Wittgenstein – Towards a Re-Evaluation

Proceedings of
the 14th International Wittgenstein-Symposium
Centenary Celebration

13th to 20th August 1989
Kirchberg am Wechsel (Austria)

Herausgeber/Editors
RUDOLF HALLER & JOHANNES BRANDL

Wien 1990
Verlag Hölder-Pichler-Tempsky
Sätze der Logik: an Alternative Conception

GÖRAN SUNDHOLM
University of Leiden

It is often claimed with reference to Wittgenstein’s later thought on the philosophy of mathematics that a metamathematical theorem, such as Gödel’s, has no more relevance for the philosophy of mathematics than any other mathematical theorem. Be that as it may: one (meta-) mathematical result, namely the Theorem of Church on the (recursive) undecidability of predicate logic without identity, seems to have a direct and lethal bearing on Wittgenstein’s Tractarian conception of logic.

The notion of a Satz der Logik undoubtedly plays a crucial role in the logical framework of the Tractatus. The usual English translation uses ‘proposition of logic’, which shows the same ambiguity as the German, namely that between ‘(unasserted) proposition which belongs to logic’ and ‘(asserted) theorem of logic’. In the Tractatus the former is undeniably the more important, but also the second reading hints at an important feature of the Sätze der Logik: they are clearly intended as the proper version of Frege’s ‘Begriffsschriftsätze’.

The Tractarian treatment of the propositions of logic can be found in the thirtyone theses from 6.1. to 6.2. In the first of these the Sätze der Logik are identified as the tautologies, i.e. those propositions that are logically true, come what may, independently of what is the case. Subsequently a number of characterizing marks are singled out. First, the Sätze der Logik say, or state, nothing (6.11). This mark is further amplified in a parenthetical rider which identifies the Sätze der Logik with the analytic propositions. The sense in which Wittgenstein uses ‘analytic’ is not altogether clear here. Kant gave three explanations of analyticity in the Kr.d.r.V. and other philosophers, such as Bolzano and Frege, offered yet further. Frege’s notion is proof-theoretical and can certainly be ruled out as far as the Tractatus is concerned, but Bolzano’s notion is very similar to that of a tautology. Given the location of the rider in the text of the Tractatus, though, the second Kantian explanation of an analytic judgement as an elucidation, rather than an expansion, of our knowledge seem to serve Wittgenstein’s purpose rather well. I shall, however, have occasion below to return to the Kantian explanations in connection with the alternative conception of the Sätze der Logik. Secondly, in 6.111 we learn that a Satz der Logik is without (factual) content. The laws of logic are not a part of natural science. Indeed, thirdly, their correct explanation must assign to them a unique status (6.112).

Apart from these marks which serve to fix the location of the Sätze der Logik, Wittgenstein imposes two further marks of a more epistemological nature, concerning the possibility of identifying a Satz der Logik as such. It must, fourthly, be possible to recognize mechanically, by calculation from the symbol alone, whether a proposition belongs to logic or not (6.113, 6.126, 6.1262). Therefore, fifthly, a Satz der Logik coincides with its own proof (6.1265), since the process of calculation is shown in the symbol itself (6.126, 6.1261).

Thus we have found five distinguishing marks of the Sätze der Logik. They must be non-stating and have no factual content. Furthermore, they have a unique position and can be recognized as logical from the symbol alone. Finally, they are their own proofs. Reflection on these marks shows that the Tractarian conception of logic is irreparably flawed. Given the Theorem of Church mentioned above it is clear that Wittgensteins’ Sätze der Logik do not fulfill the distinguishing marks he imposes. In particular, the possibility to decide, by a mechanical calculation from the symbol alone, whether a
proposition is logically true or not, fails. Church’s Thesis does hardly seem at issue here, whence the identification between recursiveness and decidability is permissible and it is clear that the logical language of the Tractatus is meant to comprise also of multiple generality. (In 6.1232 the Axiom of Reducibility, which contains nested iterated quantification, is singled out as a Satz.) Accordingly, within the Tractarian framework, one cannot decide mechanically if a Satz belongs to logic or not. Indeed, the decision procedure provided by Wittgenstein at 6.1203 is, as he observes, applicable solely to propositions lacking quantifiers.

The circumstance that Church’s Theorem refutes the Tractarian conception is, of course, well-known and has been noticed by, e.g., Anscombe, Black and Fogelin in their commentaries. Indeed, with the benefit of hindsight it is not surprising that tautologicity, i.e. logical truth, and mechanical recognition are incompatible. The Tractatus is the first major exponent of the “ontological turn” in logical theory. Previous writers in the objectivistic tradition, e.g. Bolzano, Frege and Russell, still retained strong epistemological interests and inference was one of their main concerns. Wittgenstein, on the other hand, is interested only in the relation of logical consequence between propositions, i.e. contents of judgements or assertions, and not in inference where one passes from judgements (assertions) to another judgement (assertion) (recall the criticism of the Urteilsstrich at 4.442). The relation of logical consequence is ultimately defined in terms of the truth of propositions and thus the main task of logic is no longer to characterize valid inference but to lay down what propositions are true given the truth of certain propositions. In view of the fact that the truth of a proposition consists in das Bestehen of the Sachverhalt it presents, logic is reduced to an immensely complex network of relations between Sachverhalte and these relations are completely independent of any judging or inferring activity. Thus, prima facie, it is not to be expected that one should be able to decide mechanically whether these relations obtain or not.

