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Physica B 203 (1994) 219–225

PHYSICA B

Reflectionless tunneling through a double-barrier NS junction

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Abstract

The resistance is computed of an $\text{NI}_1\text{NI}_2\text{S}$ junction, where N is the normal metal, S the superconductor, and I_i the insulator or tunnel barrier (transmission probability per mode Γ_i). The ballistic case is considered, as well as the case that the region between the two barriers contains disorder (mean free path l , barrier separation L). It is found that the resistance at fixed Γ_2 shows a *minimum* as a function of Γ_1 , when $\Gamma_1 \approx \sqrt{2}\Gamma_2$, provided $l \gtrsim \Gamma_2 L$. The minimum is explained in terms of the appearance of transmission eigenvalues close to one, analogous to the “reflectionless tunneling” through a NIS junction with a disordered normal region. The theory is supported by numerical simulations.

1. Introduction

Reflectionless tunneling is a novel quantum interference effect which occurs when dissipative normal current is converted into dissipationless supercurrent at the interface between a normal metal (N) and a superconductor (S) [1]. Experimentally, the effect is observed as a peak in the differential conductance around zero voltage or around zero magnetic field [2]. Its name refers to the fact that, for full-phase coherence, the Andreev-reflected quasiparticle can tunnel through the potential barrier at the NS interface without suffering reflections. (The potential barrier can be the insulator (I) in an NIS junction, or the Schottky barrier in a semiconductor–superconductor junction.) Application of a voltage or magnetic field destroys the phase coherence between electrons and holes, and thus reduces the conductance of the junction. We now have a good theoretical understanding of the effect, based on a combination of numerical [3, 4], and analytical work [5–10]. The basic requirement for reflectionless tunneling is that the normal region has a resistance which is larger than the resistance of the interface. In that case the disorder is able to open a fraction of the tunneling channels, i.e. it induces the appearance of transmission eigenvalues close to one [10]. As a result of these open channels, the resistance has a linear dependence on the transparency of the interface, instead of the quadratic de-

pendence expected for Andreev reflection [11] (which is a two-particle process).

The purpose of this work is to present a study of reflectionless tunneling in its simplest form, when the resistance of the normal metal is due to a second tunnel barrier, in series with the barrier at the NS interface. This allows an exact calculation, which shows many of the features of the more complicated case when the resistance of the normal region is due to disorder. Furthermore, the double-barrier geometry provides an experimentally realizable model system, for example, in tunneling from an STM into a superconductor via a metal particle [12].

The outline of this paper is as follows. In Section 2 we consider the problem of a $\text{NI}_1\text{NI}_2\text{S}$ junction without disorder. We compute the resistance of the junction as a function of the transmission probabilities per mode Γ_1 and Γ_2 of the two barriers. The resistance at fixed Γ_2 shows a *minimum* as a function of Γ_1 when $\Gamma_1 \approx \sqrt{2}\Gamma_2 \equiv \Gamma$. The resistance in the minimum depends *linearly* on $1/\Gamma$, in contrast to the quadratic dependence in the case of a single barrier. In Section 3 we apply a recent scaling theory [9], to find the influence on the resistance minimum of disorder in the region between the barriers (length L , mean free path l). The resistance minimum persists as long as $l \gtrsim \Gamma L$. In the diffusive regime ($l \ll L$), our results agree with a previous Green's function calculation by Volkov et al. [7]. The analytical results are supported by numerical simulations, using the recursive Green's function technique [13]. We conclude in Section 4

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2. NINIS junction without disorder

We consider a $\text{NI}_1\text{NI}_2\text{S}$ junction, where N is the normal metal, S is the superconductor, and I_i the insulator or tunnel barrier (see inset of Fig. 1). The transmission probability per mode of I_i is denoted by Γ_i . For simplicity, we neglect the mode dependence of Γ_i . In this section, we assume ballistic motion between the barriers. (The effect of disorder in the normal region is considered in Section 3.) A straightforward calculation yields the transmission probabilities T_n of the two barriers in series,

$$T_n = (a + b \cos \phi_n)^{-1}, \quad (2.1)$$

where

$$a = 1 + \frac{2 - \Gamma_1 - \Gamma_2}{\Gamma_1 \Gamma_2}, \quad (2.2a)$$

$$b = \frac{2(1 - \Gamma_1)^{1/2}(1 - \Gamma_2)^{1/2}}{\Gamma_1 \Gamma_2}, \quad (2.2b)$$

