the logica yearbook
1998

Edited by Timothy Childers
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Identity:
Absolute, Criterial, Propositional

GÖRAN SUNDHOLM*

1. Frege’s treatment of predication is marked by a certain tension which comes to the fore in the way he treats of, respectively, number-words and identity. His (negative) answer to the question ‘Drückt das Zahlwort “Ein” eine Eigenschaft von Gegenständen aus?’ is that count-nouns are unsaturated and stand in need of completion by means of a general concept. Indeed, Frege notes, ein Weiser is not somebody who is one and who is wise. Similarly, the exclamation: “There are two!” makes no sense without it being possible to answer the counter-question “Two what?” The answer will be given using a term ‘α’ which expresses a type (Russell, Whewell). In order to know a type (understand a type-word) one must, of course, know its criterion of application. This, however, on its own, is not sufficient: “No entity without identity,” quips Quine, but Frege himself had seen the need for an additional criterion of identity.

Consider the two types $\mathbb{N}^+ \times \mathbb{N}^+$ of ordered pairs of positive integers and $\mathbb{Q}^+$ of positive rationals. Formally they have the same application criterion:

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* Text of an invited contribution (to LOGICA '98, Castle Liblice, Czech Republic, June 23-26, 1998.
2 Alternative terminology here includes: category (Dummett), set (Cantor), sort (Locke) and universal (general) concept (St. Thomas Aquinas). The computer scientist’s $p : \alpha$ and $p = q : \alpha$
will be used to indicate that $p$ is an element of $\alpha$, and that $p$ and $q$ are equal such elements, respectively. The set theoretic $p \in \alpha$ and $p = q \in \alpha$
would serve equally well.
4 The term “criterion of identity” is due to Frege, Gl §62. Michael Dummett, Frege: Philosophy of Language, Duckworth, London 1973, passim, adds “criterion of application”. The criteria of application and identity are rules for how to make certain judgements with respect to the type—or category—$\alpha$. Per Martin-Löf pointed out, in unpublished lectures at Leyden, autumn 1993, that the term “criterion” is particularly apt, since it derives from the Greek for judgement.
\[
p : \mathbb{N}^+ \quad q : \mathbb{N}^+
\]
\[
\frac{\langle p, q \rangle : \alpha}{\langle p, q \rangle : \alpha}
\]

\langle 2, 3 \rangle and \langle 4, 6 \rangle are equal elements of type \( \mathbb{Q}^+ \), but not of the type \( \mathbb{N}^+ \times \mathbb{N}^+ \).

In order to individuate the types in question different criteria of identity are needed: the type \( \mathbb{N}^+ \times \mathbb{N}^+ \) is individuated by the identity criterion

\[
p : \mathbb{N}^+ \quad q : \mathbb{N}^+ \quad r : \mathbb{N}^+ \quad s : \mathbb{N}^+ \quad p = r : \mathbb{N}^+ \quad q = s : \mathbb{N}^+
\]
\[
\langle p, q \rangle = \langle r, s \rangle : \mathbb{N}^+ \times \mathbb{N}^+
\]

and the type \( \mathbb{Q}^+ \) by the criterion

\[
p : \mathbb{N}^+ \quad q : \mathbb{N}^+ \quad r : \mathbb{N}^+ \quad s : \mathbb{N}^+ \quad p \times s = q \times r : \mathbb{N}^+
\]
\[
\langle p, q \rangle = \langle r, s \rangle : \mathbb{Q}^+
\]

2. Frege’s conception of predication, identity and quantification, on the other hand, is \textit{absolute} with respect to the universe of objects and not restricted to types:

(1) Predicates are regimented as functions which have uniform sense throughout the universe and with values among only the two truth-values \textit{das Wahre} and \textit{das Falsche}.

(2) Types have no special status, but are ordinary predicates: 
“\( p \) is an \( \alpha \)” is regimented as “\( \alpha(a) \)”.

Hence,

(3) the identity predicate \( \xi = \eta \) is an ordinary two-place function and its rules of inference are uniform throughout the universe of objects:

\[
\begin{align*}
= & I) \quad a = a & \text{true} \quad \text{And} \quad =E) \quad a = b & \text{true} \quad \Phi[a/x] & \text{true} \\
& \Phi[b/x] & \text{true.}^5
\end{align*}
\]

\[^5\text{When matters of notation are not at issue, I allow myself to accommodate the formal languages of Frege, Russell and Whitehead to modern standards. The use of natural deduction rules at this point is certainly unFregean; \textit{mutatis mutandis}, though, the point made remains valid.}\]
(4) Restricted identity “a is the same a as b” is regimented in terms of unrestricted identity “a(a) and a(b) and a = b”.

(f) The restricted quantification “All a are F” is regimented in terms of the unrestricted quantification “∀x(a(x) ⇒ F(x))”.

