
</tr>
<tr>
<td>TUCKALS3</td>
<td>2 3 3</td>
<td>.32</td>
<td>883</td>
<td>.36</td>
</tr>
</tbody>
</table>

**COMPUTATION OF NUMBER OF PARAMETERS:**

A + B + C + core - transformational freedom

<table>
<thead>
<tr>
<th>MODELS</th>
<th>A<em>B</em>C + core - transformational freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>TUCKALS2</td>
<td>I<em>P + J</em>Q + K<em>Q</em>P - K*P<strong>2 - Q</strong>2</td>
</tr>
<tr>
<td>TUCKALS3</td>
<td>I<em>P + J</em>Q + K<em>R + P</em>Q*R - P<strong>2 - Q</strong>2 - R**2</td>
</tr>
<tr>
<td>PARAFAC</td>
<td>I<em>S + J</em>S + K*S + S - S - S - S</td>
</tr>
</tbody>
</table>
## Varimax Rotation: Deciding On Weights

<table>
<thead>
<tr>
<th>Relative Weights</th>
<th>Varimax Value</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Core</td>
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<tr>
<td>unrotated</td>
<td>2.136</td>
</tr>
<tr>
<td>0</td>
<td>2.782</td>
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<tr>
<td>0.5</td>
<td>2.665</td>
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<tr>
<td>1.0</td>
<td>2.603</td>
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<tr>
<td>1.5</td>
<td>2.561</td>
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<tr>
<td>2.0</td>
<td>2.519</td>
</tr>
<tr>
<td>2.5</td>
<td>2.443</td>
</tr>
<tr>
<td>3.0</td>
<td>2.180</td>
</tr>
<tr>
<td>3.5</td>
<td>2.134</td>
</tr>
<tr>
<td>4.0</td>
<td>2.099</td>
</tr>
<tr>
<td>4.5</td>
<td>2.071</td>
</tr>
<tr>
<td>5.0</td>
<td>2.049</td>
</tr>
<tr>
<td>5.5</td>
<td>2.030</td>
</tr>
<tr>
<td>100</td>
<td>1.895</td>
</tr>
<tr>
<td>1000</td>
<td>1.843</td>
</tr>
<tr>
<td>10000</td>
<td>1.842</td>
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<tr>
<td>0.5</td>
<td>2.629</td>
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<tr>
<td>0.5</td>
<td>2.522</td>
</tr>
<tr>
<td>1.0</td>
<td>2.522</td>
</tr>
<tr>
<td>1.0</td>
<td>2.582</td>
</tr>
</tbody>
</table>
Joint biplot for Brands and Attributes
First versus Third Component
for Second Component of Doctors

16/05/08 10:48:17
## Components for Brands and Attributes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Unrotated Components (Orthonormal)</th>
<th>Components After Varimax Rotation of the Core Matrix</th>
<th>Components After Joint Varimax Rotation of Components and the Core</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3</td>
<td>1 2 3</td>
<td>1 2 3</td>
</tr>
<tr>
<td><strong>Brands:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>.595 -.701</td>
<td>.836 -.385</td>
<td>.919 .034</td>
</tr>
<tr>
<td>G</td>
<td>.098 .213</td>
<td>-.001 .234</td>
<td>-.175 .371</td>
</tr>
<tr>
<td>D</td>
<td>-.609 -.385</td>
<td>-.390 -.606</td>
<td>-.220 -.414</td>
</tr>
<tr>
<td>E</td>
<td>-.462 -.083</td>
<td>-.384 -.270</td>
<td>-.074 -.716</td>
</tr>
<tr>
<td>B</td>
<td>.089 .304</td>
<td>-.048 .031</td>
<td>-.159 .259</td>
</tr>
<tr>
<td>C</td>
<td>.105 .285</td>
<td>-.025 .302</td>
<td>-.184 .258</td>
</tr>
<tr>
<td>F</td>
<td>.183 -.367</td>
<td>.011 .410</td>
<td>-.107 .209</td>
</tr>
<tr>
<td><strong>Attributes:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inexp</td>
<td>.481 .536 -.012</td>
<td>.573 -.436 -.001</td>
<td>.709 .100 .080</td>
</tr>
<tr>
<td>NoSiEf</td>
<td>-.350 -.222 .127</td>
<td>-.361 .188 -.149</td>
<td>-.419 -.005 .111</td>
</tr>
<tr>
<td>Safe</td>
<td>-.467 -.251 .229</td>
<td>-.461 .225 -.266</td>
<td>-.535 -.001 .218</td>
</tr>
<tr>
<td>Prophy</td>
<td>-.490 .575 -.421</td>
<td>-.424 -.736 .160</td>
<td>-.006 .863 -.064</td>
</tr>
<tr>
<td>RelPain</td>
<td>.291 -.302 -.122</td>
<td>.194 .301 .249</td>
<td>.050 -.327 -.284</td>
</tr>
<tr>
<td>FlexDo</td>
<td>.130 .195 .762</td>
<td>.302 .036 -.737</td>
<td>.177 -.261 .732</td>
</tr>
<tr>
<td>RelSym</td>
<td>.245 -.278 -.267</td>
<td>.130 .234 .371</td>
<td>.037 -.224 -.397</td>
</tr>
<tr>
<td>Heals</td>
<td>.160 -.253 -.296</td>
<td>.048 .188 .373</td>
<td>-.013 -.145 -.395</td>
</tr>
</tbody>
</table>
## Core Array