and ϕ_n is the phase accumulated between the barriers by mode $n = 1, 2, \dots, N$ (with N the number of propagating modes at the Fermi level). If we substitute $\Gamma_i = 1/\cosh^2 \alpha_i$ ($\alpha_i \geq 0$), the coefficients a and b can be rewritten as

$$a = \frac{1}{2} + \frac{1}{2} \cosh 2\alpha_1 \cosh 2\alpha_2, \quad (2.3a)$$

$$b = \frac{1}{2} \sinh 2\alpha_1 \sinh 2\alpha_2. \quad (2.3b)$$

Since the transmission matrix t is diagonal, the transmission probabilities T_n are identical to the eigenvalues of tt^\dagger .

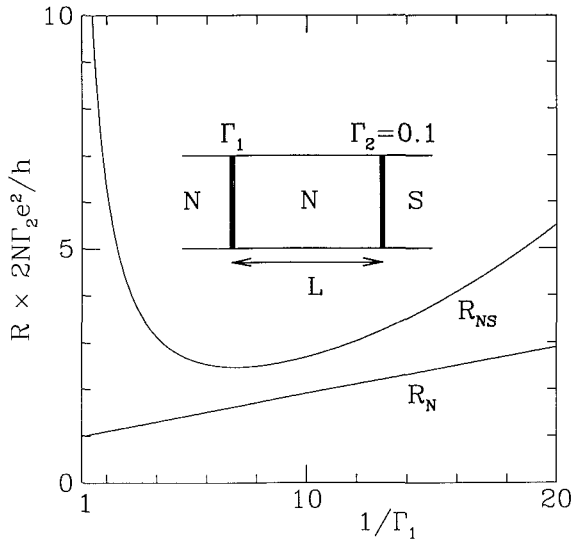


Fig. 1. Dependence of the resistances R_N and R_{NS} of ballistic NININ and NINIS structures, respectively, on barrier transparency Γ_1 , while transparency $\Gamma_2 = 0.1$ is kept fixed [computed from Eqs. (2.6) and (2.7)]. The inset shows the NINIS structure considered.

We use the general relationship between the conductance $G_{NS} \equiv G_{\text{NINIS}}$ of the NINIS junction and the transmission eigenvalues of the normal region [14],

$$G_{NS} = \frac{4e^2}{h} \sum_{n=1}^N \frac{T_n^2}{(2 - T_n)^2}, \quad (2.4)$$

which is the analogue of the Landauer formula,

$$G_N = \frac{2e^2}{h} \sum_{n=1}^N T_n, \quad (2.5)$$

for the conductance $G_N \equiv G_{\text{NININ}}$ in the normal state. We assume that $L \gg \lambda_F$ (λ_F is the Fermi wavelength) and $N\Gamma_i \gg 1$, so that the conductance is not dominated by a single resonance. In this case, the phases ϕ_n are distributed uniformly in the interval $(0, 2\pi)$ and we may replace the summations in Eqs. (2.4) and (2.5) by integrals over ϕ : $\sum_{n=1}^N f(\phi_n) \rightarrow (N/2\pi) \int_0^{2\pi} d\phi f(\phi)$. The result is

$$G_{NS} = \frac{4e^2 N}{h} \frac{\cosh 2\alpha_1 \cosh 2\alpha_2}{(\cosh^2 2\alpha_1 + \cosh^2 2\alpha_2 - 1)^{3/2}}, \quad (2.6)$$

$$G_N = \frac{4e^2 N}{h} (\cosh 2\alpha_1 + \cosh 2\alpha_2)^{-1}. \quad (2.7)$$

These expressions are symmetric in the indices 1 and 2; it does not matter which of the two barriers is closest to the superconductor.

In the same way we can compute the entire distribution of the transmission eigenvalues, $\rho(T) \equiv \sum_n \delta(T - T_n) \rightarrow (N/2\pi) \int_0^{2\pi} d\phi \delta(T - T(\phi))$. Substituting $T(\phi) = (a + b \cos \phi)^{-1}$ from Eq. (2.1), we find

$$\rho(T) = \frac{N}{\pi T} (b^2 T^2 - (aT - 1)^2)^{-1/2} \quad (2.8)$$

for $(a + b)^{-1} < T < (a - b)^{-1}$.