3. Frege’s arguments that type distinctions (and restrictions) are superfluous are not strong; in fact, given his exacting standards, they are uncommonly weak. One uses what Quine has dubbed don’t care cases.

So long as the only objects dealt with in arithmetic are the integers, the letters a and b in ‘a + b’ indicate only integers; the plus-sign need be defined only between integers. Every extension of the field to which the objects indicated by a and b belong obliges us to give a new definition of the plus-sign. … It is thus necessary to lay down rules from which it follows, e.g., what ‘n+1’ stands for, if ‘n’ is to stand for the Sun. What rules we lay down is a matter of comparative indifference; but it is essential that we should do so—that ‘a + b’ should always have a Bedeutung, whatever signs for definite objects may be inserted in place of ‘a’ and ‘b’.

So, according to Frege (and his follower Quine), we do not need any type distinctions in the universe, because one can always effect a don’t care extension of a type restricted function or predicate, using a throw-away value for the indifferent cases. However, an actual attempt to effect such an indifferent, more or less arbitrary choice of a rule reveals an awkward difficulty. Consider, say, the following definition of a very simple function, which attempts to carry out a throwaway distinction of cases:

\[ x + 1 = \text{def} \begin{cases} k + 1, & \text{if } x = k \text{ and } k \text{ is a natural number} \\ 213, & \text{if } x \text{ is not a natural number.} \end{cases} \]

6 “Die Identität ist eine so bestimmt gegebene Beziehung, dass nicht abzusehen ist, wie bei ihr verschiedene Arten vorkommen können”, Gg. II. p. 254. For an opposite view, see Aristotle, Metaphysics, Α, 2, 1018 a 35-39.


8 FB, p. 19.
This determination, however, of the don't care—throw-away—value requires a separation of cases according to whether the argument \( x \) is a natural number or not, that is, according to the very type-distinction, the avoidance of which was the whole point of Frege's exercise.\(^9\)

4. Russell partly accepts and partly rejects Frege's absolutism. The Doctrine of Types does, of course, introduce a plethora of type distinctions, but within each type Russell largely accepts the Fregean paradigm. In particular, identity of individuals is defined using second level quantification:

\[ x = y. := \forall \phi. \phi(x) \supset \phi(y) \text{ Df,} \]

where, given the Axiom of Reducibility, we may remove the exclamation marks.\(^{10}\) Furthermore, in Russell, by definition, the types serve as the ranges of significance for propositional functions and they also serve as the domains of quantification.

5. Wittgenstein, in the *Tractatus*, further challenges Frege's absolutism. Identity, in that work, has the task, not to mirror reality, but to license substitutions.\(^{11}\) The Tractarian theory looks back towards Frege's early theory of identity from the *Begriffsschrift*, where '≡' expresses, not identity of *Bedeutungen*, but sameness of content (*Inhaltsgleichheit*). There Frege wants to eat his cake and have it. For instance, according to his conventions, the Bs theorem

\[(52) \quad | \quad c \equiv d \supset (f(c) \supset f(d)),\]

should be taken in the sense of its universal closure

\[(52') \quad | \quad \forall c \forall d (c \equiv d \supset (f(c) \supset f(d))).\]

The Bs §8 explanation of *Inhaltsgleichheit*

'\( c \equiv d \)' means "'a' and 'b' have the same content"

turns (52') into

\[(52'') \quad | \quad \forall c \forall d ('a' and 'b' have the same content \supset (f(c) \supset f(d))).\]

\(^9\) Frege's other argument is no better. It is spelt out at Gg II, §65, pp. 77-78. David Bell, *Frege's Theory of Judgement*, O.U.P., 1979, pp. 45-47 offers a lucid refutation of this and the above FB argument.


\(^{11}\) 4.241-4.243.
This formula, however, does not make sense, since quotations-marks, notoriously, resist “quantifying in”.

Wittgenstein, on the other hand, explicitly acknowledges that the substitution-licensing notion of identity is not propositional in nature and accordingly removes the identities from the realm of propositions. Therefore, identities are not open to quantification in the Tractatus, since the quantifiers can be applied only to propositional Urbilder.