<table>
<thead>
<tr>
<th>Components for Brands:</th>
<th>Components for Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frontal Slice 1:</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>13.777</td>
</tr>
<tr>
<td>2</td>
<td>-3.628</td>
</tr>
<tr>
<td><strong>Frontal Slice 2:</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-2.081</td>
</tr>
<tr>
<td>2</td>
<td>-5.500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unrotated</th>
<th>Varimax Rotation of the Core Only</th>
<th>Joint Varimax Rotation of the Components and the Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>14.441</td>
<td>.045</td>
</tr>
<tr>
<td>2</td>
<td>.592</td>
<td>-9.748</td>
</tr>
</tbody>
</table>

| 1         | 2                                 | 3         |
| 1         | 14.105                            | -1.235    | .610 |
| 2         | 7.680                             | 7.997     |     |
Assessment of Goodness of Model Fit:

- Kroonenberg and De Leeuw (1980), and Kroonenberg (1983) show that
  \[ SS(\text{Residual}) = SS(\text{Total}) - SS(\text{Fit}) \]
  \[ SS \text{ Accounted For} = \frac{SS(\text{Fit})}{SS(\text{Total})} \]

- Also, as it has been shown by Ten Berge, De Leeuw, and Kroonenberg (1987), when the ALS algorithm has converged,
  \[ SS(\text{Residual}_m) = SS(\text{Total}_m) - SS(\text{Fit}_m) \]
  where \( m \) stands for any level of any mode of the data matrix.

- Using the last relationship, the relative fit of individual levels of a mode can be established. Also, whether a given level fits the model well or badly can be determined.
Model selection Tucker3 model

Deviance versus Sum of Numbers of Components
(Three-Mode Scree Plot)

Deviance (SS(Residual))

Sum of Numbers of Components (S = P + Q + R)

16/05/08 09:58:28
Tucker 3 Solutions

<table>
<thead>
<tr>
<th></th>
<th>Raw SS</th>
<th>Standardized SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS(Total)</td>
<td>1564.857</td>
<td>1.0000</td>
</tr>
<tr>
<td>A.EST.SS(Fit)</td>
<td>885.938</td>
<td>.5661</td>
</tr>
<tr>
<td>B.EST.SS(Fit)</td>
<td>1008.107</td>
<td>.6442</td>
</tr>
<tr>
<td>C.EST.SS(Fit)</td>
<td>465.270</td>
<td>.2973</td>
</tr>
<tr>
<td>SS(Fit)</td>
<td>430.970</td>
<td>.2754</td>
</tr>
<tr>
<td>SS(Residual)</td>
<td>1133.887</td>
<td>.7246</td>
</tr>
</tbody>
</table>

DF = Number of data points (minus loss of information due to preprocessing or missing data) minus the number of independent parameters

Number of independent parameters =

\[(I*P) + (J*Q) + (K*R) + (P*Q*R) - P**2 - Q**2 - R**2\]

with \(I, J, K\) the numbers of levels of 1st, 2nd, and 3rd modes, respectively,

and \(P, Q, R\) the numbers of components of 1st, 2nd, and 3rd modes, respectively.
Relating Subject Components to External Variables

- $Y$: Number of years of experience as a medical doctor (standardized)
- $X_1$: First component score for “subjects” mode,
- $X_2$: Second component score for “subjects” mode,

Linear Model: $Y = X_1 B_1 + X_2 B_2$

- Estimates: $B_1 = -1.054$, std. error = 0.997, $t = 1.058$, $p$-value = 0.29
- $B_2 = 1.350$, std. error = 0.997, $t = 1.345$, $p$-value = 0.18
- $R^2 = 0.01$, $F$-value = 1.477, $df = (2, 282)$, $p$-value = 0.23.