In Fig. 1 we plot the resistance $R_N = 1/G_N$ and $R_{NS} = 1/G_{NS}$, following from Eqs. (2.6) and (2.7). Notice that R_N follows Ohm's law,

$$R_N = \frac{h}{2Ne^2} (1/\Gamma_1 + 1/\Gamma_2 - 1), \quad (2.9)$$

as expected from classical considerations. In contrast, the resistance R_{NS} has a *minimum* if one of the Γ 's is varied while keeping the other fixed. This resistance minimum cannot be explained by classical series addition of barrier resistances. If $\Gamma_2 \ll 1$ is fixed and Γ_1 is varied, as in Fig. 1, the minimum occurs when $\Gamma_1 = \sqrt{2}\Gamma_2$. The minimal resistance R_{NS}^{min} is of the same order of magnitude as the resistance R_N in the normal state at the same value of Γ_1 and Γ_2 . (For $\Gamma_2 \ll 1$, $R_{NS}^{\text{min}} = 1.52R_N$) In particular, we find that R_{NS}^{min} depends linearly on $1/\Gamma_i$, whereas for a single barrier $R_{NS} \propto 1/\Gamma^2$.

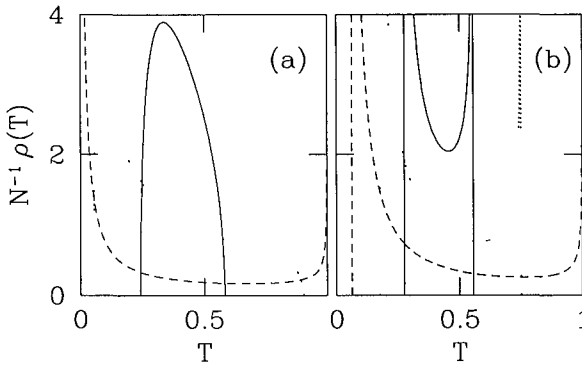


Fig. 2. Density of normal-state transmission eigenvalues for an NS junction with a potential barrier at the interface (transmission probability $\Gamma = 0.4$). The left panel (a) shows the disorder-induced opening of tunneling channels in a NIS junction (solid curve: $s = 0.04$, dotted: $s = 0.4$, dashed: $s = 5$; where $s \equiv L/l$). The right panel (b) shows the opening of channels by a second tunnel barrier (transparency Γ') in a NINIS junction (solid curve: $\Gamma' = 0.95$; dotted: $\Gamma' = 0.8$, dashed: $\Gamma' = 0.4$). The curves in (a) are computed from Ref. [9], the curves in (b) from Eq.(2.8). Notice the similarity of the dashed curves.

The linear dependence on the barrier transparency shows the qualitative similarity of a ballistic NINIS junction to a disordered NIS junction. To illustrate the similarity, we compare in Fig. 2 the densities of transmission eigenvalues through the normal region. The left panel is for an NIS junction (computed using the results of Ref. [9]), the right panel is for an NINIS junction (computed from Eq. (2.8)). In the NIS junction, disorder leads to a bimodal distribution $\rho(T)$, with a peak near zero transmission and another peak near unit transmission (dashed curve). A similar bimodal distribution appears in the ballistic NINIS junction for approximately equal transmission probabilities of the two barriers. There are also differences between the two cases: The NIS junction has a unimodal $\rho(T)$ if $L/l < 1/\Gamma$, while the NINIS junction has a bimodal $\rho(T)$ for any ratio of Γ_1 and Γ_2 . In both cases, the opening of tunneling channels, i.e., the appearance of a peak in $\rho(T)$ near $T = 1$, is the origin for the $1/\Gamma$ dependence of the resistance.