6. For Russell, each propositional function, also that of identity, has a type as its range of significance. Accordingly, his view of predication, identity and quantification is a type-restricted one. Peter Geach famously concurs with respect to the identity-predicate: according to him it also requires supplementation with a type-indication. The assertion: “They are the same” invites Geach’s inevitable counter-question: “The same WHAT?” which certainly merits an answer. Geach, however, went one step further, and held that identity is not just type-restricted: according to him, it is even type-relative. This means that for some a and b they are the same a but different ß. I cannot follow him in this: he is right concerning the restrictedness, but goes too far with relativity. Examples of purported relativity have been offered by Geach and others. Here I wish to defuse one of them. The example has already been given in section 1 above: according to the identity-relativist (2, 3) is the same rational number as (4, 6), but they are not the same ordered pairs of natural numbers. This will only work if (2, 3), say, is an element both of $N^* \times N^*$ and of $Q^*$. When the (token) ‘(2, 3)’ stands for an ordered pair in $N^* \times N^*$, however, it cannot, at the same time, stand for an element of $Q^*$, because when it stands for an element of type $N^* \times N^*$, it obeys the identity criterion of this type, and not that of $Q^*$. It is an internal property of elements of a type that they obey the identity criterion of that type and not that of another. Similarly, when students of biochemistry at Helsinki University use certain tokens of the singular term ‘Professor Göran Sundholm’ to refer to their teacher, my exact namesake, they cannot at the very same time use the very same tokens to refer to me. Such considerations will serve to defuse the standard examples of relative identity. The relativity theorists forget that criteria of identity only operate against the background of equally important, but prior, criteria of application. Indeed, as Per Martin-Löf has noted, ‘no entity without identity’ is most readily obtained by combining the type doc-

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12 6.23, 6.232, 6.2322.
15 David Wiggins, Sameness and Substance, Blackwell, Oxford, 1980, Ch. 1, gives a comprehensive list of examples of allegedly relative identity and, after careful scrutiny, rightly rejects the claim for each case.
triune ‘no entity without type’ and ‘no type without identity’, that is, the point that each type is given by, i.a., a criterion of identity.  

7. David Wiggins resists relativity: we cannot have \( p = q : \alpha \), but \( p \neq q : \beta \), where \( \alpha \) and \( \beta \) are different types (or categories). He is, however, prepared to accept the restrictedness of identity in the forms  

\[
a = b \supset \exists f (a =_f b)
\]

and  

\[
\exists f (a =_f b) \supset \forall g (g (a) = g (b)).
\]

Though at one with Wiggins in preferring restrictedness and rejecting relativity, I cannot follow him in his use of these formulae. In the second formula, types are used as if they were common propositional functions amenable to quantification. A propositional function \( P \) however has a negation \( \neg P \), which is also a propositional function. \( P (a) \) and \( \neg P (a) \) are both propositions, when \( P \) is a propositional function with respect to a certain domain \( D \) and \( a \) belongs to that domain. Types, however, are not closed under negation. There is no common identity criterion for non-crows, or non-natural numbers, or what have you.

Furthermore, even if we accepted the types as propositional functions, the existential quantification with respect to types that Wiggins employs is impredicative, and I, for one, have great difficulty in accepting such quantification as meaningful, owing to the vicious circularity in the meaning-explanations.

8. The way out of the corner into which I have painted myself is to draw the tripartite distinction of my title. For each suitable domain \( D \) we have a propositional function  

\[
\text{id}_D(x, y) : \text{Prop}, \text{ provided that } x : D \text{ and } y : D.
\]

---

\(^{16}\) In the Leyden lectures referred to above.  
\(^{17}\) Op. cit., f.n. 15.  
\(^{18}\) *Sameness and Substance*, op. cit., f.n. 15, at p. 58 and p. 18.  
\(^{20}\) In order to be suitable the domain has to be inductively generated from below, so as to avoid problems of impredicativity.
These meaning of these propositional functions is uniform in the chosen domain $D$:

the rules

\[
\begin{align*}
(id_D) & \quad a : D \quad \text{And} \quad (id_D E) \quad \text{id}_D (a, a) \text{ true} \\
& \quad \Phi [a/x] \text{ true}
\end{align*}
\]

hold irrespective of the choice of $D$.

Like any propositional function the propositional function $\text{id}_D$ has a certain range of significance which is a type, in casu the type $D$. Hence, the type $D$ is prior in the conceptual order to the propositional functions in general which have $D$ for their range of significance and to $\text{id}_D$ in particular. But the criterial identity $=$, which is used in formulating the identity criteria of the form $p = q : D$, is prior to the type $D$. Accordingly, in virtue of their different locations in the conceptual order:

\[
\text{criterial identity and propositional identity are not the same.}
\]

9. The notion of identity, on the other hand, which is used here in formulating the conclusion, is neither the criterial nor the propositional one. It is an instance of the Medieval transcendental idem and this third notion of identity is the true notion of ABSOLUTE identity, a notion for which it holds, as Wittgenstein remarked, that

\[
\text{to say of two things that they are identical is nonsense, and to say of one thing that it identical with itself says nothing at all.}^{21}
\]

---

21 5.5303. Concerning the transcendental, absolute idem, see Günther Schulemann, Die Lehre von den Transcendentalien in der Scholastischen Philosophie, Felix Meiner, Leipzig, 1929.