Charge Detection Enables Free-Electron Quantum Computation

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It is known that a quantum computer operating on electron spin qubits with single electron Hamiltonians and assisted by single-spin measurements can be simulated efficiently on a classical computer. We show that the exponential speedup of quantum algorithms is restored if single-charge measurements are added. These enable the construction of a CNOT (controlled NOT) gate for free fermions, using only beam splitters and spin rotations. The gate is nearly deterministic if the charge detector counts the number of electrons in a mode, and fully deterministic if it only measures the parity of that number.

It can count the occupation number of a spatial mode (0, 1, or 2 elections with opposite spin) If the point contact is replaced by a quantum dot with a resonant conductance, then it is possible to operate the device as a parity meter. It can distinguish occupation number one (when it is on resonance) from occupation number 0 or 2 (when it is off resonance)—but it cannot distinguish between 0 and 2. We will consider both types of charge detectors in what follows.

The no-go theorem [3,4] applies only to fermions, not to bosons. Indeed, in an influential paper [2], Knill, Laflamme, and Milburn showed that the exponential speedup over a classical algorithm cannot be reached with single-electron Hamiltonians assisted by single-spin measurements. Here we show that the full power of quantum computation is restored if single-charge measurements are added. These enable the construction of a CNOT gate for free fermions, using only beam splitters and spin rotations.

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determinant of order \(N\) can be evaluated in a time which scales polynomially with \(N\), the quantum algorithm can be simulated efficiently on a classical computer. This is the no-go theorem mentioned in the introduction.

We now add measurements of the local charge \(Q_i = n_i + n_d\) to the algorithm. The eigenvalues of \(Q_i\) are 0, 1, 2. The probability that charge one is measured is given by the expectation value of the projection operator

\[
P_i = 1 - (1 - Q_i)^2 = a_i^d a_i a_i^d a_i^t + a_i^d a_i a_i^d a_i^t \quad (1)
\]

The operator \(P_i\) is the sum of two local operators in the computational basis. The probability that \(M\) spatial modes are singly occupied therefore consists of a sum of an exponentially large number \(2^M\) of determinants, so now a classical simulation need no longer scale polynomially with the number of modes. Notice that a measurement of \(Q_i\) contains less information about the state than separate measurements of \(n_i\) and \(n_d\). The fact that partial measurements can add computational power is a basic principle of quantum algorithms [1].

Let us now see how these formal considerations could be implemented, by constructing a CNOT gate using only beam splitters, spin rotations, and charge detectors. To construct the gate we need one of two new building blocks that are enabled by charge detectors. The first building block is the Bell-state analyzer shown in Fig. 1. For this device it does not matter whether the charge detector operates as an electrometer or as a parity meter. The second building block, shown in Fig. 2, converts a charge parity measurement to a spin parity measurement. We present each device in turn and then show how to construct the CNOT gate.

The Bell-state analyzer makes it possible to teleport [19] the spin state \(|\alpha| + |\beta\rangle\) of an electron \(A\) to another electron \(A'\), using a third electron \(B\) that is entangled with \(A'\). The teleportation is performed by measuring the joint state of \(A\) and \(B\) in the Bell basis

\[
|\Psi_0\rangle = (|0\rangle - |1\rangle)/\sqrt{2} \quad (2)
\]
\[
|\Psi_1\rangle = (|0\rangle + |1\rangle)/\sqrt{2} \quad (3)
\]
\[
|\Psi_2\rangle = (|2\rangle + |3\rangle)/\sqrt{2} \quad (4)
\]
\[
|\Psi_3\rangle = (|2\rangle - |3\rangle)/\sqrt{2} \quad (5)
\]

A no-go theorem [20, 21] says that such a Bell measurement cannot be done deterministically (meaning with 100% success probability) without using interactions between the qubits. However, it has been noted that this theorem does not apply to qubits that possess an additional degree of freedom [22], and that is how we will work around it.