3. Effects of disorder

Let us now investigate what happens to the resistance minimum if the region of length L between the tunnel barriers contains impurities, with elastic mean free path l . We denote $s \equiv L/l$. When introducing disorder, it is necessary to consider ensemble-averaged quantities. To calculate the ensemble-averaged conductance $\langle G_{NS} \rangle$, we need to know the density ρ of the transmission eigenvalues T_n as a function of s . It is convenient to work with the parameterization

$$T_n = 1/\cosh^2 x_n, \quad x_n \geq 0. \quad (3.1)$$

The density of the x_n 's is defined by $\rho(x, s) \equiv \langle \sum_n \delta(x - x_n) \rangle$. From Eq. (2.1) we know that, for $s = 0$ (no disorder),

$$\begin{aligned} \rho(x, 0) &= N \int_0^{2\pi} \frac{d\phi}{2\pi} \delta(x - \operatorname{arccosh} \sqrt{a + b \cos \phi}) \\ &= \frac{N}{\pi} \sinh 2x (b^2 - (a - \cosh^2 x)^2)^{-1/2}, \end{aligned} \quad (3.2)$$

for $\operatorname{arccosh} \sqrt{a - b} \equiv x_{\min} < x < x_{\max} \equiv \operatorname{arccosh} \sqrt{a + b}$.

For $s > 0$ we obtain the density $\rho(x, s)$ from the integro-differential equation [15]

$$\begin{aligned} \frac{\partial}{\partial s} \rho(x, s) &= -\frac{1}{2N} \frac{\partial}{\partial x} \rho(x, s) \frac{\partial}{\partial x} \\ &\int_0^\infty dx' \rho(x', s) \ln |\sinh^2 x - \sinh^2 x'|, \end{aligned} \quad (3.3)$$

which is the large N -limit of the scaling equation due to Dorokhov [16] and Mello et al. [17]. This equation describes the evolution of $\rho(x, s)$ when an infinitesimal slice of disordered material is added. With initial condition (3.2) it therefore describes a geometry where all disorder is on one side of the two tunnel barriers, rather than in between. In fact, only the total length L of the disordered region matters, and not the location relative to the barriers. The argument is similar to that in Ref. [18]. The total transfer matrix M of the normal region is a product of the transfer matrices of its constituents (barriers and disordered segments): $M = M_1 M_2 M_3 \dots$. The probability distribution of M is given by the convolution $p(M) = p_1 * p_2 * p_3 * \dots$ of the distributions p_i of transfer matrices M_i . The convolution is defined as

$$p_i * p_j(M) = \int dM' p_i(MM'^{-1}) p_j(M'). \quad (3.4)$$

If for all parts i of the system, $p_i(M_i)$ is a function of the eigenvalues of $M_i M_i^\dagger$ only, the convolution of the p_i commutes [18]. The distributions p_i are then called isotropic. A disordered segment (length L , width W) has an isotropic distribution if $L \gg W$. A planar tunnel barrier does not mix the modes, so a priori it does not have an isotropic distribution. However, if the mode dependence of the transmission probabilities is neglected (as we do here), it does not make a difference if we replace its distribution by an isotropic one. The commutativity of the convolution of isotropic distributions implies that the location of the tunnel barriers with respect to the disordered region does not affect $\rho(x, s)$. The systems in Figs. 3(a)–(c) then have identical statistical properties.

Once $\rho(x, s)$ is known, the conductances $\langle G_{NS} \rangle$ and $\langle G_N \rangle$ can be determined from

$$\langle G_{NS} \rangle = \frac{4e^2}{h} \int_0^\infty dx \frac{\rho(x, s)}{\cosh^2 2x}, \quad (3.5)$$

$$\langle G_N \rangle = \frac{2e^2}{h} \int_0^\infty dx \frac{\rho(x, s)}{\cosh^2 x}, \quad (3.6)$$

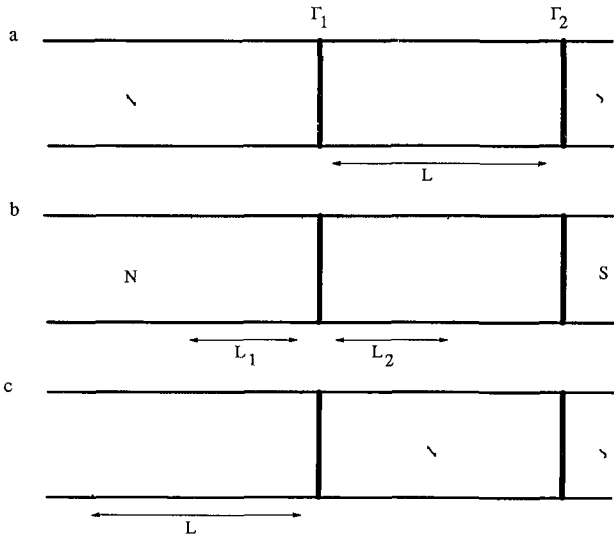


Fig. 3 The systems a, b, and c are statistically equivalent, if the transfer matrices of each of the two barriers (solid vertical lines) and the disordered regions (shaded areas, $L_1 + L_2 = L$ in case b) have isotropic distributions, in that case, the position of the disorder with respect to the barriers does not affect the eigenvalue density $\rho(x, s)$