We have seen that Wittgenstein’s candidate for the Sätze der Logik, viz. the tautologies, do no fulfill the marks imposed in the Tractatus. Accordingly it is of interest to inquire whether some other satisfactory notion can be found within the Tractarian framework. The remainder of my paper is devoted to this task.

The Doctrine of Showing is one of the three main pillars that support the elaborate edifice of the Tractatus, the other two being the Picture Theory of Linguistic Representation and Logical Atomism. What can be stated in language, as opposed to only shown, is that an object has a certain material, or external, property and similarly for relations. Wittgenstein’s terminology for the opposite group of properties and relations, the possession of which can only be shown but not stated, is not entirely stable. Broadly speaking (4.122) Wittgenstein distinguishes terminologically between, on the one hand, the cases where the non-statable property, or relation, concerns carriers among Gegenstände and Sachverhalte and, on the other hand, among facts or Sachlagen. In the former case he uses formal and in the latter structural (or internal) as a qualification. A possible reason for this terminological shift might be found by considering the linguistic counterparts to Gegenstände and Sachverhalte, namely Names and Elementarsätze. It is clear from the Tractatus that to speak of a structural property of a Name or elementary proposition would be highly inappropriate (2.0201, 3.26, 5.555-5.5571). (In some places though Wittgenstein does use internal also for a property of Gegenstände, e.g. 2.0123.)

The candidate for an alternative conception of the Sätze der Logik that I now want to consider consists of ascriptions of the form

\[(*) \text{ a is } P\]

where P is a formal property. (I confine myself to this simplest case here, but the
discussion below can without difficulty be extended to cover also ascriptions of formal or internal relations to suitable \textit{relata}.) Such ascriptions satisfy Wittgenstein’s five marks surprisingly well. First they are clearly non-stating and, secondly, have no factual content since they express something which can only be shown but not stated. Thirdly, their unique role is to lay bare the presuppositions of language, in that they provide answers to the question: What properties must language have in order to be able to say the things it can say? Fourthly, that something falls under a formal concept cannot be said, but is shown in the very sign for the object in question (4.126). The sign for a formal concept is a variable and an object falls under the concept if the symbol for the object fits the place held open by that variable. It is here, indeed, possible to read off from the symbol “a” alone whether a is P. Finally, fifthly, since an object is given with its formal properties, an ascription of the form (*) serves as its own proof, since the property P is \textit{eo ipso} contained in the object a.

Thus, incidentally, Kant’s first explanation of analytic judgements applies to our ascriptions (*): the predicate P is contained in the concept of a. Also the second explanation fits: these ascriptions do not serve to extend our knowledge, but rather to elucidate our means for presenting knowledge. Finally, it would be self-contradictory to assume that a is not P (4.123). The object a is given as a P and cannot but be P. Hence, also Kant’s third explanation fits.

The alternative conception satisfies the five Tractarian marks and Kant’s three explanations of analytic judgements. There are, however, a number of problems here. Wittgenstein clearly intends formal concepts to be decidable from the symbol alone. On the other hand his counterparts to inductive definitions, viz. the formal series (4.1252, 4.1273, 6), are held to produce formal concepts. Not every inductively defined class is decidable, though. Furthermore, it is hardly the case that all ascriptions (*) are of equal status as is demanded in 6.127 for the \textit{Sätze der Logik}. On the contrary it seems clear that there must be a certain order among them, e.g. given two ascriptions that a is a \textit{Gegenstand} and that f(x) is a propositional function, we can obtain the third that f(a) is a proposition.

Thus, ultimately, I would hold that neither the tautologies nor the ascriptions of formal properties suffice to meet all the requirements found in the \textit{Tractatus} concerning the notion of a \textit{Satz der Logik}, and that most likely there is no coherent such notion to be had.

\textbf{Notes}

I want to thank Eva Picardi and Peter Simons for helpful comments on my Kirchberg talk.

1 The inspiration for this choice can be found in Per Martin-Löf’s type theory, where judgements of the form a:A, a is an object of type A, satisfy many of the properties of the \textit{Sätze der Logik}. See, e.g. “An intuitionistic theory of types” in: H.E. Rose/J. Sheperdson (eds.), \textit{Logic Colloquium’73}, Amsterdam 1975, pp. 73-118.