In Fig. 1 we show how a deterministic Bell measurement for noninteracting electrons can be performed using three 50/50 beam splitters, three charge detectors, and two local spin rotations (represented by Pauli matrices \(\sigma_x\) and \(\sigma_z\)). The beam splitter scatters two electrons into the same arm (bunching) if they are in the singlet state (2), and into two different arms (antibunching) if they are in one of the triplet states (3)(5). This can be easily understood from the antisymmetry of the wave function under particle exchange, demanded by the Pauli principle. The singlet state is antisymmetric in the spin degree of freedom, so the spatial part of the wave function should be

\[
\Psi = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} D_{00}.
\]

The Bell-state analyzer makes it possible to teleport a single electron to another, provided the two are entangled. The Bell state is the sum of two local operators in the computational basis. The probability that charge one is measured is given by the expectation value of the projection operator

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FIG 1 Bell state analyzer for noninteracting electrons, consisting of three 50/50 beam splitters (dashed horizontal lines), two mixers (solid horizontal lines), two local spin rotations (Pauli matrices \(\sigma_x\) and \(\sigma_z\)), and three charge detectors (squares). The charge detectors may operate either as electrometers (counting the occupation \(n_i = 0, 1, 2\) in an arm) or as parity meters (measuring \(p_i = q_i \mod 2\)). The first charge detector can identify the spin singlet state \(|\Psi_1\rangle\), which is the only one of the four Bell states (2)(4) to show \((p_i = 0)\) since \((\sigma \otimes \sigma)(|\Psi_1\rangle) = -|\Psi_1\rangle\) the second charge detector can identify \(|\Psi_2\rangle\) when \(p_2 = 0\). Finally since \((\sigma \otimes \sigma \otimes \sigma)(|\Psi_2\rangle) = |\Psi_2\rangle\) the third charge detector can identify the two remaining states \(|\Psi_3\rangle\) (when \(p_3 = 1\)) and \(|\Psi_4\rangle\) (when \(p_3 = 0\)).

FIG 2 Gate that converts a charge parity measurement to a spin parity measurement. The shaded box at the right represents the circuit shown at the left. A pair of electrons is incident in arms \(a\) and \(b\). A polarizing beam splitter (double dashed line) transmits spin up and reflects spin down. A charge detector records bunching \((p = 0)\) or antibunching \((p = 1)\) and passes the electrons on to a second polarizing beam splitter. If each electron at the input is in a spin eigenstate \(|\downarrow\rangle\) or \(|\uparrow\rangle\), then output equals input and \(p\) measures the spin parity \((p = 1\text{ if the two spins are aligned, and } p = 0\text{ if they are opposite})\). The gate can be used to encode a qubit \(|\downarrow\rangle\) as the two particle state \(|\downarrow\rangle|\uparrow\rangle\) and \(|\uparrow\rangle|\downarrow\rangle\) as \(|\uparrow\rangle|\downarrow\rangle\). For that purpose the input consists of the qubit to be encoded in arm \(a\) plus an ancilla in arm \(b\) in the state \(|\downarrow\rangle|\uparrow\rangle\) and \(|\uparrow\rangle|\downarrow\rangle\) as \(|\uparrow\rangle|\downarrow\rangle\). For that purpose the input consists of the qubit to be encoded in arm \(a\) plus an ancilla in arm \(b\) in the state \(|\downarrow\rangle|\uparrow\rangle\) and \(|\uparrow\rangle|\downarrow\rangle\) as \(|\uparrow\rangle|\downarrow\rangle\). For that purpose the input consists of the qubit to be encoded in arm \(a\) plus an ancilla in arm \(b\) in the state \(|\downarrow\rangle|\uparrow\rangle\) and \(|\uparrow\rangle|\downarrow\rangle\) as \(|\uparrow\rangle|\downarrow\rangle\). For that purpose the input consists of the qubit to be encoded in arm \(a\) plus an ancilla in arm \(b\) in the state \(|\downarrow\rangle|\uparrow\rangle\) and \(|\uparrow\rangle|\downarrow\rangle\) as \(|\uparrow\rangle|\downarrow\rangle\).
symmetric, and vice versa for the triplet state.) Let \( p_i \) be the charge \( q_i \) measured by detector \( i \mod 2 \). So \( p_i = 0 \) means bunching and \( p_i = 1 \) means antibunching after beam splitter \( \iota \). The quantity

\[
\mathcal{B} = p_1 + p_1p_2 + p_1p_2p_1
\]