where we have substituted Eq. (3.1) into Eqs. (2.4) and (2.5). In Ref. [9] a general solution to the evolution equation was obtained for arbitrary initial condition. It was shown that Eq. (3.3) can be mapped onto Euler's equation of hydrodynamics

$$\frac{\partial}{\partial s} U(\zeta, s) + U(\zeta, s) \frac{\partial}{\partial \zeta} U(\zeta, s) = 0 \quad (3.7)$$

by means of the substitution

$$U(\zeta, s) = \frac{\sinh 2\zeta}{2N} \int_0^\infty dx' \frac{\rho(x', s)}{\sinh^2 \zeta - \sinh^2 x'} \quad (3.8)$$

Here, $U \equiv U_x + iU_y$ and $\zeta \equiv x + iy$. Eq. (3.7) describes the velocity field $U(\zeta, s)$ of a 2D fluid at constant pressure in the x - y plane. Its solution is¹

$$U(\zeta, s) = U_0(\zeta - sU(\zeta, s)) \quad (3.9)$$

in terms of the initial value $U_0(\zeta) \equiv U(\zeta, 0)$. The probability distribution $\rho(x, s)$ follows from the velocity field by inversion of Eq. (3.8),

$$\rho(x, s) = \frac{2N}{\pi} U_y(x - i\varepsilon, s), \quad (3.10)$$

where ε is a positive infinitesimal

¹ The implicit equation (3.9) has multiple solutions in the entire complex plane, we need the solution for which both ζ and $\zeta - sU(\zeta, s)$ lie in the strip between the lines $y = 0$ and $y = -\pi/2$

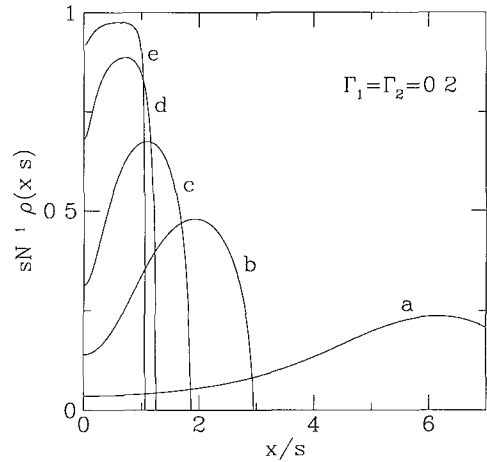


Fig. 4 Eigenvalue density $\rho(x, s)$ as a function of x (in units of $s \equiv L/l$) for $\Gamma_1 = \Gamma_2 = 0.2$. Curves a, b, c, d, and e are for $s = 0, 5, 20, 100, 1000$, respectively. In the special case of equal tunnel barriers, open channels exist already in the absence of disorder

In our case, the initial velocity field [from Eqs. (3.2) and (3.8)] is

$$U_0(\zeta) = -\frac{1}{2} \sinh 2\zeta [(\cosh^2 \zeta - a)^2 - b^2]^{-1/2} \quad (3.11)$$

The resulting density (3.10) is plotted in Fig. 4 for $\Gamma_1 = \Gamma_2 \equiv \Gamma$ and several disorder strengths. The region near $x = 0$ is of importance for the conductance (since x near zero corresponds to near-unit transmission). The number $N_{\text{open}} \equiv \rho(0, s)$ is an estimate for the number of transmission eigenvalues close to 1 (the so-called “open channels” [19]). In the absence of disorder, N_{open} is non-zero only if $\Gamma_1 \approx \Gamma_2$ (then $a - b = 1 \Rightarrow x_{\text{min}} = 0$). From Eq. (3.2) we find $N_{\text{open}} = N\Gamma/\pi$ for $s = 0$ and $\Gamma_1 = \Gamma_2 \equiv \Gamma \ll 1$. Adding disorder reduces the number of open channels. If $\Gamma_1 \neq \Gamma_2$ there are no open channels for $s = 0$ ($x_{\text{min}} > 0$). Disorder then has the effect of increasing N_{open} , such that $N_{\text{open}} \approx N/s$ if $(\Gamma_1 + \Gamma_2)s \gg 1$. The disorder-induced opening of channels was studied in Refs. [9, 10] for the case of a single-tunnel-barrier.