Conclusion: Subject components are not related to number of years of experience as a medical doctor.
Relating Residuals to External Variables:

- **Y**: Number of years of experience as a medical doctor (standardized)
- **X**: Sum of squares of residuals for each subject

**Linear Regression:**  \( Y = B X \)

- Estimated \( B = -0.006 \), \( R^2 = 0.0007 \), \( F\)-value = 0.212, d.f. = (1, 282)
- p-value = 0.646
- Conclusion: Residuals are not related to number of years of experience.
Result HiClas3-model (2D×3A×3B)

Sulcrate

Zantac, Avid, Pepcid, Losec

Cytotec

Tagamet

D1,D3

D1,D3

D1,D3

D1,D2

D1,D2

D4

No side effects
Safe

Relieves pain and symptoms, Promotes health

Prophylactic

Flexible dose

D1 (1 1) = 70
D2 (1 0) = 69
D3 (0 1) = 98
D4 (0 0) = 46

Low cost
HICLAS3: Example
(Tucker3-HICLAS)

Stimuli

Ignored in restaurant;
Disconnecting operator;
Closing store;
Missing page in book;

Subjects

Accused by instructor;
People tell lies about you;
Persistently contradicted;
Unfairly blamed for error

Response

Grimace

Hands tremble;
Perspire; Want to strike

Turn away; Lose patience;
Feel irritated; Curse

Become enraged;
Become tense;
Heart beats faster

Based on example Leuven group
Preprocessing: Double Centring
(in three-mode component analysis)

• **Double centring:**
doctors may not use the attributes uniformly across the brands and across the attributes.

• **Double centring:**
Scores in deviations from brand means \( \overline{x}_{i,k} \) and attribute means. \( \overline{x}_{.,jk} \)
Origin = zero point for both brands and attributes of each subject’s scores

\[
\begin{align*}
x_{ijk}^* &= x_{ijk} - \overline{x}_{.jk} - \overline{x}_{i,k} + \overline{x}_{..k}
\end{align*}
\]
Preprocessing: Double centring

Three-way factorial design without replacement (1 observation per cell):
Dependent variable: Brand possesses attribute (score = 1)

\[ x_{ijk} = m + a_i + b_j + c_k + ac_{ik} + bc_{jk} + ab_{ij} + abc_{ijk} \]

After centring:

\[ x_{ijk} = \text{consensus} + \text{differences} \]

Analysed with Three-mode PCA

\[ ab_{ij} = \text{consensus of doctors about attributes of brands} \]
\[ abc_{ijk} = \text{differences between doctors about attributes of brands} \]
Model selection Tucker3 model

Model complexity: (Docs = 2; Attr = 3; Brands = 3) or (Docs = 3; Attr = 3; Brands = 3)
Joint biplot
(Consensus)

First Component

Third Component

-3 -2 -1 0 1 2

-3 -2 -1 0 1 2

Tagamet
Zantac
Pepcid
Sulcrate
Cytotec
Axid
Losec
Prophylactic
Heals
RelSymptoms
RelPain
Safe
NoSideEff
LowCost
FlexDose

Joint biplot
(Consensus)
Conclusions

Where to go from here

• Irregular patterns in some doctors combined with low number of ones were excluded from the HiClas analysis while these doctors were scattered all over the plot of the doctors’ components. Thus Tucker analysis picked up some information which was not available to the HiClas analysis. Similarly for the LowCost attribute.
• Is the numerical information such as the variance somewhere to be found in the HiClas results and if so can it be used?
• Construct exactly fitting hierarchical classes models and run a Tucker3 analysis on them.
• Construct doctors/attributes/brands artificially according to a specific pattern and include them in the analysis to facilitate interpretation.
• Sort out the mathematics of the comparison between models.
HiClas3 – Three-Mode Deviance Plot

Doctors × Attributes × Brands

Model complexity: (Docs = 2; Attr = 3; Brands = 3) or
(Docs = 3; Attr = 3; Brands = 3)
Component Scores

HiClas3_332

Consensus among doctors

Individual differences between doctors
Gastro – SVD-Biplot
(means across doctors - attributes centred)

24/06/08 13:41:59