(6)
takes on the value 0, 1, 2, or 3 depending on whether the incident state is \(|\Psi_0\rangle, |\Psi_1\rangle, |\Psi_2\rangle, \) or \(|\Psi_3\rangle\), respectively. The measurement of \( \mathcal{B} \) is therefore the required projective measurement in the Bell basis. It is a destructive measurement, so it does not matter whether the charge detector operates as an electrometer (measuring \( q_i \)) or as a parity meter (measuring \( p_i \)).

In Fig. 2 we show how a charge detector operating as a parity meter can be used to measure in a nondestructive way whether two spins are the same or opposite. "Nondestructive" means without measuring whether the spin is up or down. The device consists of two polarizing beam splitters in series, with the charge detector in between (a polarizing beam splitter fully transmits \( \uparrow \) and fully reflects \( \downarrow \)). At the input two electrons are incident in different arms. Input equals output if each electron is in a spin eigenstate. The measured charge parity then records whether the two spins are the same or opposite. We will refer to this device as an "encoder," because it can deterministically entangle a qubit in the arbitrary state \( \alpha|\uparrow\rangle + \beta|\downarrow\rangle \) and an ancilla in the fixed state \( (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} \) into the two-particle entangled state \( \alpha|\uparrow\rangle|\uparrow\rangle + \beta|\downarrow\rangle|\downarrow\rangle \).

To construct a CNOT gate using the Bell-state analyzer, we follow Ref. [2], where it was shown that teleportation can be used to convert a probabilistic logical gate into a nearly deterministic one. It is well known that a probabilistic CNOT gate can be constructed from beam splitters and single-qubit operations. The design of Pittman et al. [7] has success probability \( \frac{1}{3} \) and works for fermions as well as bosons. It consumes an entangled pair of ancillas, which can be created probabilistically using a beam splitter and charge detector [14]. Because the gate is not deterministic, it cannot be used in a scalable way inside the computation. However, the CNOT gate can be repeatedly executed off-line, independent of the progress of the quantum algorithm, until it has succeeded. Two Bell measurements teleport the CNOT operation into the computation [24], when needed. In this way a quantum algorithm can be executed using only single-particle Hamiltonians and single-particle measurements.

In Fig. 3 we show how to construct a CNOT gate using the encoder. Our design was inspired by that of Pittman et al. [7], but rather than being probabilistic it is exactly deterministic. We take two encoders in series, with a change of basis on going from the first to the second encoder. The change of basis is the Hadamard transformation.
a charge detector operating as an electrometer) or exactly
deterministically (using an encoder with a charge
detector operating as a parity meter) Unlike photons, elections
interact strongly if brought close together, so there is no
need to rely exclusively on single-particle Hamiltonians.
We expect that FEQC would ultimately be used for flying
qubits [25], while other gate designs based on short-range
interactions [15,26] would be preferred for stationary
qubits.

The two ingredients of the circuits considered here,
beam splitters [27,28] and charge detectors [13,16,17],
have both been realized by means of point contacts in a
two-dimensional electron gas. The time-resolved detection
required for the operation as a logical gate has not yet
been realized. The currently achievable time resolution
for charge detection is $\mu$s [16], while the resolution
required for ballistic elections in a semiconductor is in
the ps range. That time scale is not inaccessible [29], but it
might not be possible to reach the required single-election
sensitivity due to the unavoidable shot noise in the charge
detector. In the light of this, it could be more practical to
start with isolated elections in an array of quantum dots,
rather than with flying qubits, in order to investigate the
potential and limitations of our theoretical concept on a
presently accessible time scale.

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