To test our analytical results for the eigenvalue density $\rho(x, s)$, we have carried out numerical simulations, similar to those reported in Ref. [9]. The sample was modeled by a tight-binding Hamiltonian on a square lattice with lattice constant a . The tunnel barriers were accounted for by assigning a non-random potential energy $U_B = 2.3E_F$ to a single row of sites at both ends of the lattice, which corresponds to a mode-averaged barrier transparency $\Gamma_1 = \Gamma_2 = 0.18$. The Fermi energy was chosen at $1.5u_0$, with $u_0 = \hbar^2/2ma^2$. Disorder was introduced by randomly assigning a value between $\pm \frac{1}{2}U_D$ to the on-site potential of the lattice points between the barriers. The disorder strength U_D was varied between 0 and $1.5u_0$, corresponding to s between 0 and 11.7. We considered geometries with both a square disordered region

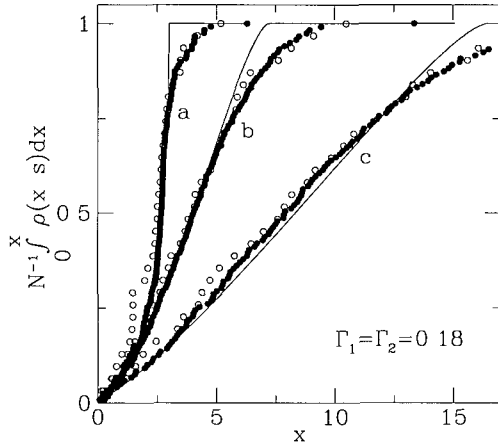


Fig. 5. Comparison between theory and simulation of the integrated eigenvalue density for $\Gamma_1 = \Gamma_2 = 0.18$. The labels a, b, c indicate, respectively, $s = 0, 3, 11.7$. Solid curves are from Eq. (3.9), data points are the x_n 's from the simulation plotted in ascending order versus n/N . Filled data points are for a square geometry, open points are for an aspect ratio $L/W = 3.8$.

(285×285 sites, $N = 119$) and a rectangular one (285×75 sites, $N = 31$), to test the geometry dependence of our results. In Fig. 5, we compare the integrated eigenvalue density $v(x, s) \equiv N^{-1} \int_0^x dx' \rho(x', s)$ with the numerical results.

The quantity $v(x, s)$ follows directly from our simulations by plotting the x_n 's in ascending order versus $n/N \equiv v$. We want to sample $v(x, s)$ at many points along the x -axis, so we need N large. Since the x_n 's are self-averaging (fluctuations are of the order of $1/N$), it is not necessary to average over many samples. The data shown in Fig. 5 are from a single realization of the impurity potential. There is good agreement with the analytical results. No geometry dependence is observed, which indicates that the restriction $L \gg W$ of Eq. (3.3) can be relaxed to a considerable extent.

Using Eqs. (3.5) and (3.8), the average conductance $\langle G_{NS} \rangle$ can be directly expressed in terms of the velocity field,

$$\langle G_{NS} \rangle = \frac{2Ne^2}{h} \lim_{\zeta \rightarrow -i\pi/4} \frac{\partial}{\partial \zeta} U(\zeta, s) \quad (3.12)$$

for $\zeta \rightarrow -i\pi/4$, $U \rightarrow iU_y$, $U_y > 0$. The implicit solution (3.9) then takes the form

$$\phi \sqrt{(2a + \sin \phi - 1)^2 - 4b^2} = 2s \cos \phi, \quad (3.13)$$

where $\phi \equiv 2sU_y \in [0, \pi/2]$. We now use that

$$\frac{\partial}{\partial \zeta} U(\zeta, s)|_{\zeta = -i\pi/4} = - \left[\frac{\partial}{\partial s} U \left(-\frac{i\pi}{4}, s \right) \right] / U \left(-\frac{i\pi}{4}, s \right)$$

[see Eq. (3.7)]. Combining Eqs. (3.12) and (3.13) we find

$$\langle G_{NS} \rangle = \frac{2Ne^2}{h} (s + 1/Q)^{-1}, \quad (3.14)$$

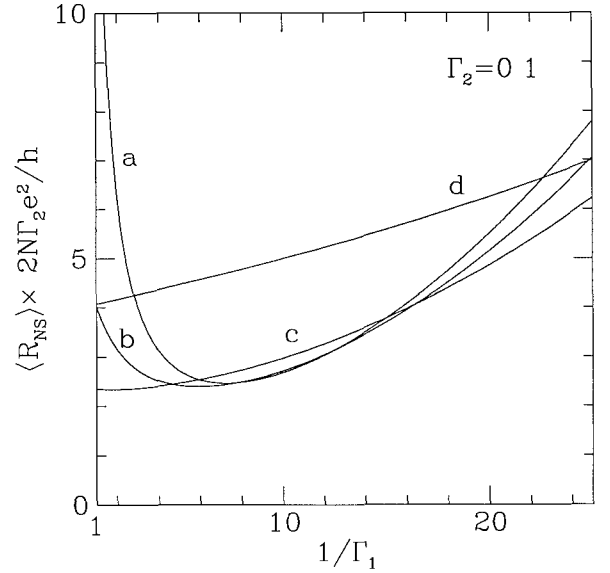


Fig. 6. Dependence of the ensemble averaged resistance $\langle R_{NS} \rangle$ for a disordered NINIS junction on barrier transparency Γ_1 , while $\Gamma_2 = 0.1$ is kept fixed [computed from Eqs. (3.14) and (3.15)]. Curves a, b, c, d are for $s = 0, 2, 7, 30$, respectively. The resistance minimum persists for small disorder.

where the effective tunnel rate Q is given in terms of the angle ϕ in Eq. (3.13) by

$$Q = \frac{\phi}{s \cos \phi} \times \left(\sin \phi + \frac{\phi^2}{4s^2} [\sin \phi + (1 - 2/\Gamma_1)(1 - 2/\Gamma_2)] \right) \quad (3.15)$$

Eqs. (3.13)–(3.15) completely determine the conductance of a double-barrier NS-junction containing disorder.

In Fig. 6, we plot $\langle R_{NS} \rangle$ for several values of the disorder, keeping $\Gamma_2 = 0.1$ fixed and varying the transparency of barrier 1. For weak disorder ($\Gamma_2 s \ll 1$), the resistance minimum is retained, but its location moves to larger values of Γ_1 . On increasing the disorder, the minimum becomes shallower and eventually disappears. In the regime of strong disorder ($\Gamma_2 s \gg 1$), the resistance behaves nearly Ohmic.

We stress that these results hold for arbitrary $s \equiv L/l$, all the way from the ballistic into the diffusive regime. Volkov et al. [7] have computed $\langle G_{NS} \rangle$ in the diffusive limit $s \gg 1$. In that limit our Eqs. (3.13) and (3.15) take the form

$$\frac{s \cos \phi}{\phi} = \frac{1}{\Gamma_1} \sqrt{1 - \left(\frac{\phi}{\Gamma_2 s} \right)^2} + \frac{1}{\Gamma_2} \sqrt{1 - \left(\frac{\phi}{\Gamma_1 s} \right)^2}, \quad (3.16)$$

$$\frac{1}{Q} = \sum_{i=1}^2 \frac{1}{\Gamma_i} \left[1 - \left(\frac{\phi}{\Gamma_i s} \right)^2 \right]^{-1/2}, \quad (3.17)$$

in precise agreement with Ref [7]. Nazarov's circuit theory [10], which is equivalent to the Green's function theory of Ref [7], also leads to this result for $\langle G_{\text{NS}} \rangle$ in the diffusive regime.

Two limiting cases of Eqs (3.16) and (3.17) are of particular interest. For strong barriers, $\Gamma_1, \Gamma_2 \ll 1$, and strong disorder, $s \gg 1$, one has the two asymptotic formulas

$$\langle G_{\text{NS}} \rangle = \frac{2Ne^2}{h} \frac{\Gamma_1^2 \Gamma_2^2}{(\Gamma_1^2 + \Gamma_2^2)^{3/2}}, \quad \text{if } \Gamma_1, \Gamma_2 \ll 1/s, \quad (3.18)$$

$$\langle G_{\text{NS}} \rangle = \frac{2Ne^2}{h} (s + 1/\Gamma_1 + 1/\Gamma_2)^{-1}, \quad \text{if } \Gamma_1, \Gamma_2 \gg 1/s \quad (3.19)$$

Eq (3.18) coincides with Eq (2.6) in the limit $\alpha_1, \alpha_2 \gg 1$ (recall that $\Gamma_i \equiv 1/\cosh^2 \alpha_i$). This shows that the effect of disorder on the resistance minimum can be neglected as long as the resistance of the junction is dominated by the barriers. In this case $\langle G_{\text{NS}} \rangle$ depends linearly on Γ_1 and Γ_2 only if $\Gamma_1 \approx \Gamma_2$. Eq (3.19) shows that if the disorder dominates, $\langle G_{\text{NS}} \rangle$ has a linear Γ dependence regardless of the relative magnitude of Γ_1 and Γ_2 .

4. Conclusions

In summary, we have derived an expression for the conductance of a ballistic NINIS junction in the limit $NI \gg 1$ that the tunnel resistance is much smaller than h/e^2 . In this regime the double-barrier junction contains a large number of overlapping resonances, so that in the normal state the resistance depends monotonically on $1/\Gamma$. In contrast, the NINIS junction shows a resistance minimum when one of the barrier transparencies is varied while the other is kept fixed. The minimal resistance (at $\Gamma_1 \simeq \Gamma_2 \equiv \Gamma$) is proportional to $1/\Gamma$, instead of the $1/\Gamma^2$ dependence expected for two-particle tunneling into a superconductor. This is similar to the reflectionless tunneling which occurs in an NIS junction. Using the results of the ballistic junction, we have described the transition to a disordered NINIS junction by means of an evolution equation for the transmission eigenvalue density [9]. We found that the resistance minimum is unaffected by disorder, as long as $l \gg L/\Gamma$, i.e., as long as the barrier resistance dominates the junction resistance. As the disorder becomes more dominant, a transition to a monotonic Γ dependence takes place. In the limit of diffusive motion between the barriers, our results agree with Ref [7].

Throughout this paper we have assumed zero temperature, zero magnetic field, and infinitesimal applied voltage. Each of these quantities is capable of destroying the phase coherence between the electrons and the Andreev-reflected holes, which is responsible for the resistance minimum. As far as the temperature T and voltage V are concerned, we require $k_B T, eV \ll \hbar/\tau_{\text{dwell}}$ for the appearance of a resistance

minimum, where τ_{dwell} is the dwell time of an electron in the region between the two barriers. For a ballistic NINIS junction, we have $\tau_{\text{dwell}} \sim L/v_F \Gamma$, while for a disordered junction $\tau_{\text{dwell}} \sim L^2/v_F \Gamma l$ is larger by a factor L/l . It follows that the condition on temperature and voltage becomes more restrictive if the disorder increases, even if the resistance remains dominated by the barriers. As far as the magnetic field B is concerned, we require $B \ll \hbar/eS$ (with S the area of the junction perpendicular to B) if the motion between the barriers is diffusive. For ballistic motion the trajectories enclose no flux, so no magnetic field dependence is expected.

A possible experiment to verify our results might be scanning tunneling microscopy of a metal particle on top of a superconducting substrate [12]. The metal–superconductor interface has a fixed tunnel probability Γ_2 . The probability Γ_1 for an electron to tunnel from STM to particle can be controlled by varying the distance. (Volkov has recently analyzed this geometry in the regime that the motion from STM to particle is diffusive rather than by tunneling [20].) Another possibility is to create an NINIS junction using a two-dimensional electron gas in contact with a superconductor. The tunnel barriers could then be implemented by means of two gate electrodes. In this way both transparencies might be tuned independently.

Acknowledgement

This research was motivated by a discussion with D. Esteve, which is gratefully acknowledged. Financial support was provided by the “Nederlandse organisatie voor Wetenschappelijk Onderzoek” (NWO) by the “Stichting voor Fundamenteel Onderzoek der Materie” (FOM), and by the “Human Capital and Mobility” programme of the European Community.

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